CTU in Prague
Faculty of Civil Engineering,
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"Photogrammetry 1“

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Introduction

Photogrammetry **part of the field of geodesy and cartography, which deals with the determination of geometric and positional information from image records, most often from photographic images**

Remote Sensing (RS) **deals with non-contact detection of surface cover types and their condition**

Laser scanning **3D scanning technology that uses laser measurements to determine the spatial coordinates of detailed points on an object.**

Introduction

Photogrammetry, RS and laser scanning are methods that provide mass localized information for technology

GIS

Basic sources of primary information about the territory supply:

- **geodetic methods** accuracy in the order of **mm - cm**
- **photogrammetry** accuracy in **mm, cm - dm**
- **RS** accuracy in the order of **m - km**

History

Al-Hassan bin Al -Haithm (965-1039)

*****

in 1032 he was the first to describe the *"camera obscura" - the central projection principe*

History

-Leonardo da Vinci (1452-1519) described the "*pinhole camera*" for the construction of central projections
-1605 Galileo Galilei invents the telescope
-1657 Schott Kasper builds the first portable *box* camera
-in 1777 the invention of the light-sensitive compound, AgCl (C.H.Scheele)
-theories of reconstruction of acquired perspective images: Taylor (1715) and J.H.Lambert (1759)
-invention of photography: Niepce and Daquerre (1839)
-negative-positive: Talbot 1841
-the title of the photo comes from J.Herschel
-the first aerial photographs were taken by the famous French photographer G.F.Tournachon (called Nadar) in 1858
-the first phototeodolite was constructed according to the design of A.Laussedat (1859); used in France for mapping in 1861
- "photogrammetry" dates back to 1858, when the German A.Meydenbauer used the term
-G.Eastman, 1884 (paper film) and its introduction in 1889 (celluloid film, first roll film camera)
-C.Pulfrich (Zeiss Jena) in 1901 constructed the first device for stereoscopic measurement, *the stereocomparator*

History

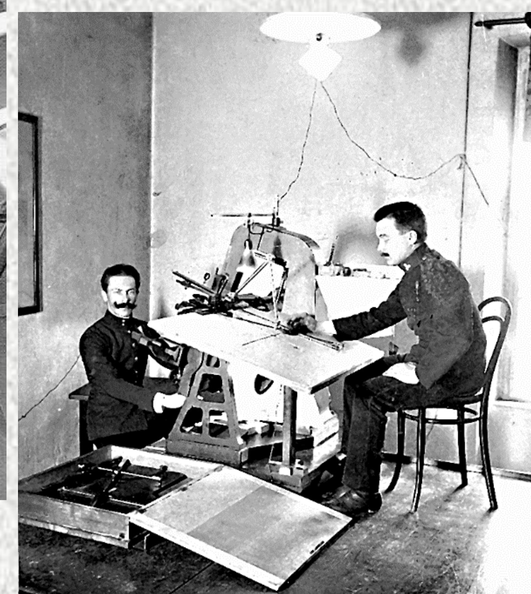
-E.Orel, 1909-1911 constructed the first "*Autostereograph*",since 1909 in the Carl Zeiss Jena works as "*Stereoautograph*"
-Th.Scheimpflug constructed in 1911 the first rectifier for transforming an inclined image of flat terrain to the scale of a map
-W.Wright was the first to take pictures from a plane in 1903
-imaging from aircraft found its application with the advent of World War I.
-In 1935, the first Kodakchrome colour film was released
-during the World War II. some new cameras and instruments were constructed and methods of using photogrammetry were developed, but mainly for military purposes
-further development of photogrammetry occurred again after 1945 (analogue plotters)
-space technology
-Seventies - analytical plotters (used computers)
-Eighties - development of digital technologies
-1990s - full transition to digital technology

History

Karel Kořistka (1825 1906) professor of mathematics and geodesy, the first rector of the Royal Czech Polytechnic Institute in Prague in the school year 1864-65. Pioneer of photogrammetry in the Czech lands

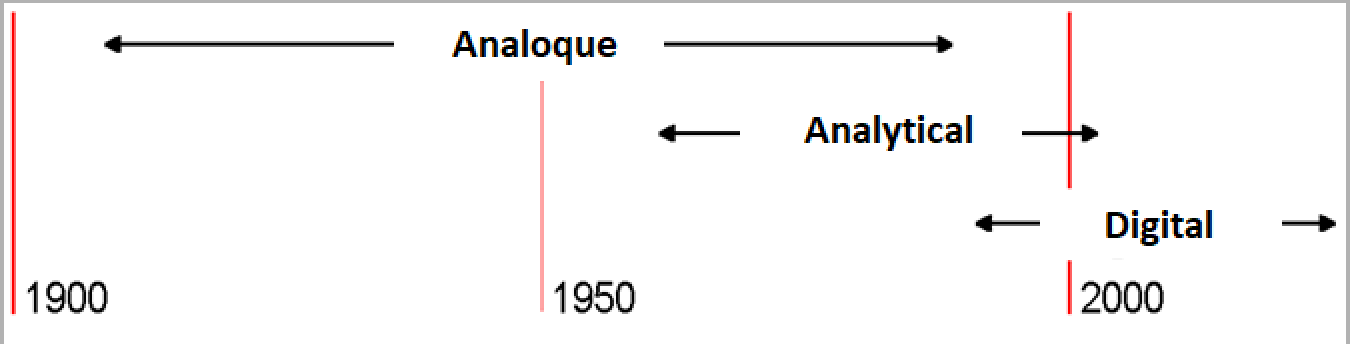
K.Kořistka got acquainted with photogrammetry on a study trip in 1862 directly with A.Laussedat and after his return from this trip he used it in Prague

World War I.



Photogrammetry - subdivision

• Basic stages of development: technology



Basic criteria for dividing photogrammetry

a) according to the position of the position
terrestrial, air, satellite

b) by number and configuration of images
single-frame and multi-frame: stereo or intersection

b) according to the processing technology
analogue, analytical, digital

Use of photogrammetry

State map works (in CZ, State Administration of Land Surveying and Cadastre (ČÚZK) topographic maps (1:10 000)

Military topographic maps (in CZ , Army of the Czech Republic)

Information systems (state administration, GIS information layers) , digital models (**DMR and DMP)**

Monument care (monument care workers, architects) documentation, documents

Construction (design and construction companies) documents, documentation, determination of deformations

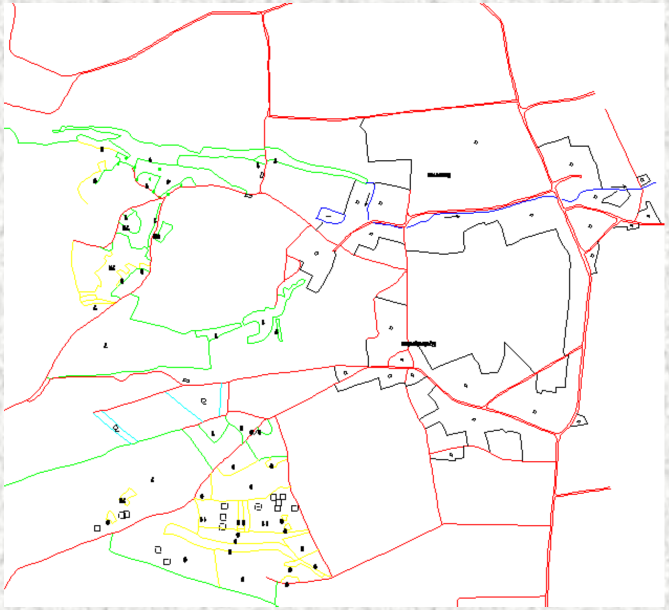
Environment (in CZ, administrations of National park, protected natural area, forestry maps (in CZ, Forest Management Institute ,ÚHÚL), vegetation delimitation

Come in:

- spatial planning - 3D building models
- water management - floods, runoff profiles
- inventory and monitoring - mines, quarries, landfills
- mechanical engineering - precision control, deformation
- rehabilitation medicine, biomechanical applications …etc.

Use of photogrammetry in mapping

Original photogrammetric vertical image is similar to map



Reasons for using photogrammetry

- **minimising field work**
- **economics**
- **speed**

- **overall time saving**
- **cost saving**
- **documentary value of the images** (time series)
- **higher resolution of the images compared to the map** (digital orthophoto)

Basics of photogrammetry

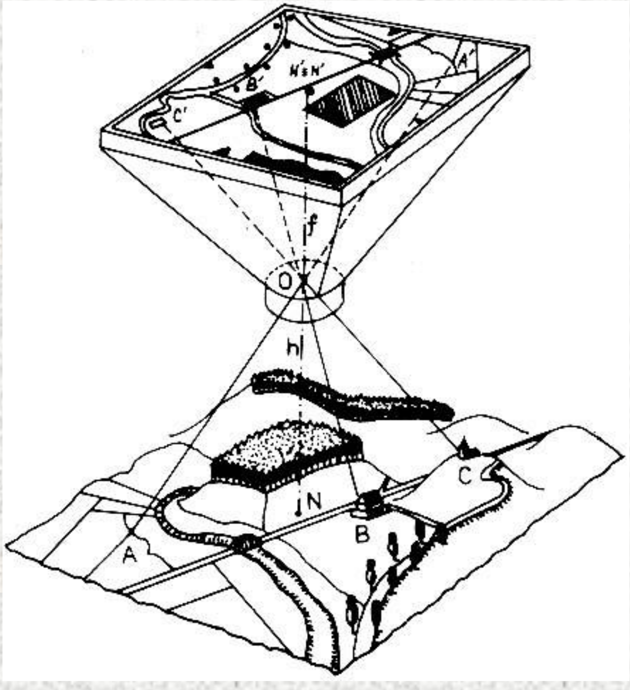
Basics of photogrammetry

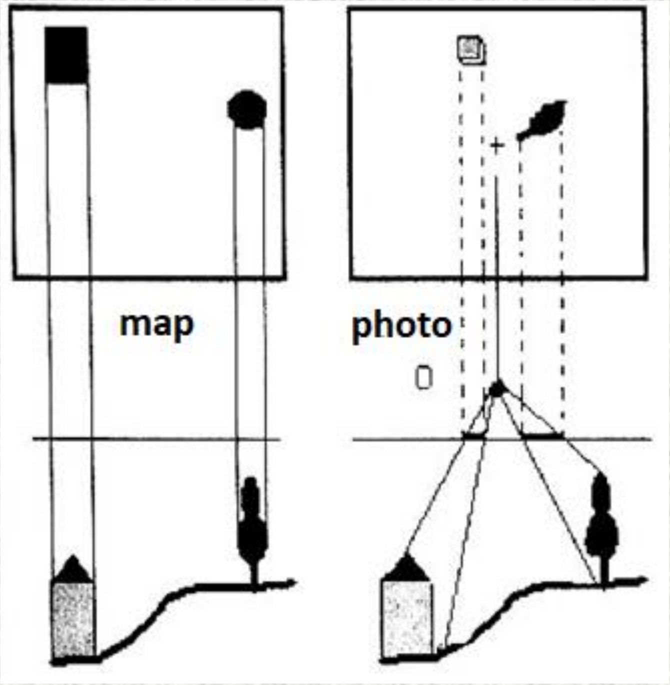
photographic image =central projection of the displayed object

Input data:

image - source of information

necessary additional data - geodetic coordinates of the ground control points, elements of internal and external orientation

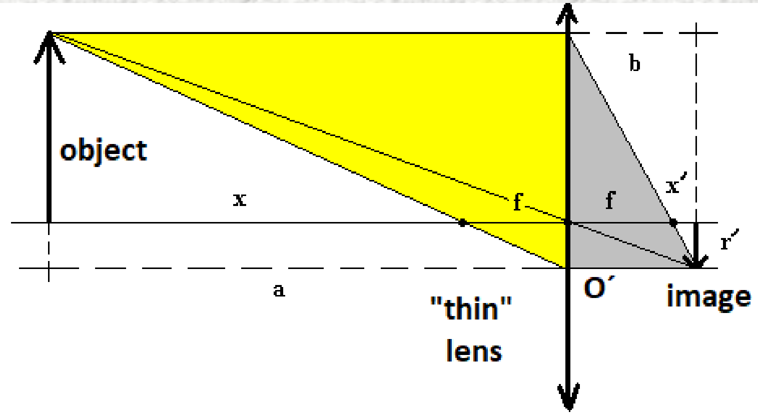




Theory - optics

Ideal Lens Imaging

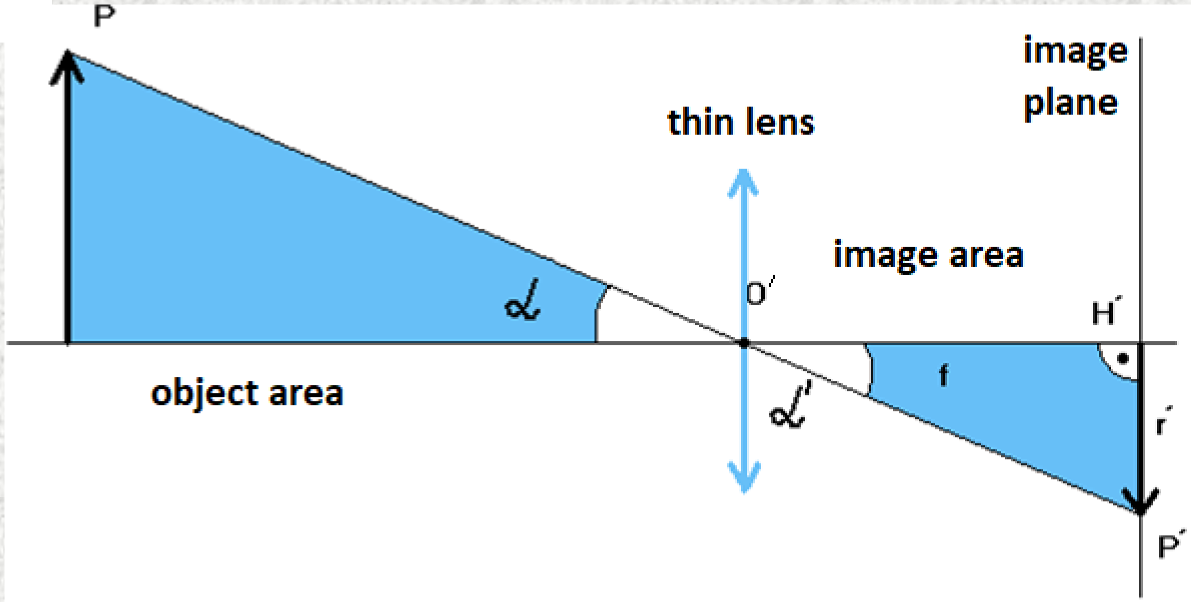
projection centre O´
main frame point H´
Focal length *f*



for an ideal projection:

$\alpha = \alpha'$ $\quad r' = f \cdot tg\ \alpha$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$



Theory - optics

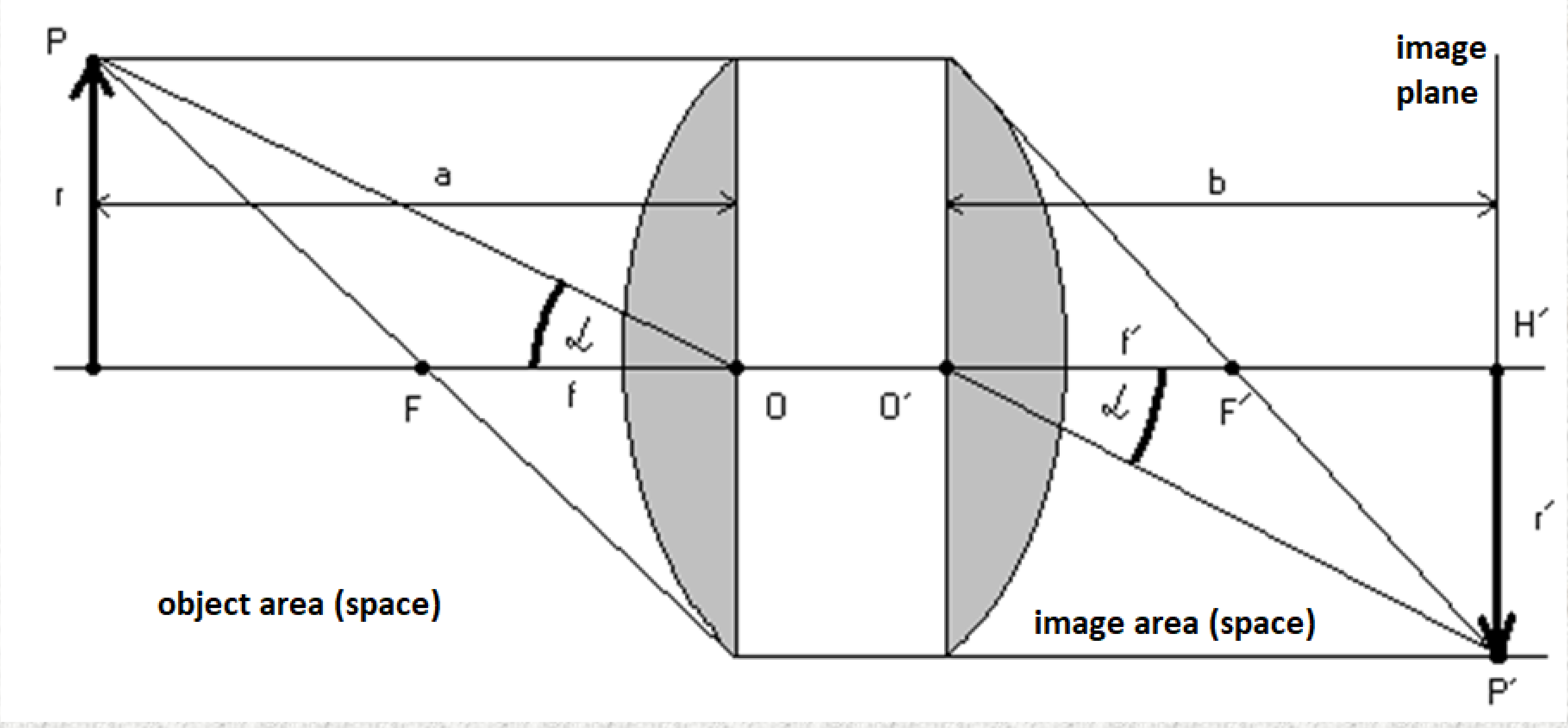
A real lens

aperture - reduces the amount of rays that form an image

O Input pupila - image of the aperture in the subject space

O´ output pupila - image of the aperture in the image space

f camera (lens) constant - distance O´H´

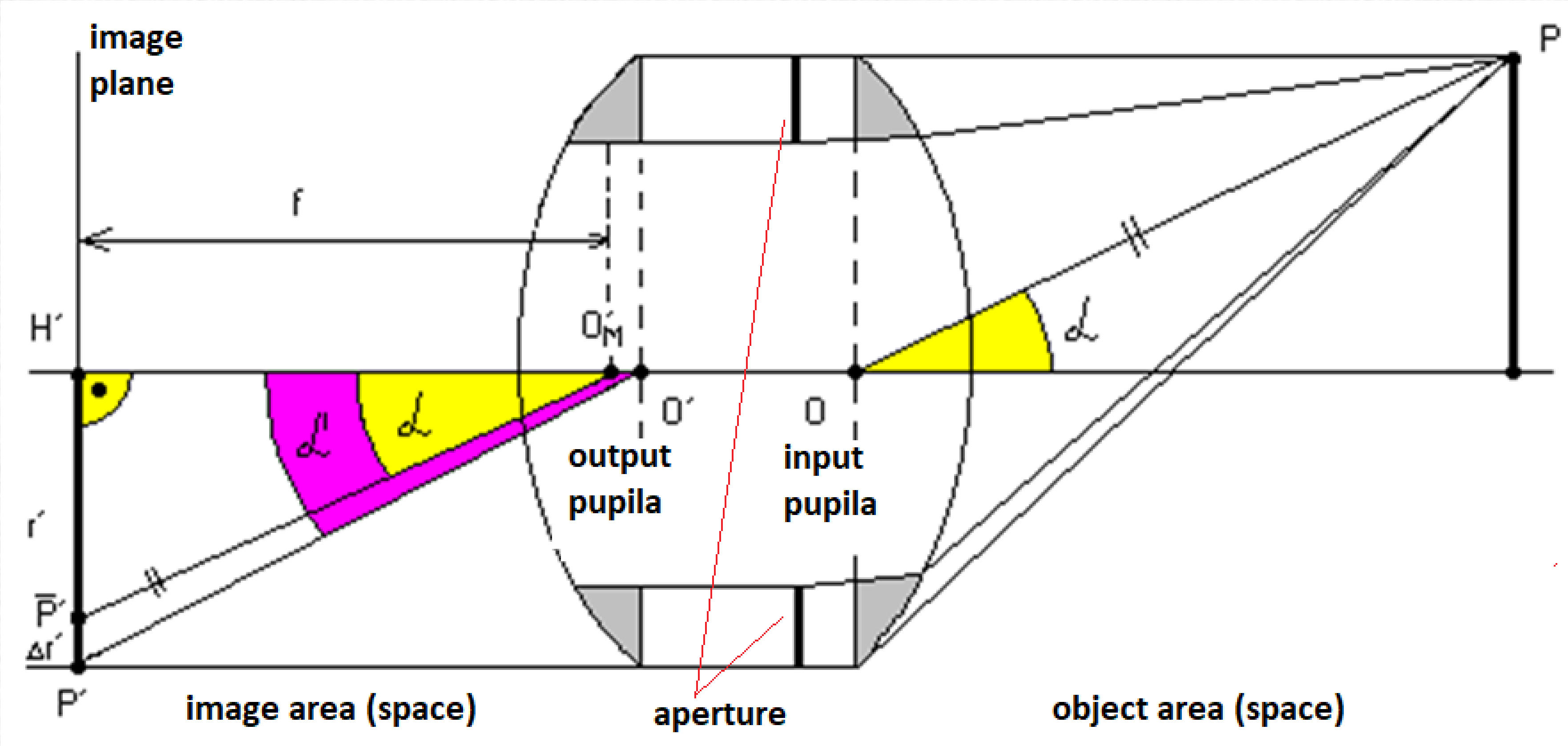


Theory - optics

$$\alpha \neq \alpha'$$

<u>***Real lens view:***</u> $r' = f \cdot tg\ \alpha + \Delta r'$

$\Delta r'$ - effect of lens distortion



K.Kraus, 1994, Photogrammetry, Band I.,Dümmler/Bonn, ISBN 3-427-78645-5

Depth of field

$$y_{\min} = \frac{f^2}{\left(\frac{f}{A}\right) \cdot \Delta u} = \frac{f^2}{n \cdot \Delta u}$$

where *n* is the aperture number and *u* is the dispersion ring (unsharpness), which should not be larger than the diameter of the measuring mark in old analoque plotters (0.02-0.05mm); *A* is the diameter of the input pupila.

type of chamber	f [mm]	n	y_{min} (u=0.05 and 0.02mm) [m]
PhoTheo19/1318	195	25	30; 76
UMK 10/1318	100	8, 16, 32	25, 13, 6; 63, 32, 16

Parameters of old phototheodolites

Summary of effects on the geometry of the lens view

Aberrations

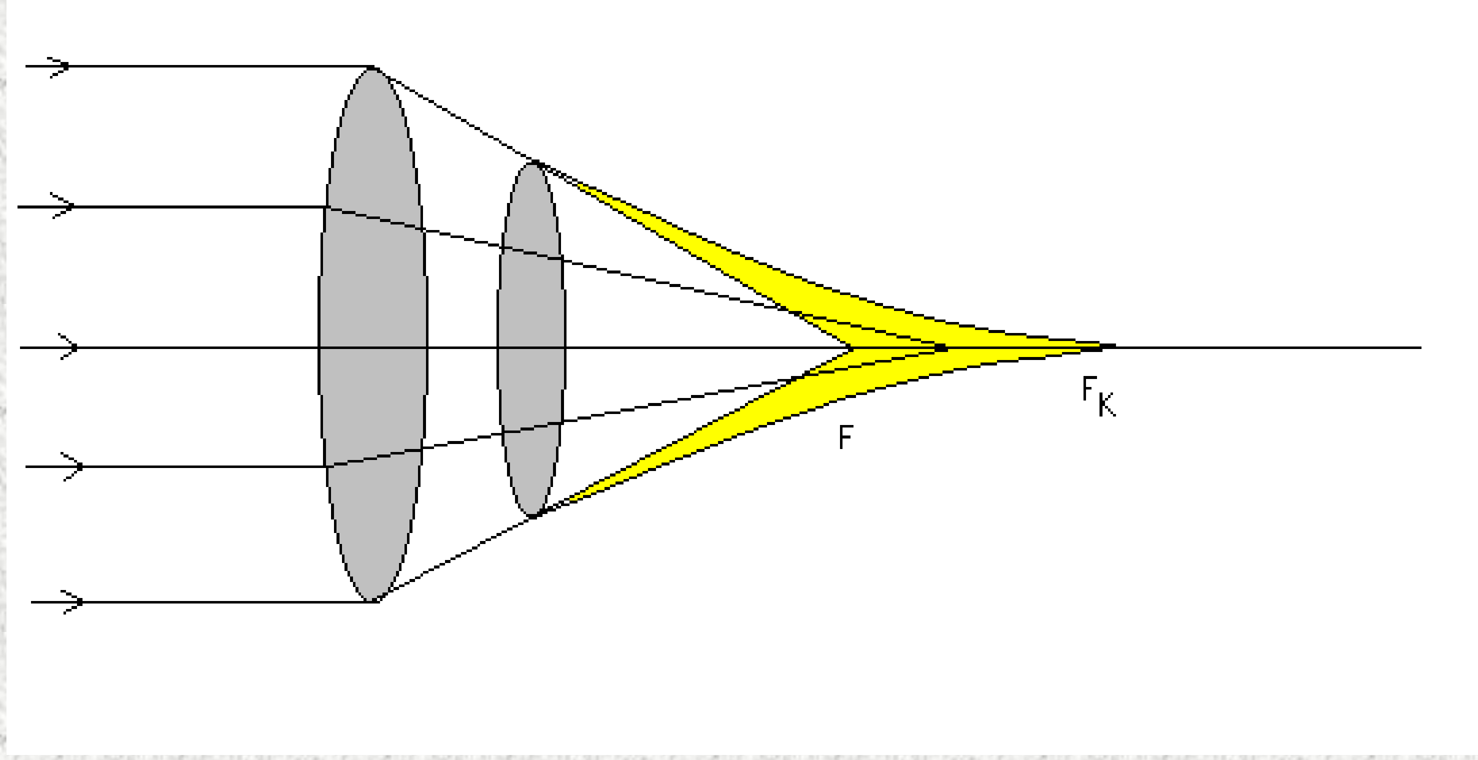
Optical defects can be subdivided into:

a) monochrome
b) coloured

Furthermore, defects arising from:

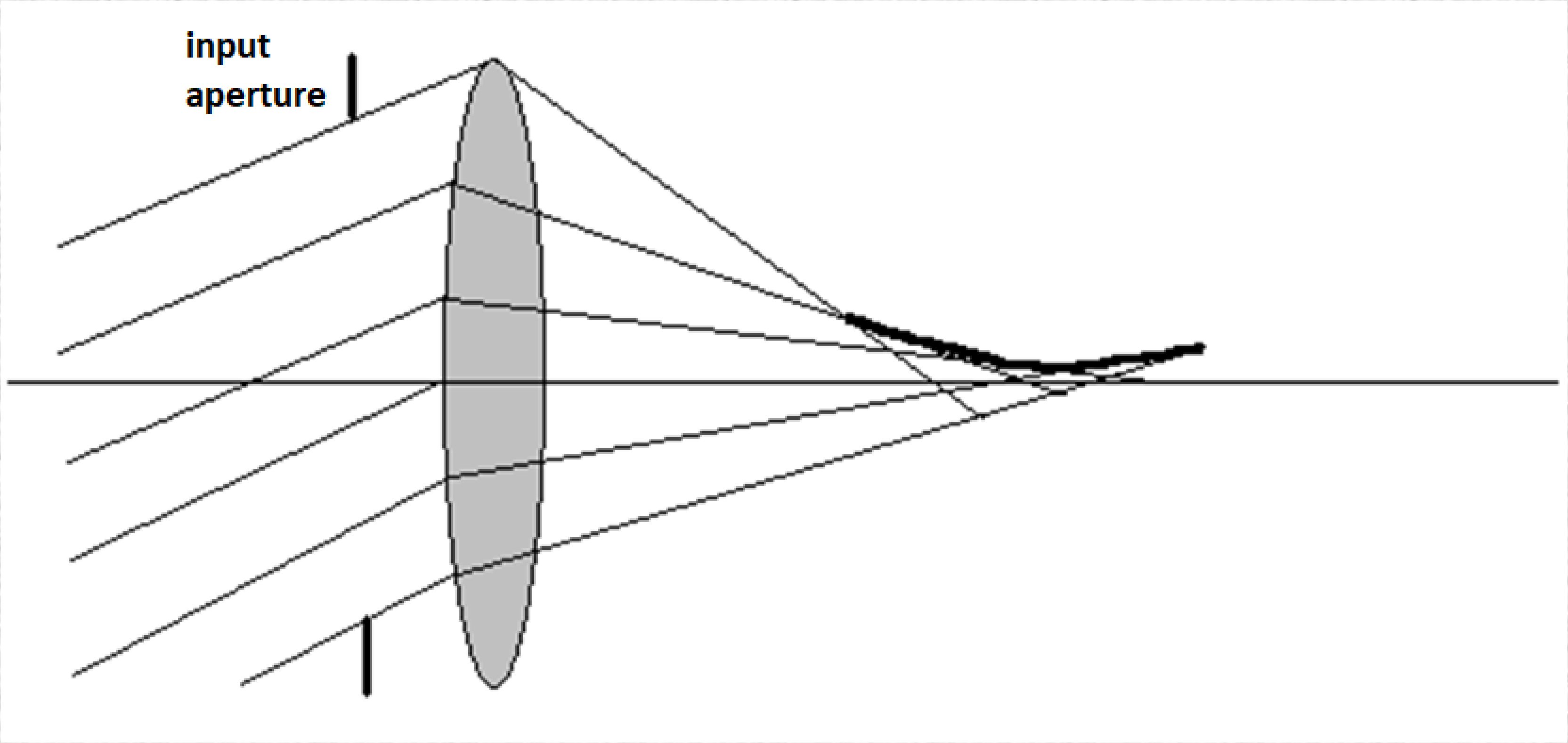
c) point imaging (spherical defect, astigmatism and coma)
d) imaging of the subject (field blurring and image distortion)

Lens defects



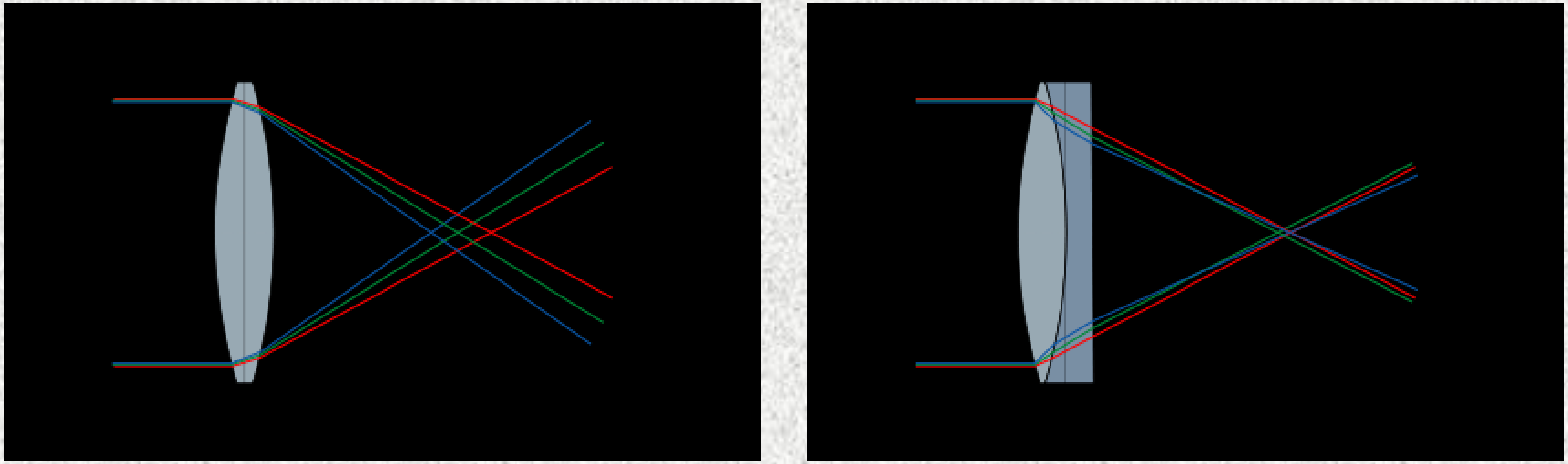
Lens view - spherical defect (aberration)

Lens defects



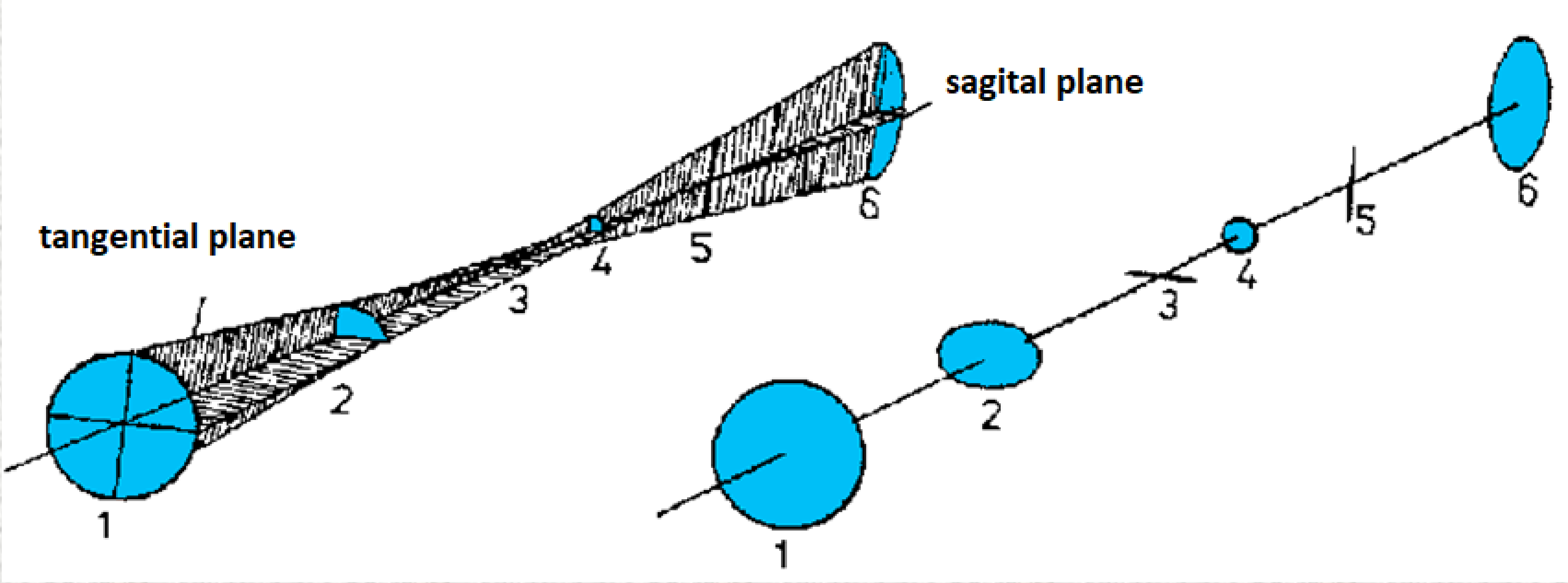
Lens view - aspherical defect (aberration)

Lens defects



Lens view - chromatic (colour) defect (aberration)
and its suppression

Lens defects



Lens View - Astigmatism

Lens distortion

- **radial**
- **tangential**

$$x' = x'_{\text{measured}} + \Delta x'$$

$$y' = y'_{\text{measured}} + \Delta y'$$

Thus, in general, the effect of **radial distortion** can be expressed by a polynomial

$$r'^2 = x'^2 + y'^2$$

$$\Delta x' = a_0 + a_1 x' + a_2 y' + \ldots$$

$$\Delta y' = b_0 + \ldots$$

$$\Delta x' = x' \cdot \left(a_1 r'^2 + a_2 r'^4 + a_3 r'^6\right) + b_1\left(r'^2 + 2x'^2\right) + 2b_1 x' y'$$

$$\Delta y' = y' \cdot \left(a_1 r'^2 + a_2 r'^4 + a_3 r'^6\right) + b_2\left(r'^2 + 2y'^2\right) + 2b_2 x' y'$$

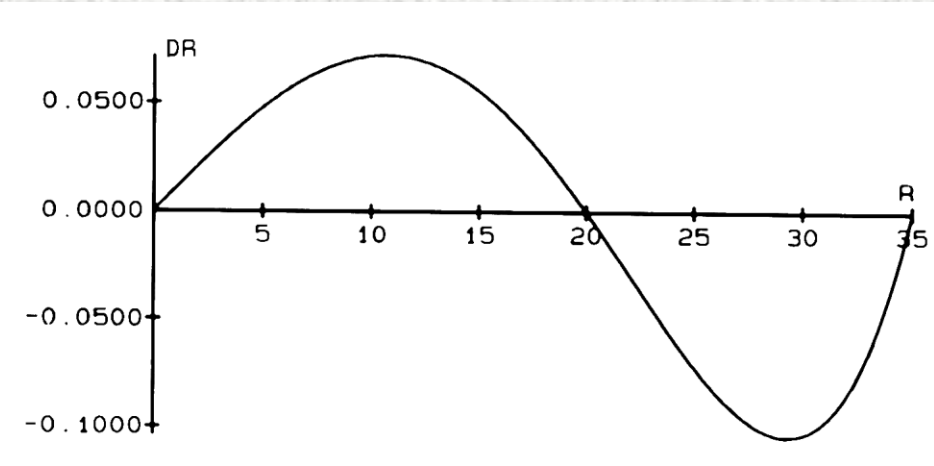
This form is too detailed, for most lenses one can make do with just the coefficients a_1 , a_2 .

Réseau system RolleiMetric

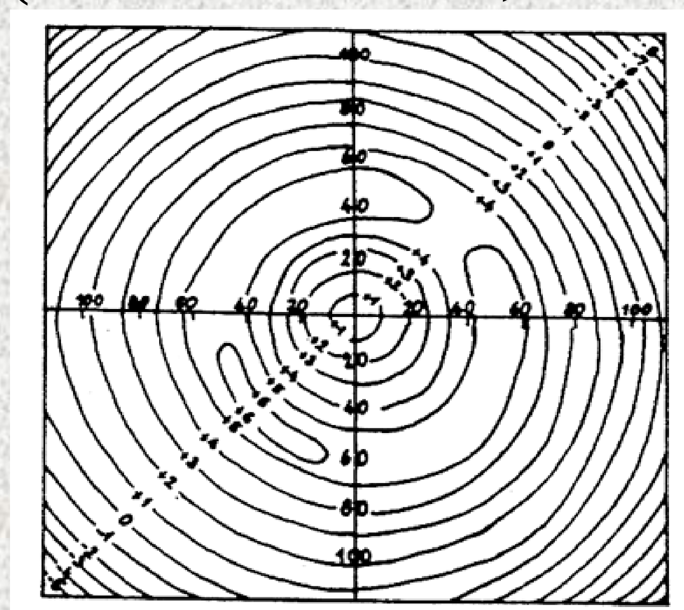
$$\Delta r' = a_1 \cdot r' \cdot \left(r'^2 - r_0'^2\right) + a_2 \cdot r' \cdot \left(r'^4 - r_0'^4\right)$$

tangential distortion is usually an order of magnitude smaller and can be expressed polynomially

Lens distortion



Lens projection - radial distortion (RolleiMetric 6006, 40mm lens)



Lens projection - radial distortion expressed by isolines

Deformation (shrinking) of photographic material

The deformation of photographic material is subdivided into:

a) regular over the whole image area - this deformation can be easily detected by comparing known original dimensions

b) differential - it is a regular deformation, which is different in the direction of the x' and y' axis

c) irregular - this deformation cannot be practically excluded without a special device (*réseau* - grid).

Material	average shrinking s = image size[mm]	values for image 13x18cm
glass plate	max. 3-5m	3-5m
acetate pad	410^{-5} . s	7m
PET pad	2.510^{-5} . s	4.5m

deformation of photographic material

Photographic material deflection

Pad	Note.	Thickness [mm]	Flatness [μm]
glass plates	flat ultraflat cut glass	1.3-3.0 1.3-3.0 6.0	30-50 25 5-10
PET film (polyester terephthalate)	mechanical pressure or vacuum pressing of material	0.06 ÷ 0.003 up to 0.18 ÷ 0.005	5-20 according to the type of adhesion of the material to the frame

Parameters of the photographic material

Coordinate systems

Generally, two types of coordinate systems are used in our photogrammetry:

a) main coordinate systems

b) - image coordinate system

- model coordinate system

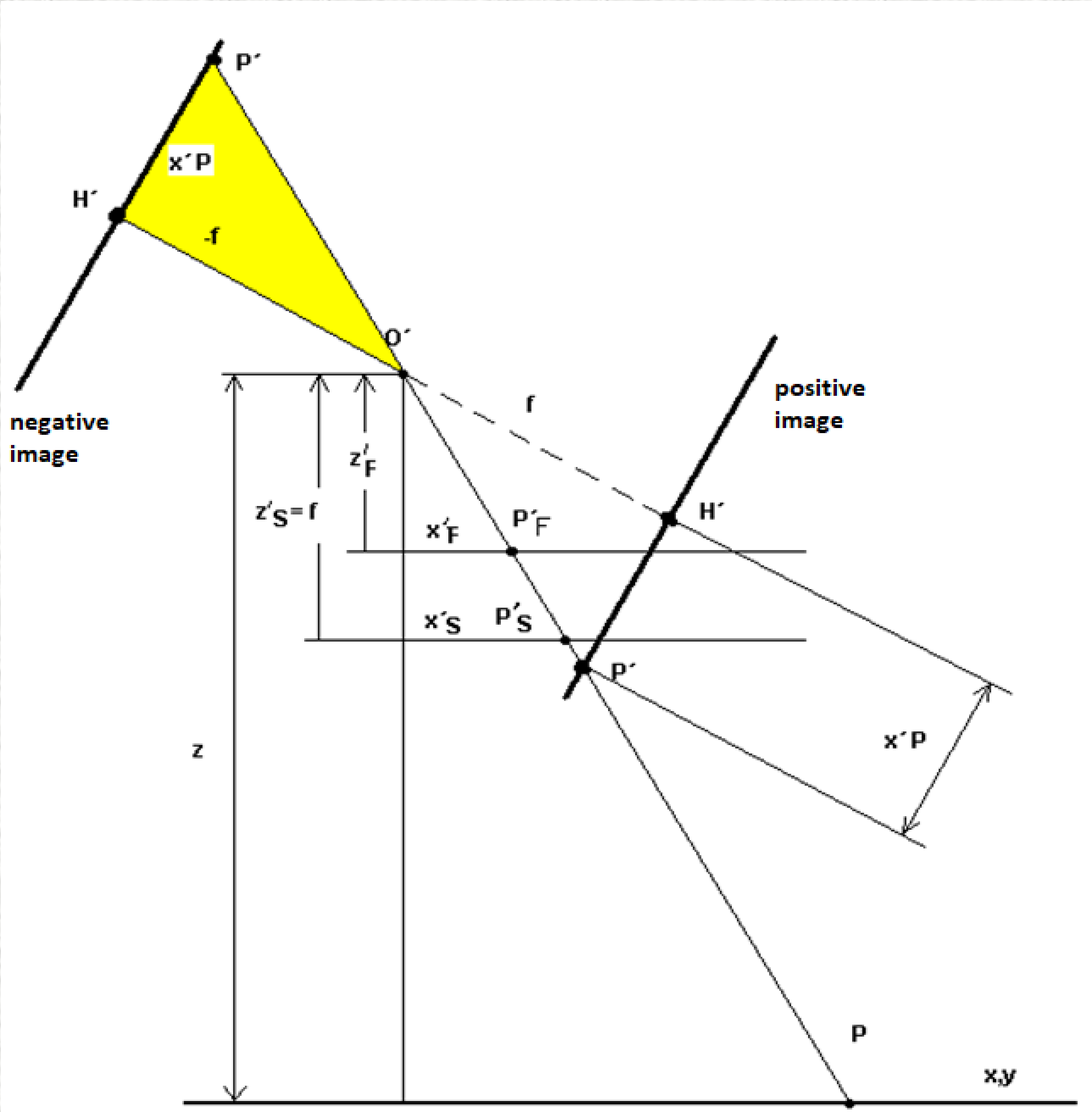
- system of geodetic coordinates

(b) **auxiliary coordinate systems**

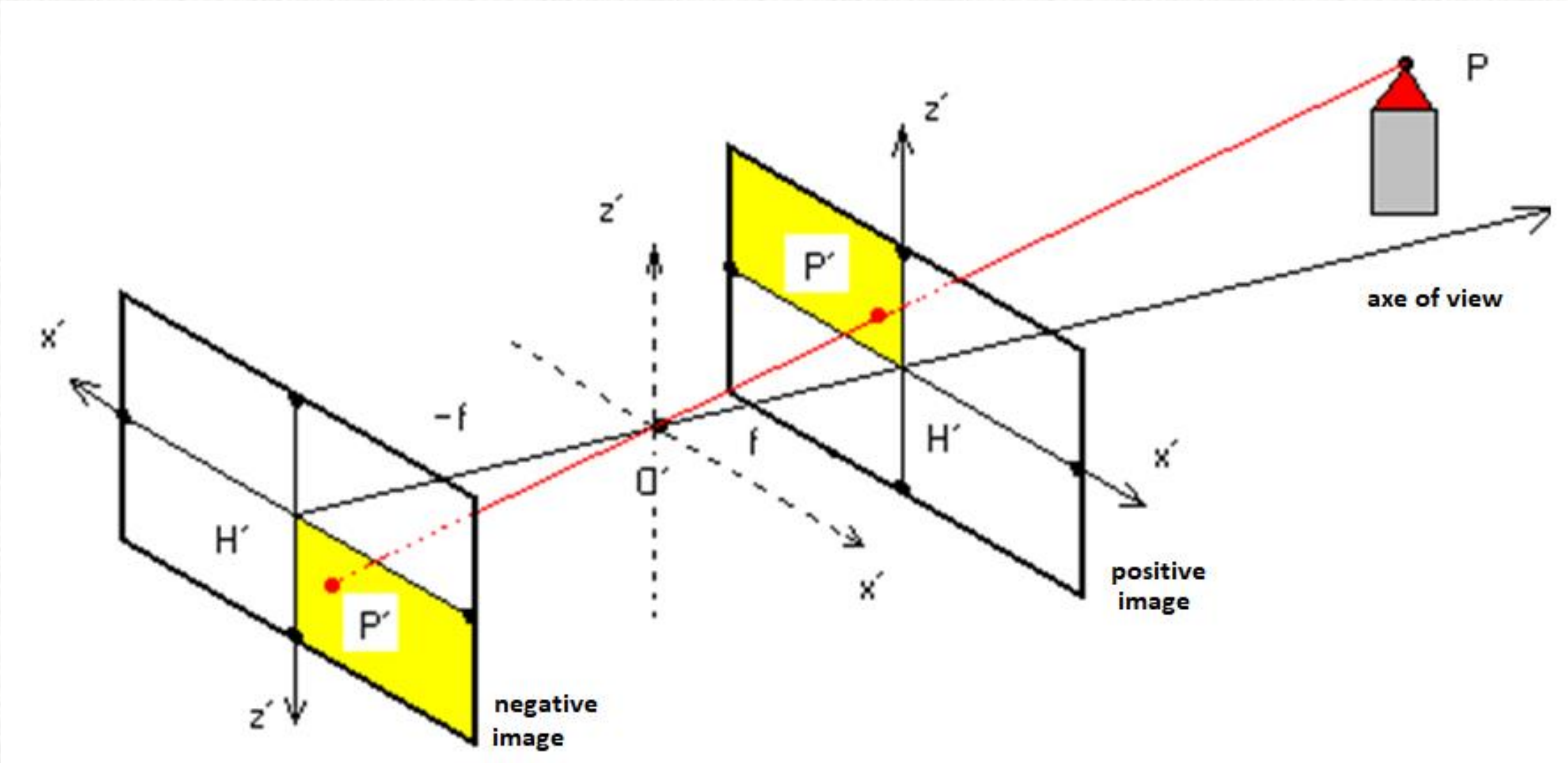
- fictitious image coordinate system

- the frame coordinate system of an exactly vertical image

Coordinate systems



Elements of interior orientation

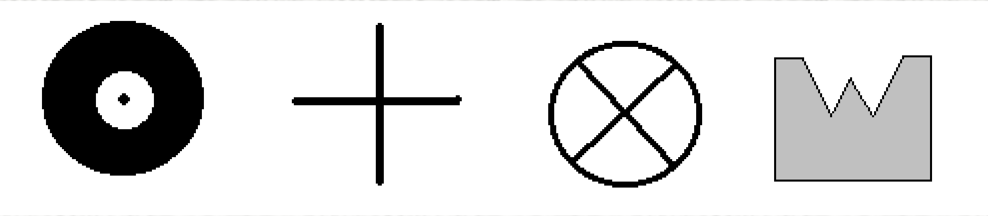


Camera constant, negative and positive, terrestrial configuration

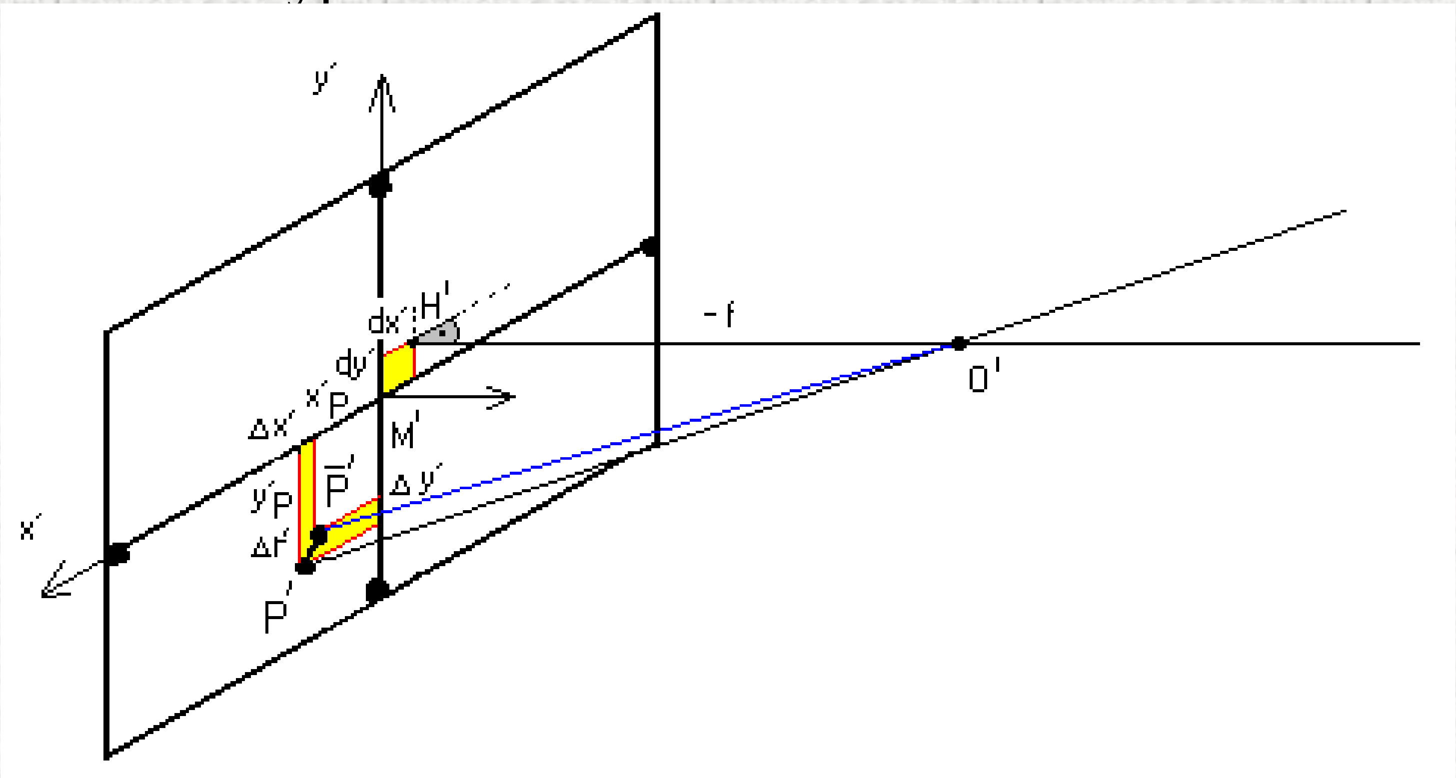
Elements of interior orientation: ***x0´,y0´, f***

(and known parametres of distortion)

Elements of internal orientation



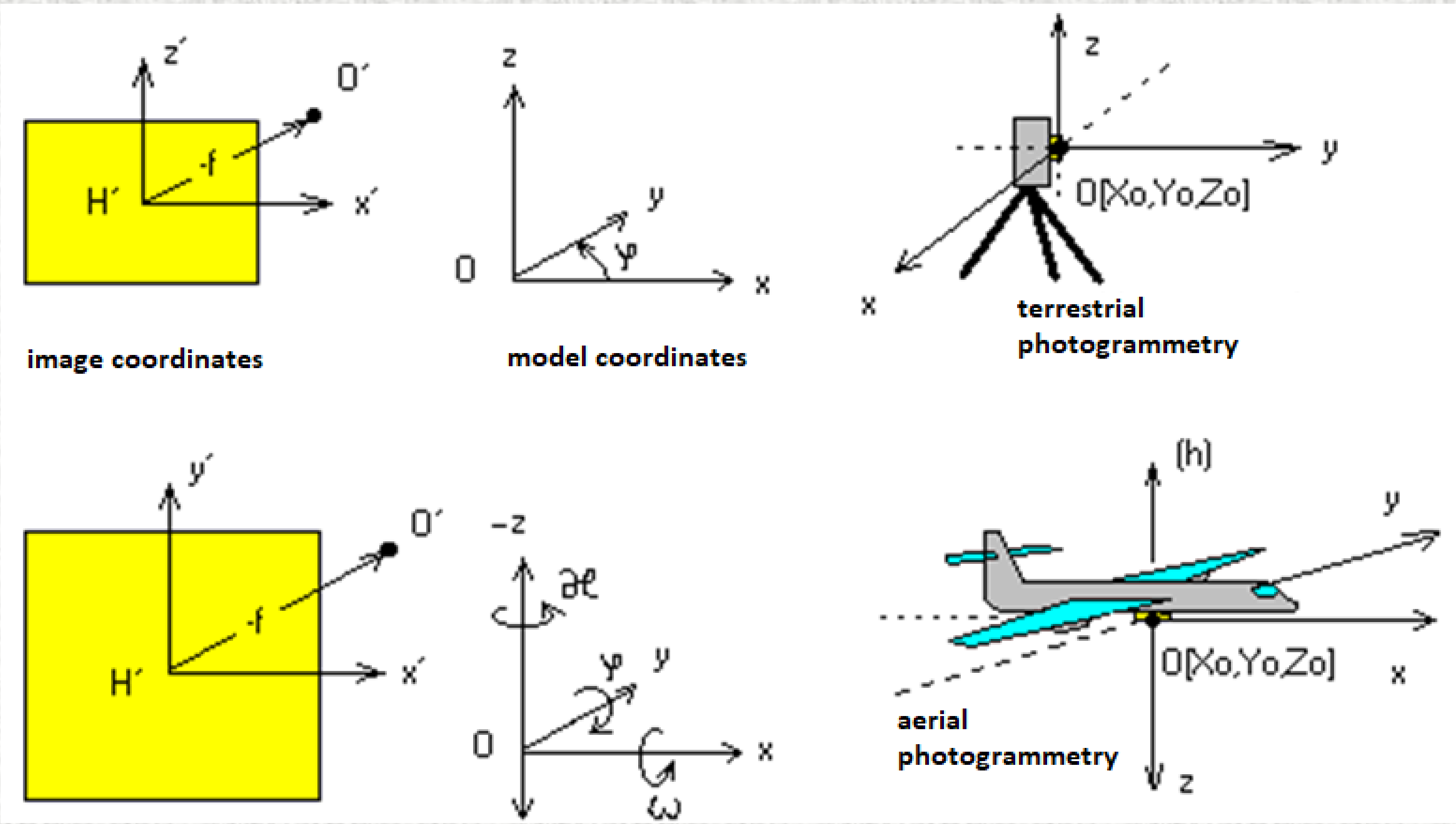
Different types of fiducial marks



Definition of the elements of the internal orientation in the general configuration (image coordinates x´,y´); The axis of view is the perpendicular to the image plane passing through the object projection centre.

Coordinate systems and conversions

Elements of internal and external orientation



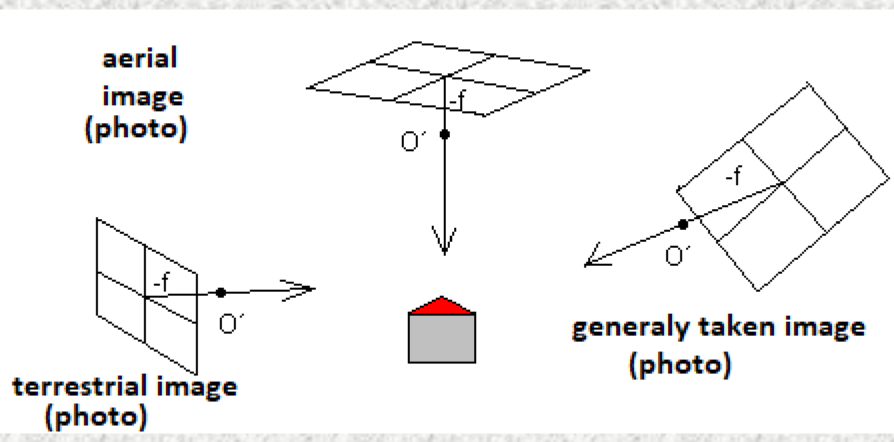
Internal and external orientation elements for terrestrial and aerial photogrammetry

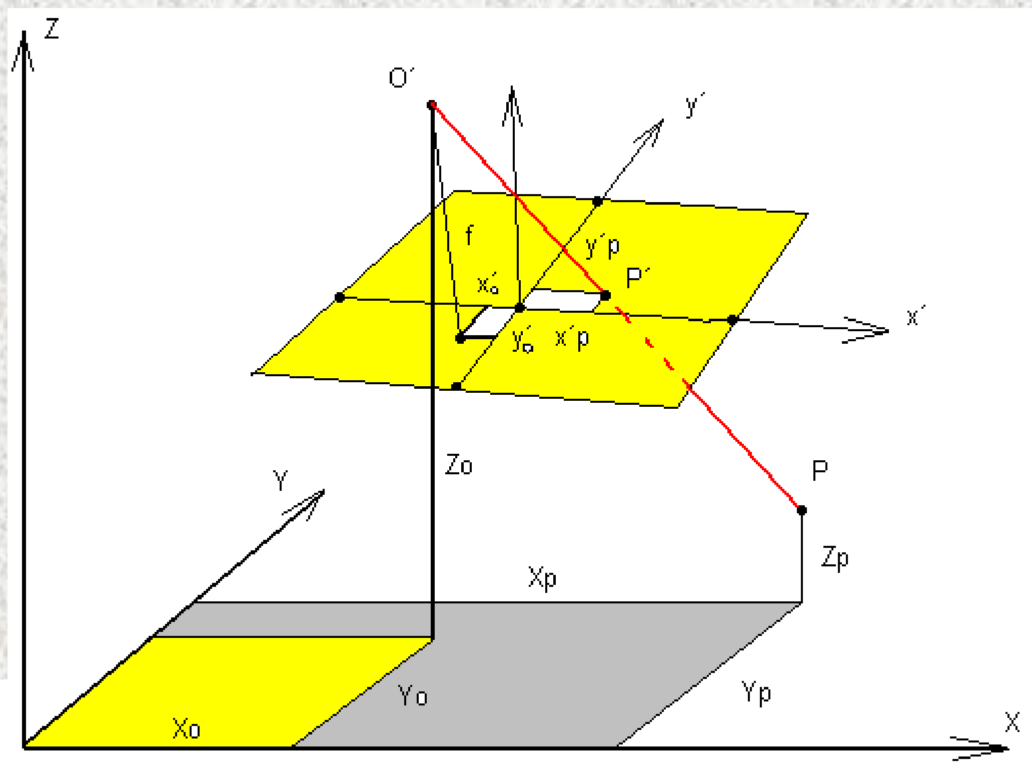
Coordinate systems

Image coordinate system: label: x , y , (z = -f)
Model coordinate system : designations: x , y , z
Geodetic system: X , Y , Z

Auxiliary systems:
Fictional image coordinate system: x_F , y_F , z_F
Vertical image coordinate system: x_S , y_S , z_S

Classic terrestrial, aerial
and generaly taken image





Coordinate systems in
aerial photogrammetry

Converting image information to geodetic systems

1) Restoration of internal orientation elements

2) External orientation

External orientations can be solved classically in two steps as:

1. **relative orientation** (mutual orientation between the two stereo images, formation of an arbitrary spatially oriented stereo model)
2. **absolute orientation** (rotation, shifting, and scale adjustment of the model into the geodetic reference system)

3. in one step **using the Bundle Block Adjustment Method (most used today, advanced computer technology required)**

(*Bündelblockausgleichung, D; svazkové vyrovnání bloku, CZ; compensation en bloc par gerbes perspectives, F*)

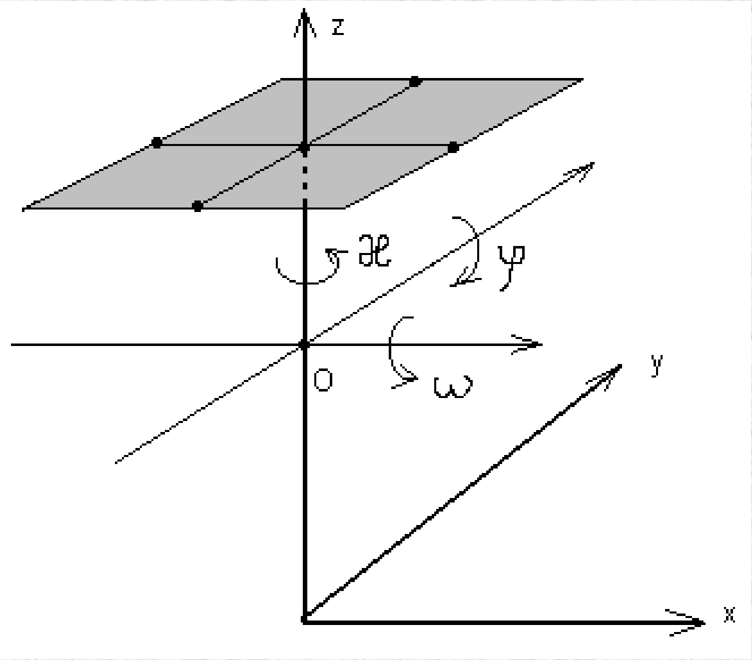
Rotation matrix

The interconversion of coordinate systems generally involves ***rotation, shifting*** and ***scaling***. While shifting and scaling are relatively simple operations, spatial rotation is more complex

Rotation in plane

$$X = x \cdot \cos\alpha - y \cdot \sin\alpha$$
$$Y = x \cdot \sin\alpha + y \cdot \cos\alpha$$

$$\mathbf{X} = \mathbf{R} \cdot \mathbf{x}., \qquad \mathbf{R} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix}$$



Rotation in space

$$\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

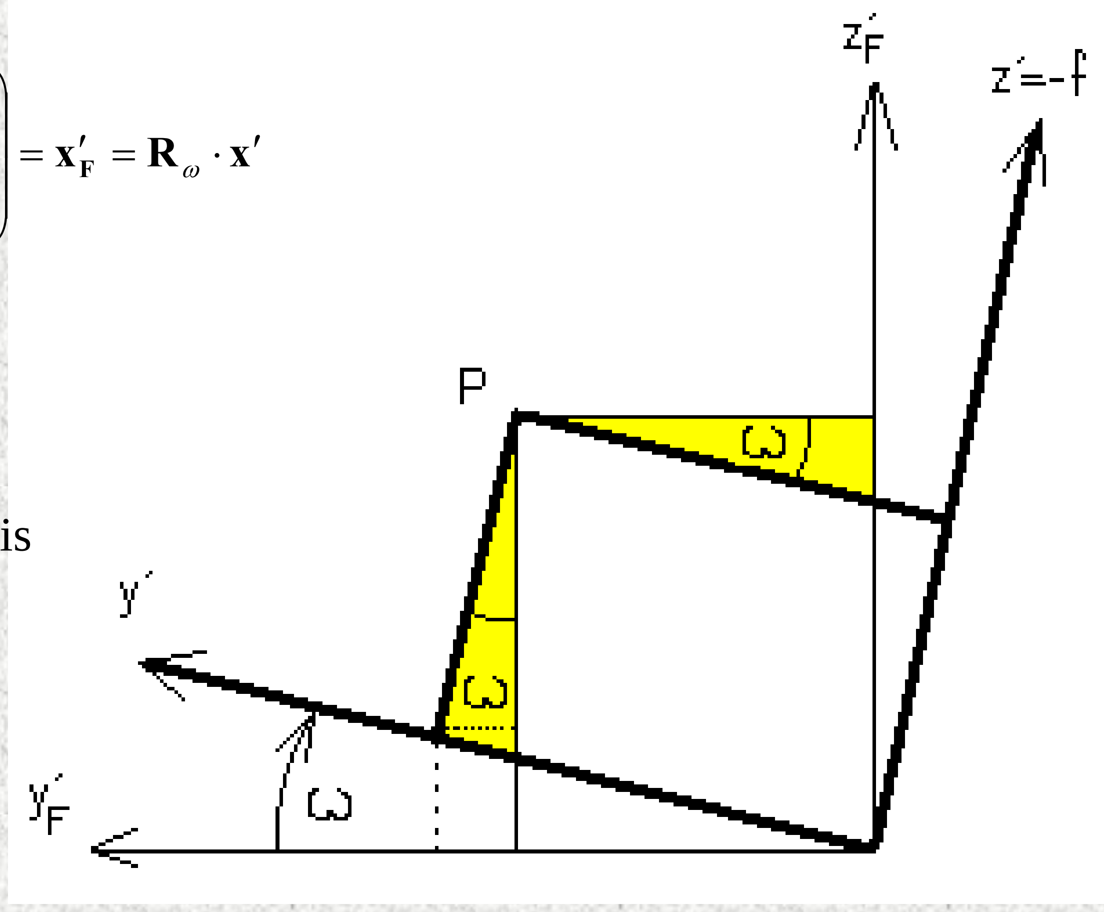
(primary, secondary and tertiary rotation)

Rotation matrix

Rotation about the primary *x´* axis

$$x'_F = x'$$
$$y'_F = y' \cdot \cos\omega - z' \cdot \sin\omega$$
$$z'_F = y' \cdot \sin\omega + z' \cdot \cos\omega$$

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \mathbf{x}'_\mathbf{F} = \mathbf{R}_\omega \cdot \mathbf{x}'$$



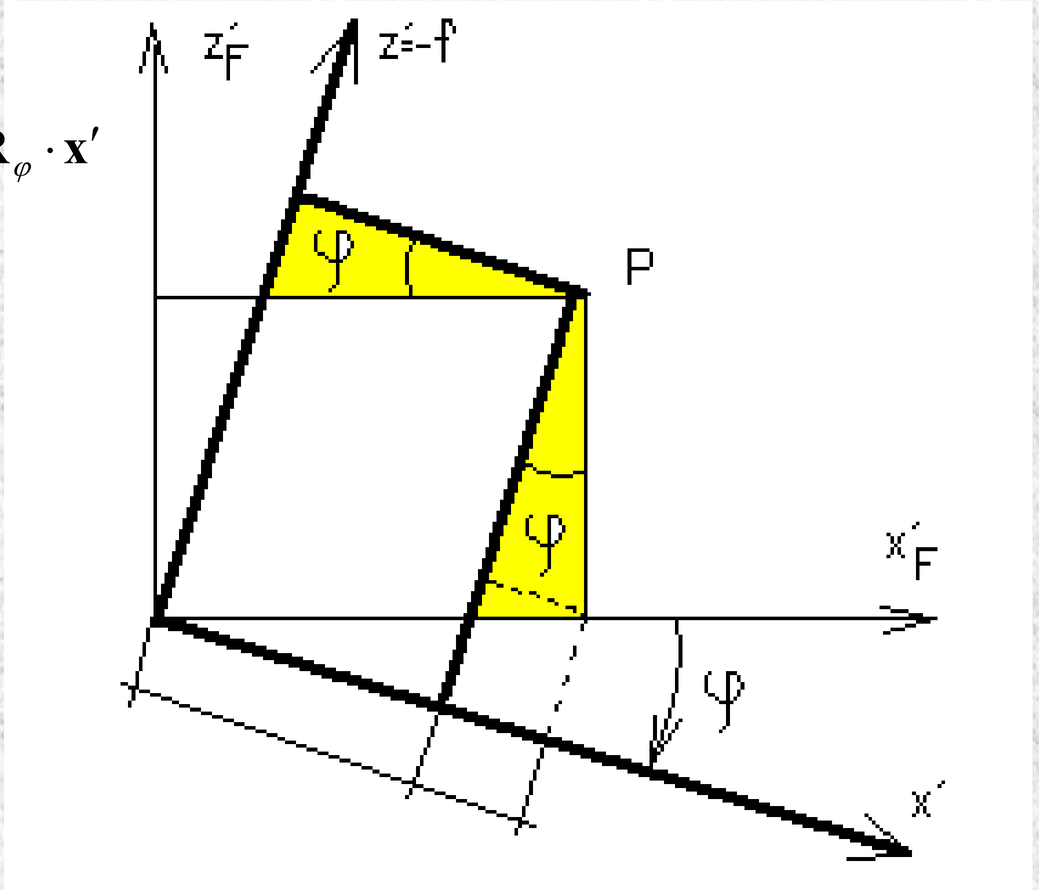
Rotation of the system around the x´ axis by an angle

Rotation matrix

Rotation about the secondary y´ axis

$$x'_F = x' \cdot \cos\varphi + z' \cdot \sin\varphi$$
$$y'_F = y'$$
$$z'_F = -x' \cdot \sin\varphi + z' \cdot \cos\varphi$$

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \mathbf{x}'_{\mathbf{F}} = \mathbf{R}_{\varphi} \cdot \mathbf{x}'$$



Rotation of the system about the y´ axis by an angle

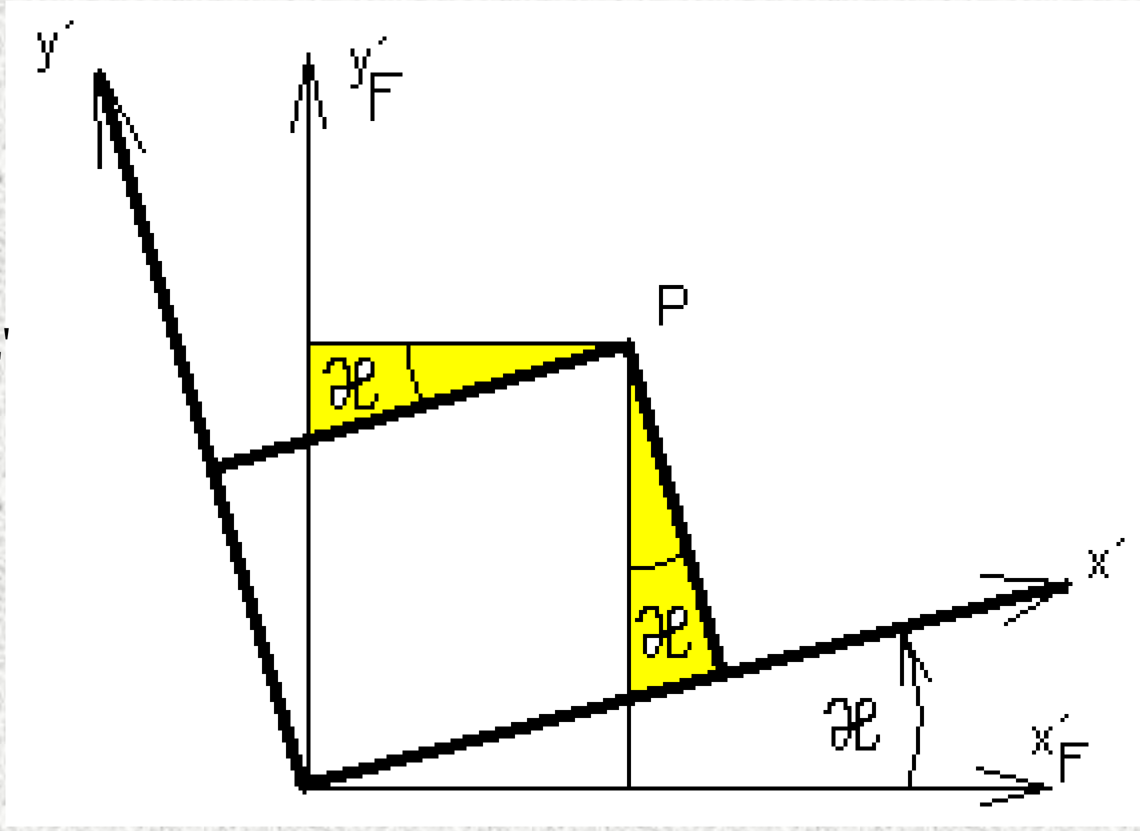
Rotation matrix

Rotation about the tertiary axis z´

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \begin{pmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \mathbf{x}'_{\mathbf{F}} = \mathbf{R}_{\kappa} \cdot \mathbf{x}'$$

$$x'_F = x' \cdot \cos\kappa - y' \cdot \sin\kappa$$
$$y'_F = x' \cdot \sin\kappa + y' \cdot \cos\kappa$$
$$z'_F = z'$$

Rotation of the system around the z' axis by an angle



Rotation matrix

Resulting rotation matrix *R*

$$\mathbf{R}_{\omega\varphi} = \mathbf{R}_{\omega} \cdot \mathbf{R}_{\varphi} \qquad \mathbf{R}_{\omega\varphi\kappa} = \mathbf{R}_{\omega\varphi} \cdot \mathbf{R}_{\kappa}$$

$$\mathbf{R}_{\omega\varphi\kappa} = \begin{pmatrix} \cos\varphi\cos\kappa & -\cos\varphi\sin\kappa & \sin\varphi \\ \sin\omega\sin\varphi\cos\kappa + \cos\omega\sin\kappa & -\sin\omega\sin\varphi\sin\kappa + \cos\omega\cos\kappa & -\sin\omega\cos\varphi \\ -\cos\omega\sin\varphi\cos\kappa + \sin\omega\sin\kappa & \cos\omega\sin\varphi\sin\kappa + \sin\omega\cos\kappa & \cos\omega\cos\varphi \end{pmatrix}$$

$$\tan\omega = -\frac{r_{23}}{r_{33}}, \sin\varphi = r_{13}, \tan\kappa = -\frac{r_{12}}{r_{11}}$$

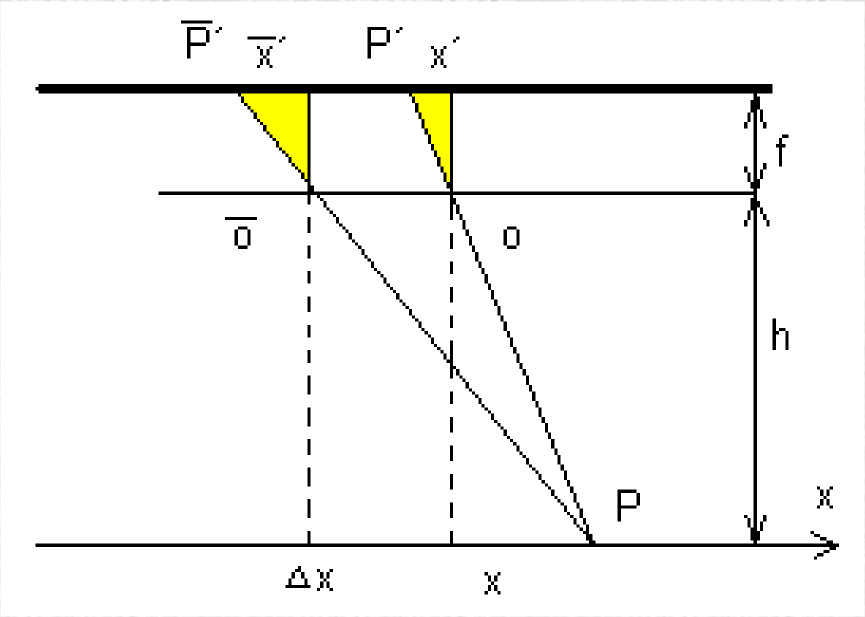
$$\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

Here it is important to note that r_{13} = sin > 0 in the 1st and 2nd quadrants and further that r_{13} = sin < 0 in the 3rd and 4th quadrants. Thus, the rotation is not uniquely determined. The quadrants of the other two rotations ω,κ are uniquely determined given the expressions from which we calculate them. ***Thus we get a total of two sets of rotations ω,φ,κ to a single rotation matrix R.***

Shift in space

- ***linear*** changes

Effect of changing the x-coordinate



$$x = \frac{z}{z'} x' = \frac{h}{f} x'$$

$$x' = \frac{f}{h} x$$

$$\Delta x' = \frac{f}{h} \cdot \Delta x, \quad \Delta y' = 0, \quad \Delta z' = \Delta f = 0$$

In this case, *f/h* is the scale of the image. Changes of *x*, *y*, *z* in the scale of the image are often referred to traditionally as ***db$_x$* , *db$_y$* , *db$_z$*** , because of the same meaning on old analog machines. The expression goes to the shape:

$$\Delta x' = db_x, \Delta y' = 0, \Delta z' = \Delta f = 0$$

Shift in space

Effect of y-coordinate change (same as for x-coordinate)

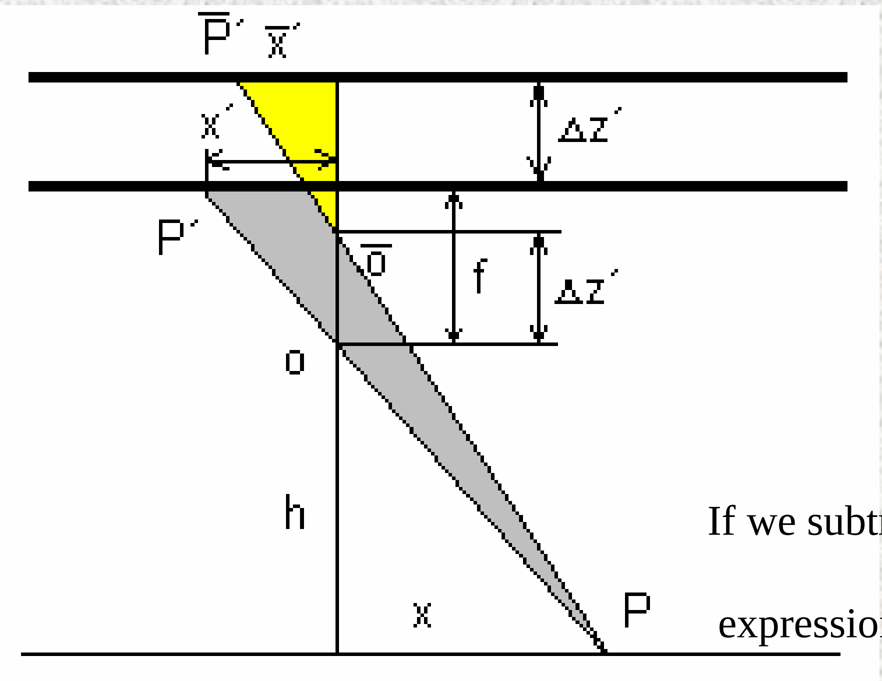
$$\Delta x' = 0, \quad \Delta y' = \frac{f}{h} y, \quad \Delta z' = \Delta f = 0$$

In this case, *f/h* is the scale of the image. Changes of *x*, *y*, *z* in the scale of the image are often referred to traditionally as $\boldsymbol{db_x}$, $\boldsymbol{db_y}$, $\boldsymbol{db_z}$, because of the same meaning on old analoque plotters. The expression transitions to the shape:

$$\Delta x' = 0, \quad \Delta y' = db_y, \quad \Delta z' = \Delta f = 0$$

Shift in space

The effect of changing the coordinate z-coordinate
- However, when the flight altitude changes, there are changes in both image coordinates



$$x = \frac{z}{z'}x' = \frac{h}{f}x', \qquad y = \frac{z}{z'}y' = \frac{h}{f}y'$$

$$x' = \frac{f}{h}x, \qquad y' = \frac{f}{h}y$$

$$\overline{x}' = \frac{f}{h+\Delta z}\cdot x, \qquad \overline{y}' = \frac{f}{h+\Delta z}\cdot y$$

If we subtract the equations, we get after adjustment

$$\frac{1}{a \pm x} \cong \frac{1}{a}\left(1 \mp \frac{x}{a}\right)$$

expression for differences in frame coordinates :

$$\Delta x' = x' - \overline{x}' = \frac{x'}{h}\cdot \Delta z, \qquad \Delta y' = y' - \overline{y}' = \frac{y'}{h}\cdot \Delta z$$

$$\Delta z = \Delta z' \cdot \frac{h}{f} \qquad \Delta x' = \frac{x'}{f}db_z, \quad \Delta y' = \frac{y'}{f}db_z, \quad \Delta z' = \Delta f = 0$$

Scale change

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = m \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

or by using affine transformation

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathbf{M} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \mathbf{M} = \begin{pmatrix} m_X & 0 & 0 \\ 0 & m_Y & 0 \\ 0 & 0 & m_Z \end{pmatrix}$$

Photogrammetrical – mathematical fundamentals

Photogrammetrical series (Gruber´s series)

derivation of the procedure

Mathematical conversions

x , y , z=(-f) → x$_F$, y$_F$, z$_F$ → x , y , z → X , Y , Z

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \mathbf{R} \cdot \begin{pmatrix} x' \\ y' \\ z' = -f \end{pmatrix},$$

$$\begin{pmatrix} x' \\ y' \\ z' = -f \end{pmatrix} = \mathbf{R}^T \cdot \begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix}$$

Further, the similarity applies:

$$\frac{x'_F}{z'_F} = \frac{x'_S}{z'_S} = \frac{x'_S}{-f} = \frac{x - x_0}{z - z_0}, \quad \frac{y'_F}{z'_F} = \frac{y'_S}{z'_S} = \frac{y'_S}{-f} = \frac{y - y_0}{z - z_0}$$

By substitution we get a **collinear relation**:

$$x'_s = -f \frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f}, y'_s = -f \frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f}$$

Mathematical conversions

$$x = x_0 + (z - z_0)\frac{r_{11}(x' - x'_0) + r_{12}(y' - y'_0) - r_{13} f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33} f}$$

$$y = y_0 + (z - z_0)\frac{r_{21}(x' - x'_0) + r_{22}(y' - y'_0) - r_{23} f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33} f}$$

a more common form because we can conveniently assign corrections to the image coordinates and linearize the relationship :

$$x' = x'_0 - f\,\frac{r_{11}(x - x_0) + r_{21}(y - y_0) + r_{31}(z - z_0)}{r_{13}(x - x_0) + r_{23}(y - y_0) + r_{33}(z - z_0)}$$

$$y' = y'_0 - f\,\frac{r_{12}(x - x_0) + r_{22}(y - y_0) + r_{32}(z - z_0)}{r_{13}(x - x_0) + r_{23}(y - y_0) + r_{33}(z - z_0)}$$

Direct relationship between image and geodetic coordinates

collinear image-model relationship

$$\frac{x'-x'_0}{-f}=\frac{x-x_0}{z-z_0},\quad \frac{y'-y'_0}{-f}=\frac{y-y_0}{z-z_0}$$

$$\begin{pmatrix} X-X_0 \\ Y-Y_0 \\ Z-Z_0 \end{pmatrix} = \mathbf{R}\cdot\begin{pmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{pmatrix},\qquad \mathbf{R}=\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

the model coordinate system can be converted to a geodetic system by rotations around three axes

$$\begin{pmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{pmatrix} = \mathbf{R}^T\cdot\begin{pmatrix} X-X_0 \\ Y-Y_0 \\ Z-Z_0 \end{pmatrix}$$

<u>the direct relationship between image and geodetic coordinates, which is the basis of all contemporary photogrammetry</u>:

$$x'=x'_0-f\,\frac{r_{11}(X-X_0)+r_{21}(Y-Y_0)+r_{31}(Z-Z_0)}{r_{13}(X-X_0)+r_{23}(Y-Y_0)+r_{33}(Z-Z_0)}$$

$$y'=y'_0-f\,\frac{r_{12}(X-X_0)+r_{22}(Y-Y_0)+r_{32}(Z-Z_0)}{r_{13}(X-X_0)+r_{23}(Y-Y_0)+r_{33}(Z-Z_0)}$$

Direct relationship - transformation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} + m \cdot R \cdot \begin{bmatrix} x' - x'_o \\ y' - y'_o \\ -f \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} + m \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \cdot \begin{bmatrix} x' - x'_o \\ y' - y'_o \\ -f \end{bmatrix}$$

$$\begin{aligned} X &= X_o + m \cdot \left(r_{11} \cdot \left(x' - x'_o\right) + r_{12} \cdot \left(y' - y'\right) - r_{13} \cdot f\right) \\ Y &= Y_o + m \cdot \left(r_{21} \cdot \left(x' - x'_o\right) + r_{22} \cdot \left(y' - y'\right) - r_{23} \cdot f\right) \\ Z &= Z_o + m \cdot \left(r_{31} \cdot \left(x' - x'_o\right) + r_{32} \cdot \left(y' - y'\right) - r_{33} \cdot f\right) \end{aligned}$$

$$m = \frac{Z - Z_o}{r_{31} \cdot \left(x' - x'_o\right) + r_{32} \cdot \left(y' - y'\right) - r_{33} \cdot f}$$

$$X = X_o + (Z - Z_o) \cdot \frac{r_{11} \cdot \left(x' - x'_o\right) + r_{12} \cdot \left(y' - y'\right) - r_{13} \cdot f}{r_{31} \cdot \left(x' - x'_o\right) + r_{32} \cdot \left(y' - y'\right) - r_{33} \cdot f}$$

$$Y = Y_o + (Z - Z_o) \cdot \frac{r_{21} \cdot \left(x' - x'_o\right) + r_{22} \cdot \left(y' - y'\right) - r_{23} \cdot f}{r_{31} \cdot \left(x' - x'_o\right) + r_{32} \cdot \left(y' - y'\right) - r_{33} \cdot f}$$

Photogrammetric series

Definition:

Photogrammetric series are expressions which (with a degree of precision given by the linearization of a complete relation) express the effect of elements of exterior orientation on image coordinates.

Rotation matrix linearization (up to 2°-3°) $\cos(\alpha) \cong 1$ and next $\sin(\alpha) \cong d\alpha$

$$\mathbf{dR} = \begin{pmatrix} 1 & -d\kappa & d\varphi \\ d\kappa & 1 & -d\omega \\ -d\varphi & d\omega & 1 \end{pmatrix}$$

$$x = x_0 + (z - z_0)\frac{r_{11}(x' - x'_0) + r_{12}(y' - y'_0) - r_{13}f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33}f}$$

$$y = y_0 + (z - z_0)\frac{r_{21}(x' - x'_0) + r_{22}(y' - y'_0) - r_{23}f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33}f}$$

Photogrammetric series

$$x'_s = -f\,\frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f}, \quad y'_s = -f\,\frac{r_{21}x' + r_{22}y' - r_{23}f}{r_{31}x' + r_{32}y' - r_{33}f}$$

for small angles using a linearized rotation matrix

$$x'_s = -f\,\frac{x' - y'd\kappa' - f\,d\varphi'}{-x'd\varphi' + y'd\omega' - f} \quad y'_s = -f\,\frac{x'd\kappa' + y' + f\,d\omega'}{-x'd\varphi' + y'd\omega' - f}$$

The next procedure is analogous for x'_s, y'_s

$$\left(-x'd\varphi' + y'd\omega' - f\right)x'_s = -f\left(x' - y'd\kappa' - f\,d\varphi'\right)$$

after division by -*f*

$$\left(1 - \frac{y'd\omega'}{f} + \frac{x'd\varphi'}{f}\right)x'_s = \left(x' - y'd\kappa' - f\,d\varphi'\right)$$

Photogrammetric series

$$x'_s = \left(x' - y'd\kappa' - f\,d\varphi'\right)\left[1 - \left(\frac{y'd\omega'}{f} - \frac{x'd\varphi'}{f}\right)\right]^{-1}$$

$$x'_s = A \cdot [1 - B]^{-1} \approx A \cdot (1 + B) = A + AB$$

$$\Delta x' = x'_s - x' = -y'd\kappa' - \left(f + \frac{x'^2}{f}\right)d\varphi' + \frac{x'y'}{f}d\omega'$$

$$\Delta y' = y'_s - y' = x'd\kappa' - \frac{x'y'}{f}d\varphi' + \left(f + \frac{y'^2}{f}\right)d\omega'$$

Photogrammetric series

Finally, these expressions need to be supplemented by the effect of translation

$$\Delta x' = -y'd\kappa' - \left(f + \frac{x'^2}{f} \right) d\varphi' + \frac{x'y'}{f} d\omega' + db'_x + \frac{x'}{f} db'_z$$

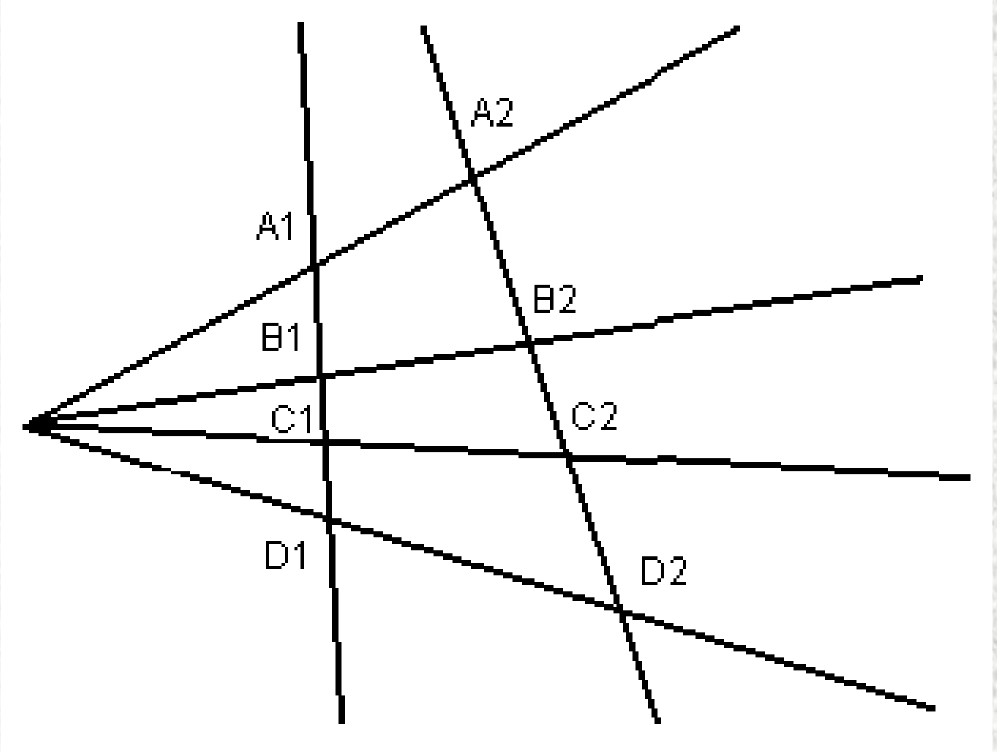
$$\Delta y' = x'd\kappa' - \frac{x'y'}{f} d\varphi' + \left(f + \frac{y'^2}{f} \right) d\omega' + db'_y + \frac{y'}{f} db'_z$$

The term is called "**complete photogrammetric series**", or historically Gruber series, and is used in simplified theoretical derivations. The meaning and use of series was quite fundamental, especially in the era of analogue photogrammetry.

Methods of photogrammetry

Single-image photogrammetry

- relationship between two planes



Papp's theorem:

The binary ratio of the quadruple points remains in the plane of the map and the image..

$$\frac{\frac{A_1C_1}{B_1C_1}}{\frac{A_1D_1}{B_1D_1}} = \frac{\frac{A_2C_2}{B_1C_2}}{\frac{A_2D_2}{B_2D_2}}$$

Mathematical expression
is the **collinear transformation**

$$X = \frac{a_1 x' + a_2 y' + a_3}{c_1 x' + c_2 y' + 1}$$

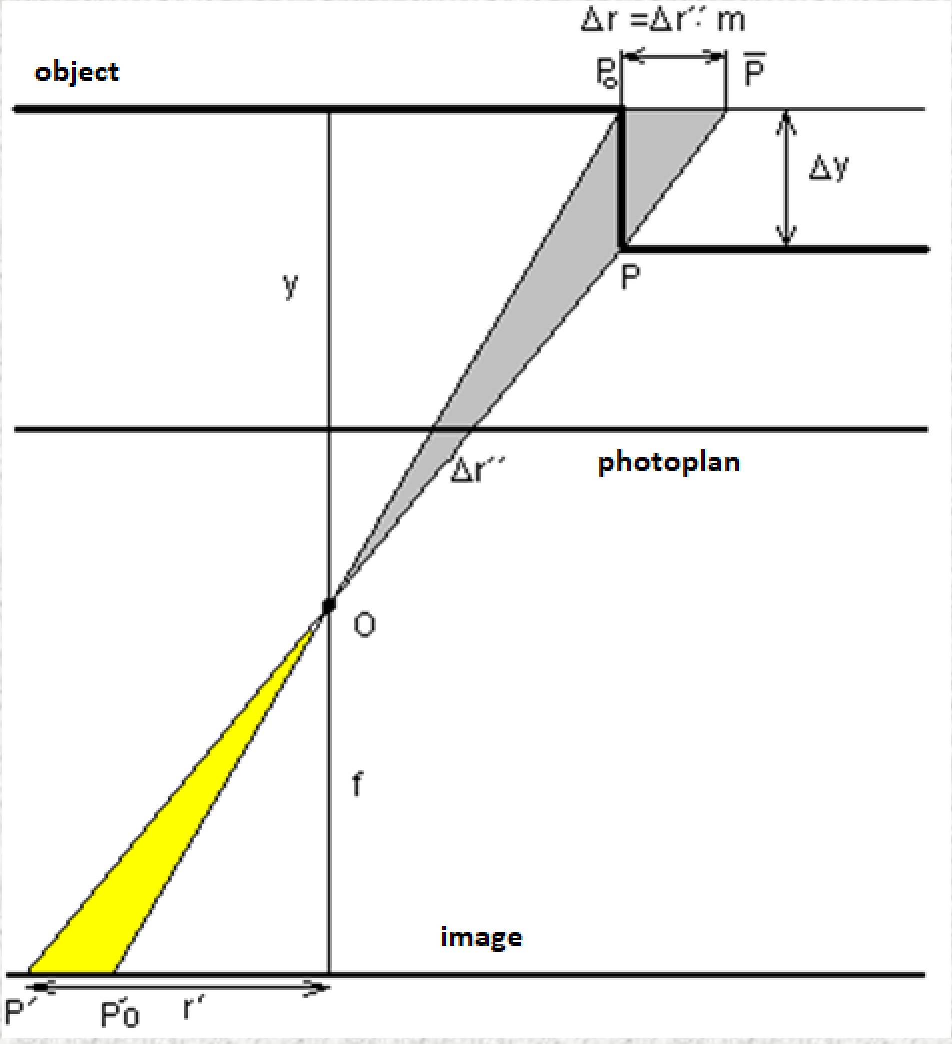
$$Y = \frac{b_1 x' + b_2 y' + b_3}{c_1 x' + c_2 y' + 1}$$

Single-image photogrammetry

- Using single image photogrammetry it is possible to generate photo-plans
- …but for flat surfaces only because of central projection

Influence of spatial object distribution terrestrial case

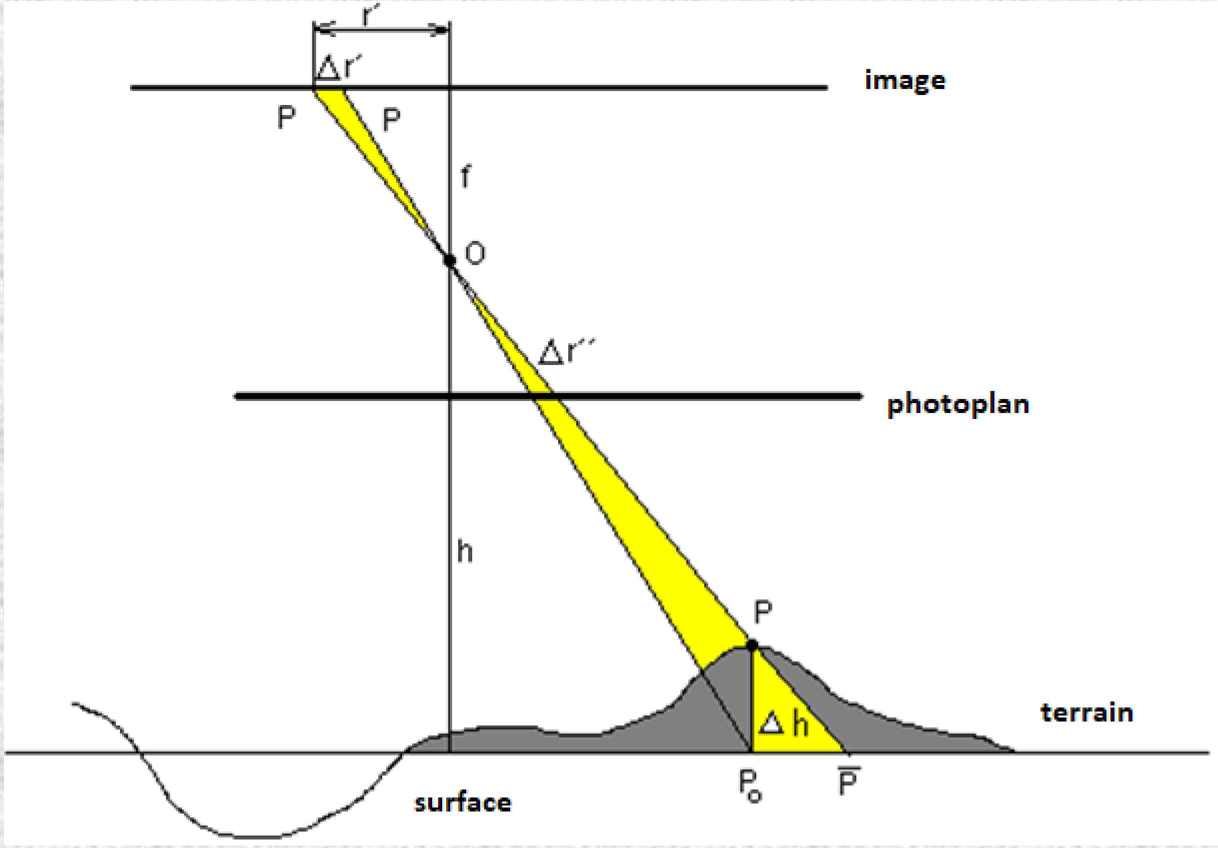
$$\Delta r = \Delta r'' \cdot m_F , \qquad \frac{\Delta r}{\Delta y} = \frac{r'}{f} , \qquad \Delta r'' = \frac{\Delta y \cdot r'}{f \cdot m_F}$$



$$\Delta y_{\max} = \frac{f \cdot m_F \cdot \Delta r''_{\max}}{r'}$$

Influence of spatial terrain distribution aerial case

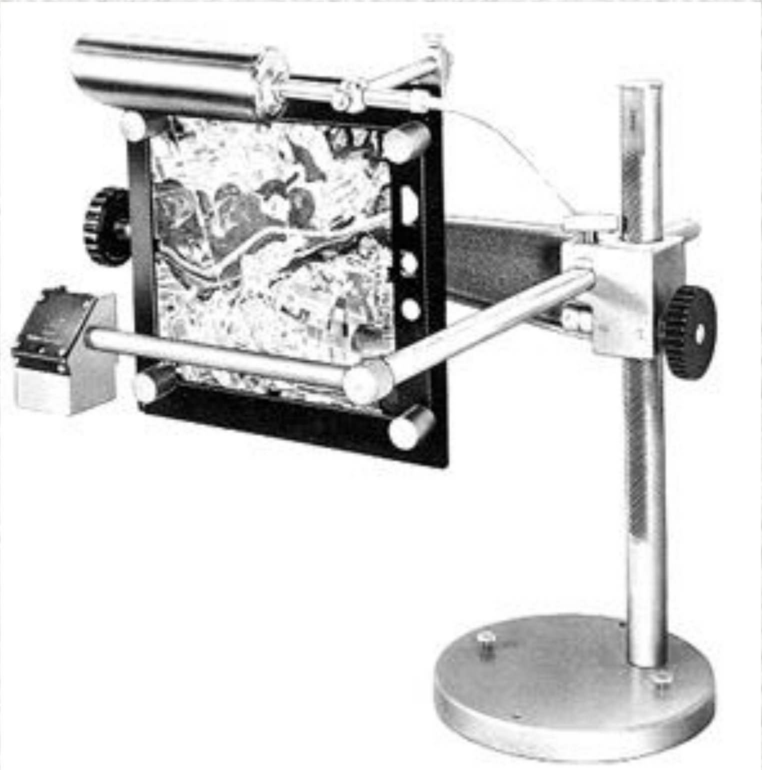
$$\Delta r'' = \frac{\Delta h \cdot r'}{f \cdot m_F} \qquad \Delta h_{\max} = \frac{f \cdot m_F \cdot \Delta r''_{\max}}{r'}$$

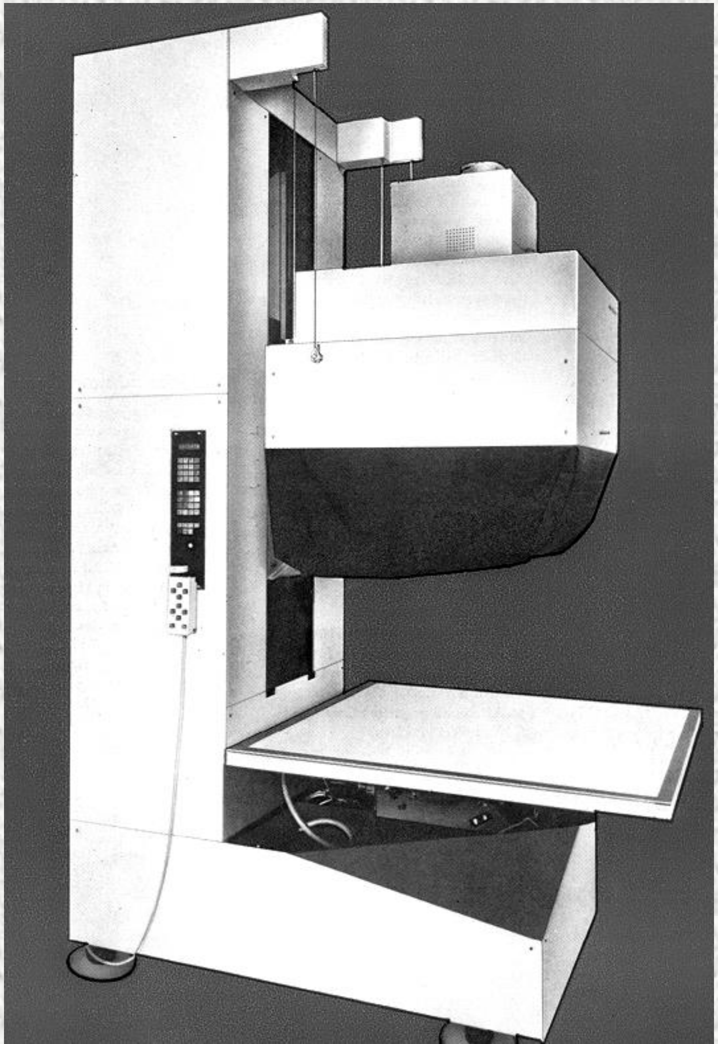


Methods of evaluation of image photogrammetry

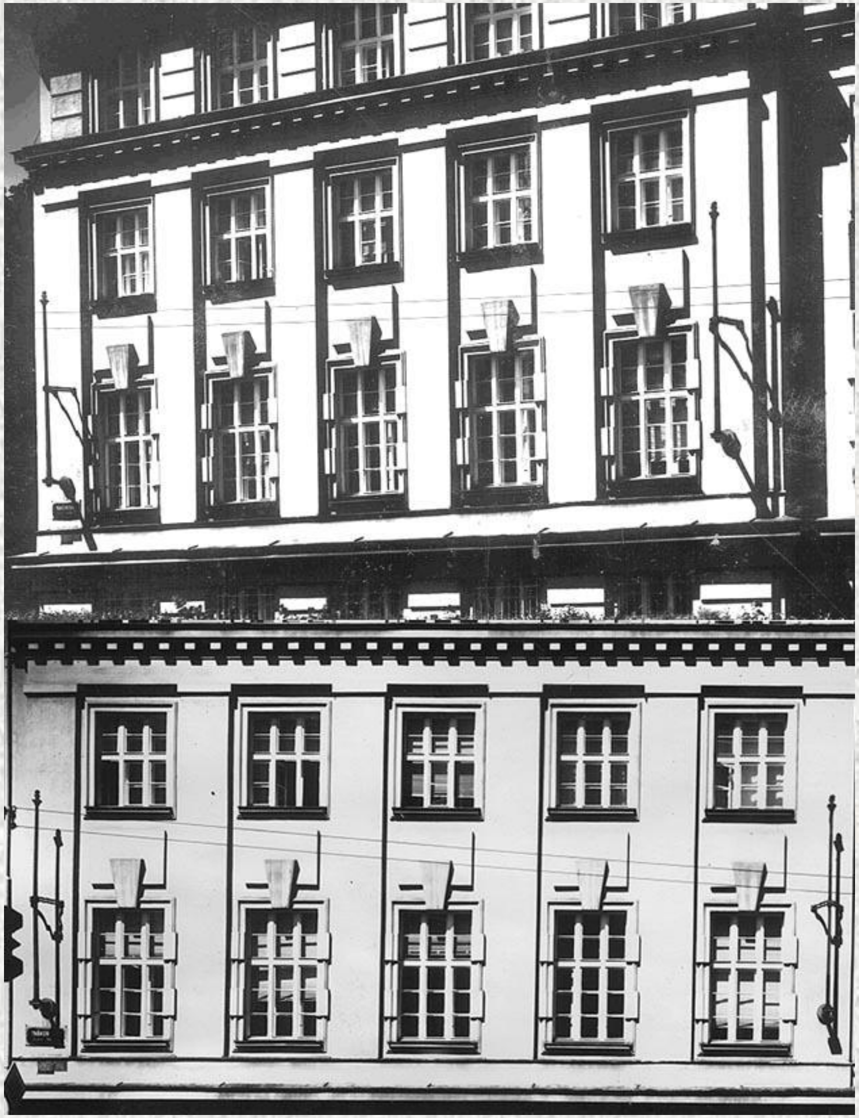
Instrument for simple plotting the image content on the map (used until the 1960s)

Last manufactured precise rectifier, Rectimat C, Zeiss Jena, 1988





Digital redrawing



Original image and photo plan, taken by the analoque metric camera (left),
an image taken with an ordinary digital camera and its digitally redrawn form - an image deformation due to uncorrected radial distortion is clearly visible (right)

Matrix solution

$$X = \frac{a_1 x' + a_2 y' + a_3}{c_1 x' + c_2 y' + 1}$$

$$Y = \frac{b_1 x' + b_2 y' + b_3}{c_1 x' + c_2 y' + 1}$$

$$X = a_1 x' + a_2 y' + a_3 \qquad\qquad - c_1 x' X - c_2 y' X$$

$$Y = \qquad\qquad b_1 x' + b_2 y' + b_3 - c_1 x' Y - c_2 y' Y$$

$$\begin{pmatrix} x_1' & y_1' & 1 & 0 & 0 & 0 & -x_1' X_1 & -y_1' X_1 \\ 0 & 0 & 0 & x_1' & y_1' & 1 & -x_1' Y_1 & -y_1' Y_1 \\ x_2' & y_2' & 1 & 0 & 0 & 0 & -x_2' X_2 & -y_2' X_2 \\ 0 & 0 & 0 & x_2' & y_2' & 1 & -x_2' Y_2 & -y_2' Y_2 \\ x_3' & y_3' & 1 & 0 & 0 & 0 & -x_3' X_3 & -y_3' X_3 \\ 0 & 0 & 0 & x_3' & y_3' & 1 & -x_3' Y_3 & -y_3' Y_3 \\ x_4' & y_4' & 1 & 0 & 0 & 0 & -x_4' X_4 & -y_4' X_4 \\ 0 & 0 & 0 & x_4' & y_4' & 1 & -x_4' Y_4 & -y_4' Y_4 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ b_1 \\ b_2 \\ b_3 \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \\ X_4 \\ Y_4 \end{pmatrix}$$

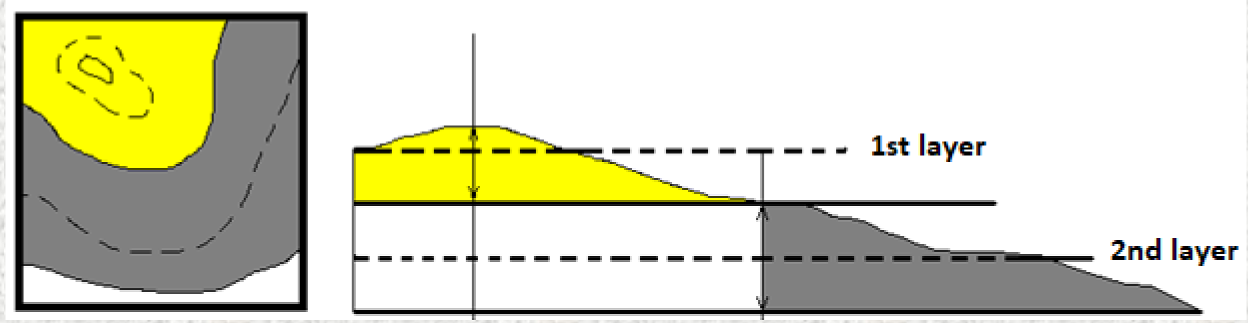
$$\mathbf{A} \cdot \mathbf{a} = \mathbf{X}$$

$$\mathbf{a} = \mathbf{A}^{-1} \cdot \mathbf{X}$$

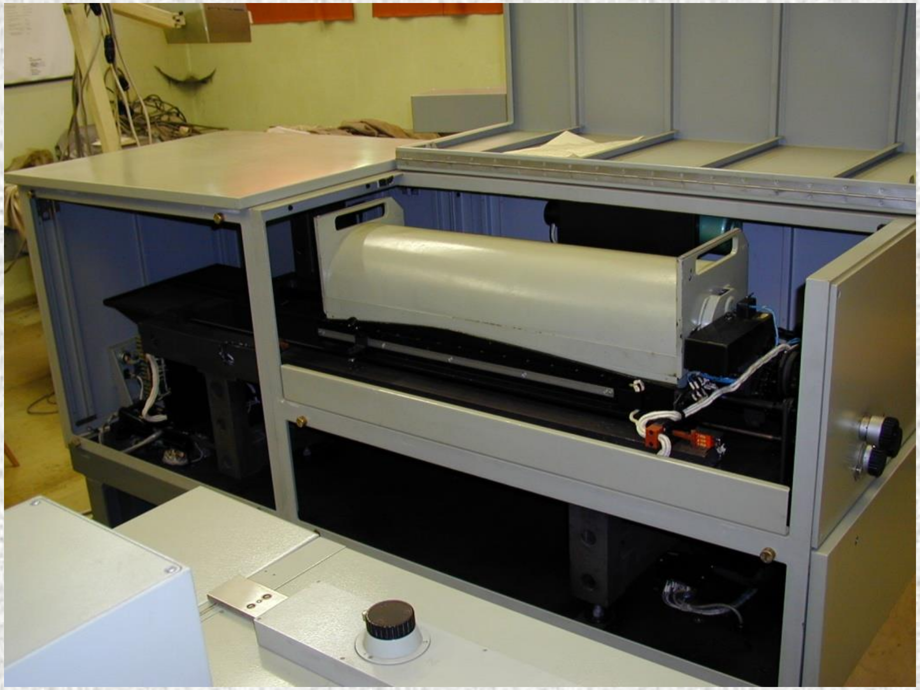
Image easy rectification can only be used with flat object or terrain (surface)

Typically, the terrain (object) is not flat; if we need to create a plan / photoplan, there are three possibilities:

1)Redrawing by layers (not used today)

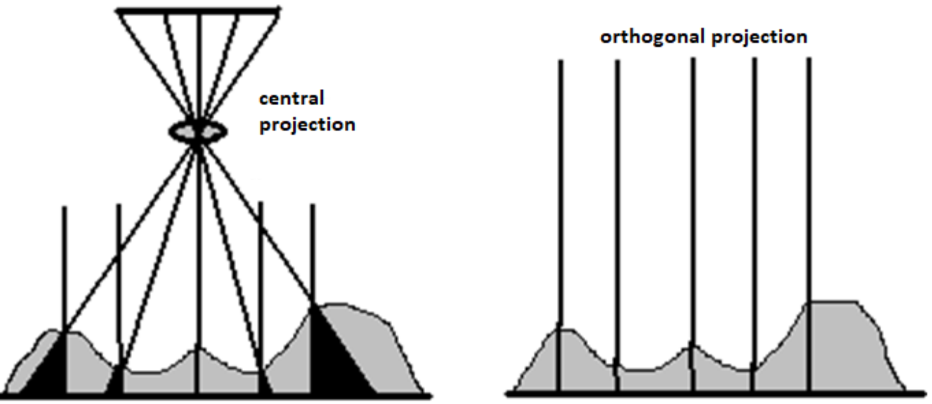


2)Differential analoque redrawing (not used today)

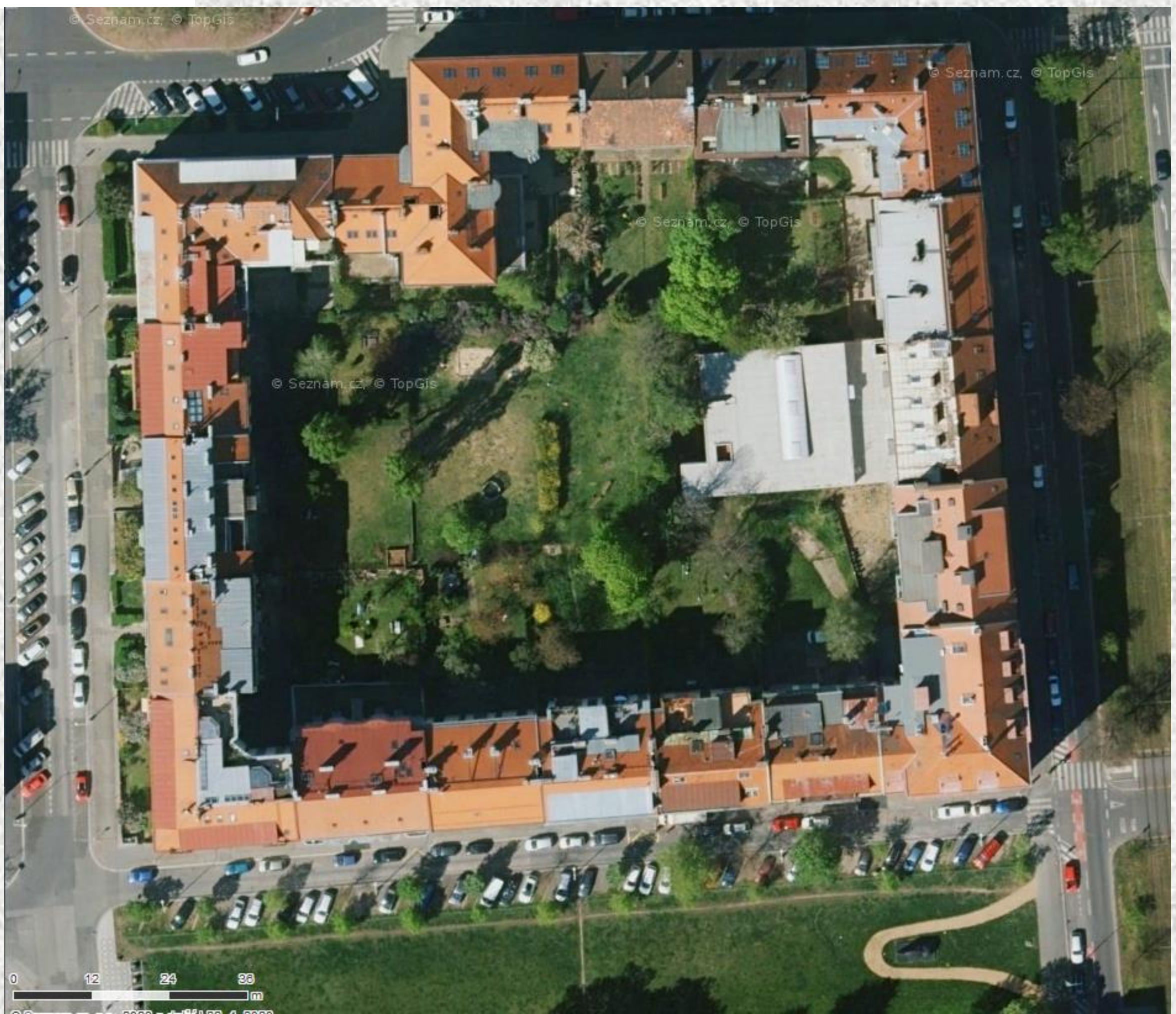


3)Digital orthophoto (today most used photogrammetrical output)

Digital orthophoto



Converting the central projection to orthogonal - to orthophoto - allows you to work with the image information as a map and insert it as a data layer in GIS.

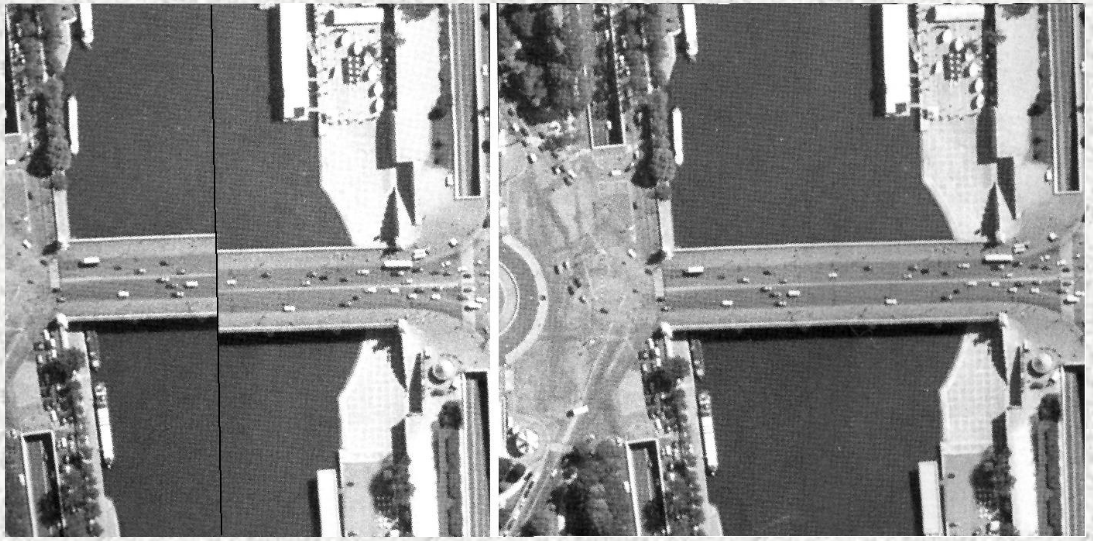


Digital orthophoto

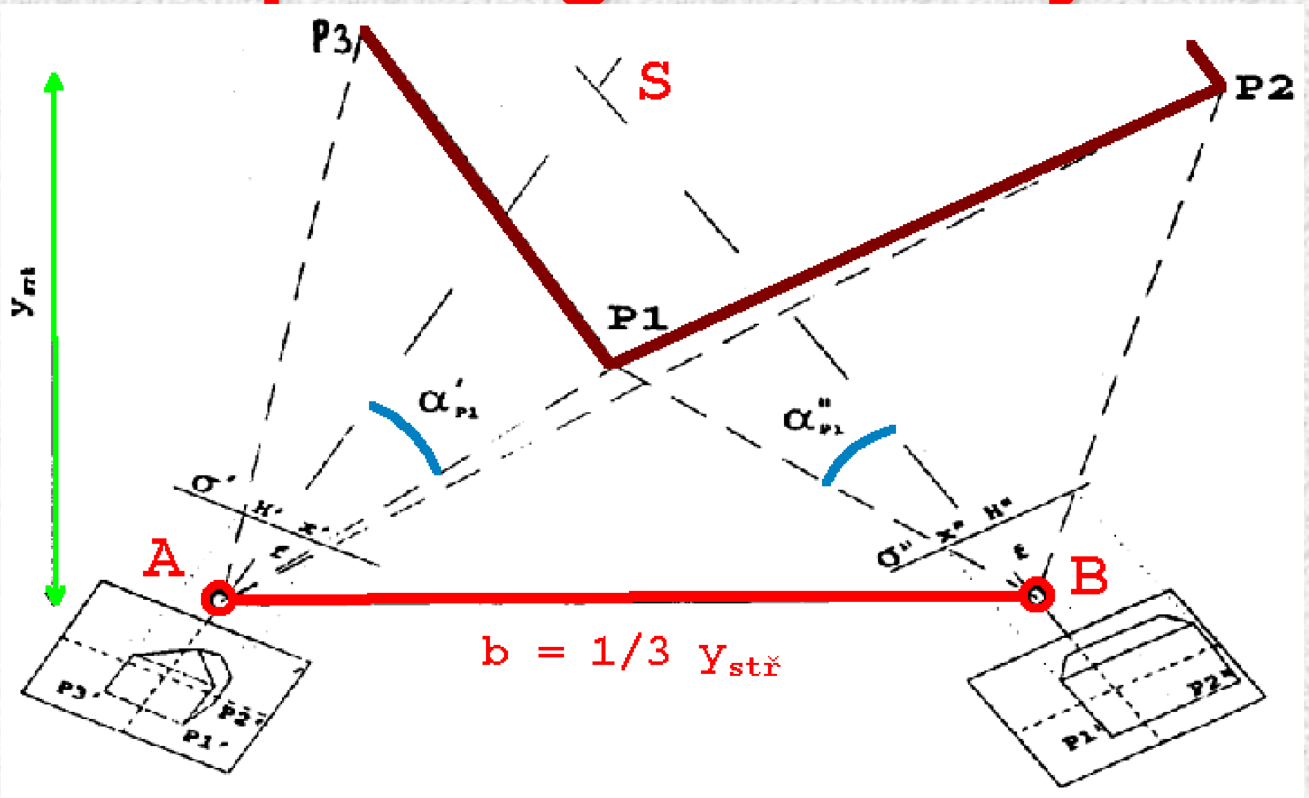
Digital orthophoto is a photogrammetric product - image conversion with central projection to orthogonal projection

for orthophoto creation: image with known elements of interior and exterior orientation and DTM are required

The problem of mosaicking - seamless orthophoto



Multi-image photogrammetry
Intersection photogrammetry



The oldest photogrammetric 3D method

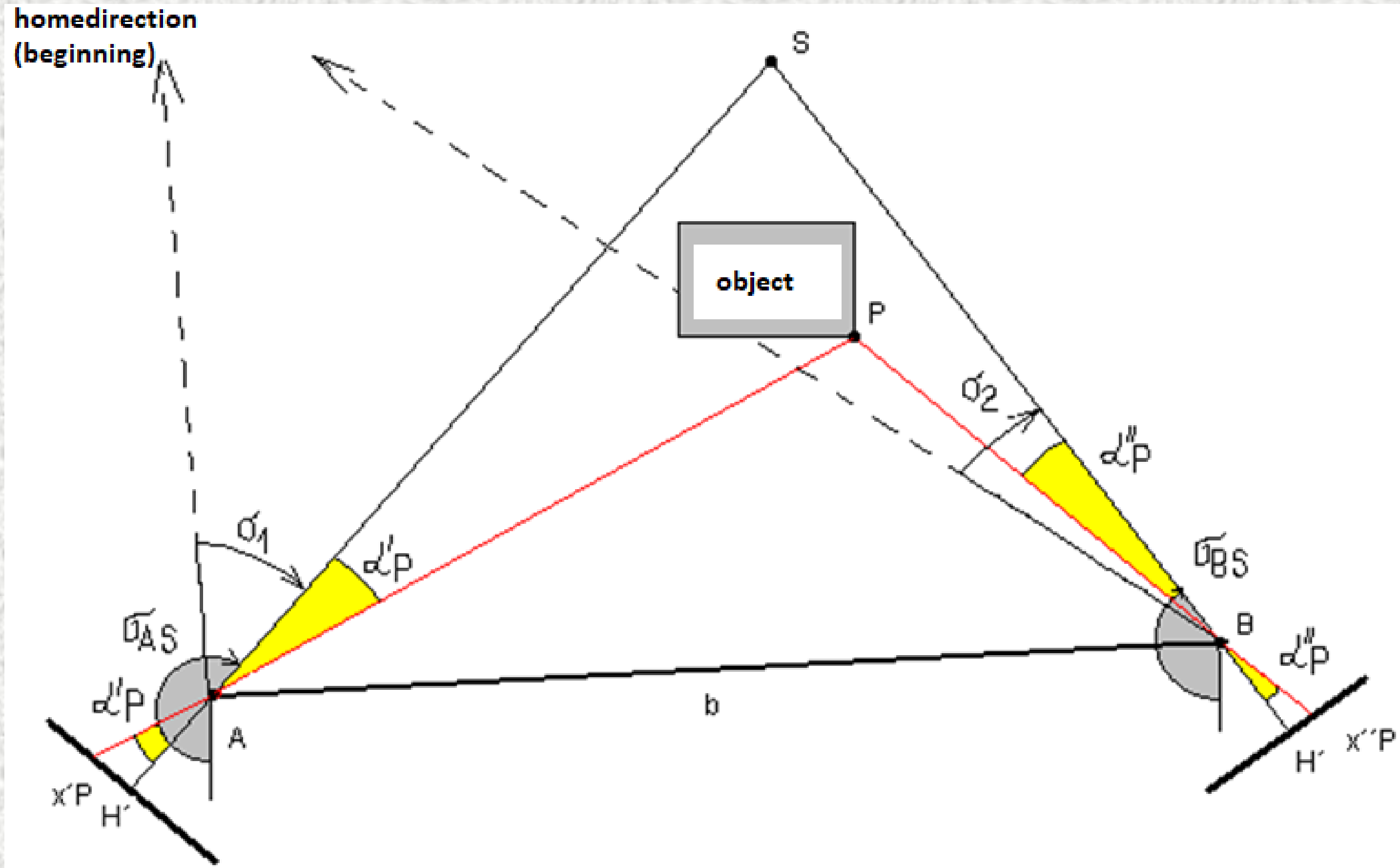
- based on intersection of rays; only for artificialy or naturally signalized points! It is necessay to find there on at least two images.
- today only in digital form

Historical solution

$$tg\alpha' = \frac{x'}{f}, \quad tg\beta' = \frac{z'}{\sqrt{\left(f^2 + x'^2\right)}} = \frac{z'}{f}\cos\alpha'$$

$$tg\alpha' = \frac{x'}{f}, \quad tg\beta' = \frac{z'}{\sqrt{\left(f^2 + x'^2\right)}} = \frac{z'}{f}\cos\alpha'$$

$$x' = f\,\frac{x'_F}{f\cdot\cos\omega - z'_F\cdot\sin\omega}, \quad z' = f\,\frac{f\cdot\sin\omega + z'_F\cdot\cos\omega}{f\cdot\cos\omega - z'_F\cdot\sin\omega}$$



At the place, it is necessary to have two points in coordinates (A, B) and targetting with phototeodolite on a point with known coordinates (S)

Today's solution

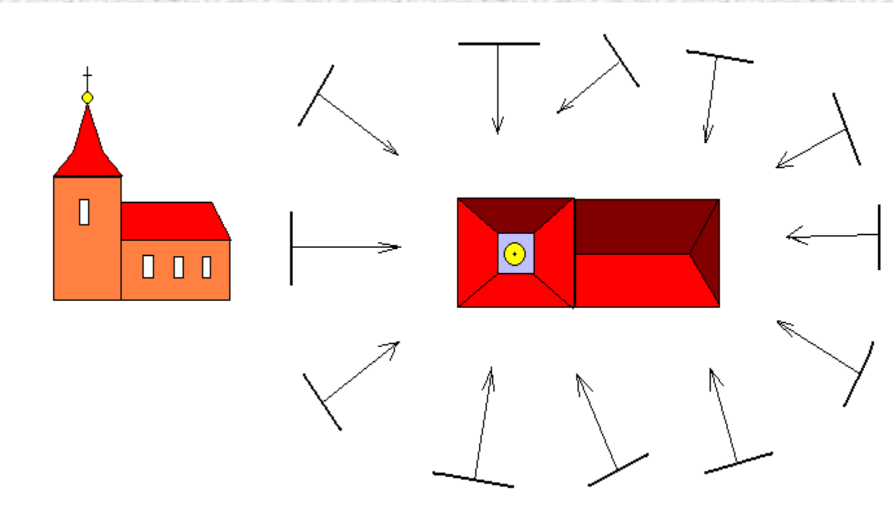
$$X = X_0 + (Z - Z_0)\frac{r_{11}(x' - x'_0) + r_{12}(y' - y'_0) - r_{13} f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33} f}$$

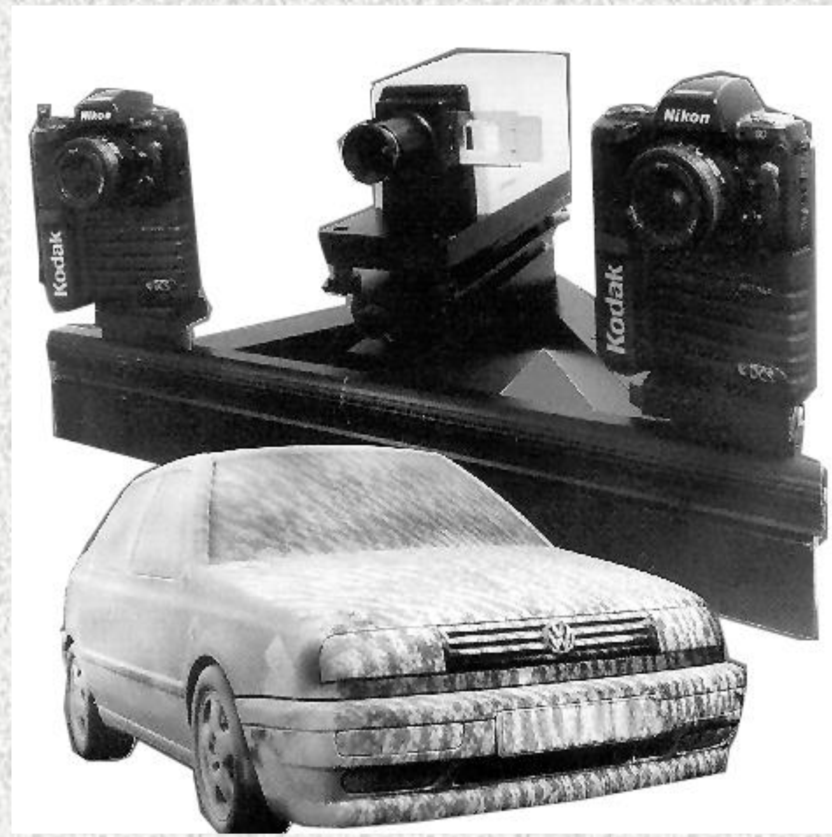
$$Y = Y_0 + (Z - Z_0)\frac{r_{21}(x' - x'_0) + r_{22}(y' - y'_0) - r_{23} f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33} f}$$

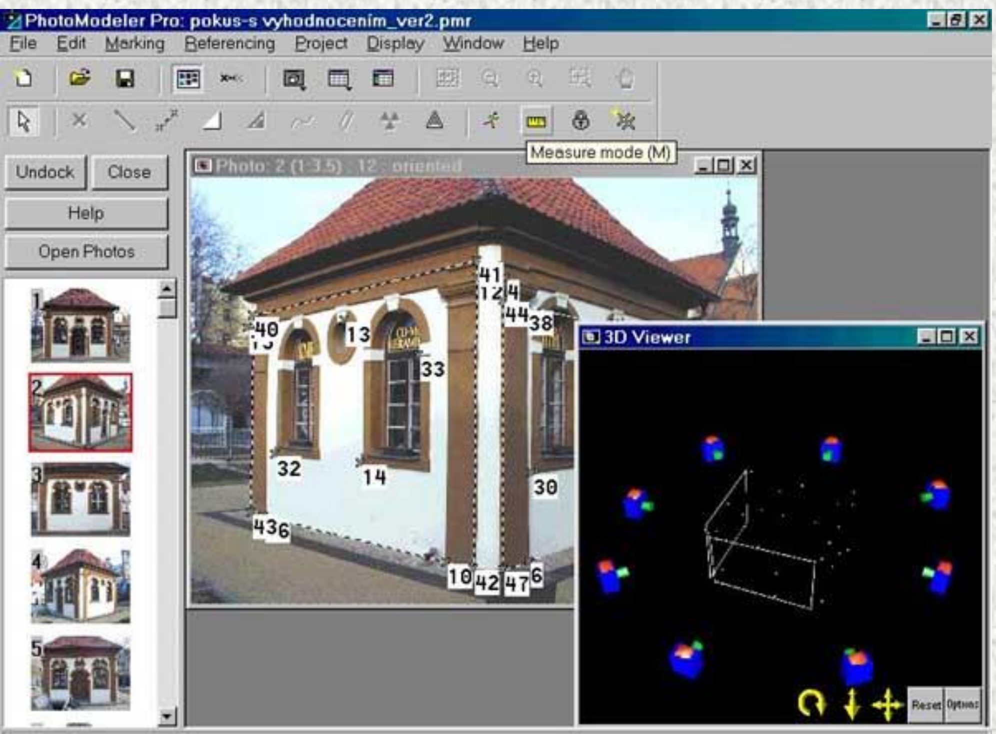
$$\begin{pmatrix} x' - x'_0 + \Delta x' \\ y' - y'_0 + \Delta y' \\ -f \end{pmatrix} = m \cdot \boldsymbol{R}^T \cdot \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix}$$

The process uses iterration on a computer; it isn´t necessary to measure any information at the place, but GCP´s or several distances are welcome (for scale of created model)

Intersection photogrammetry

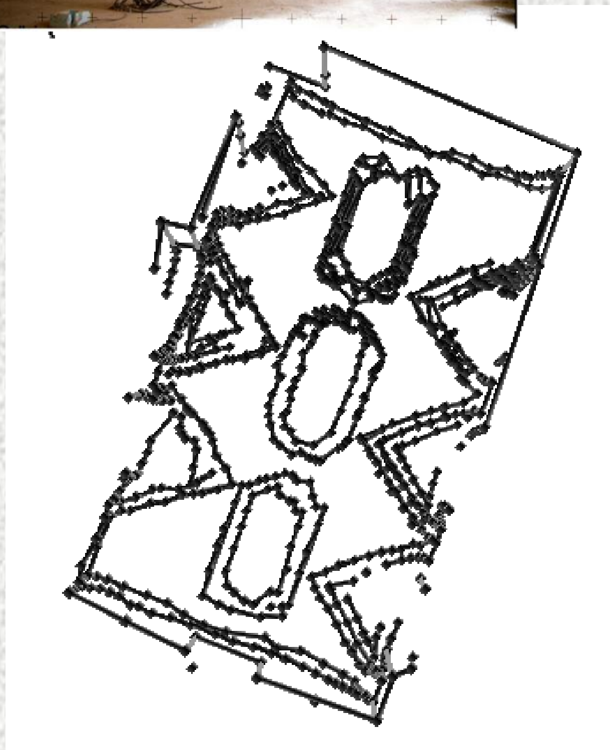
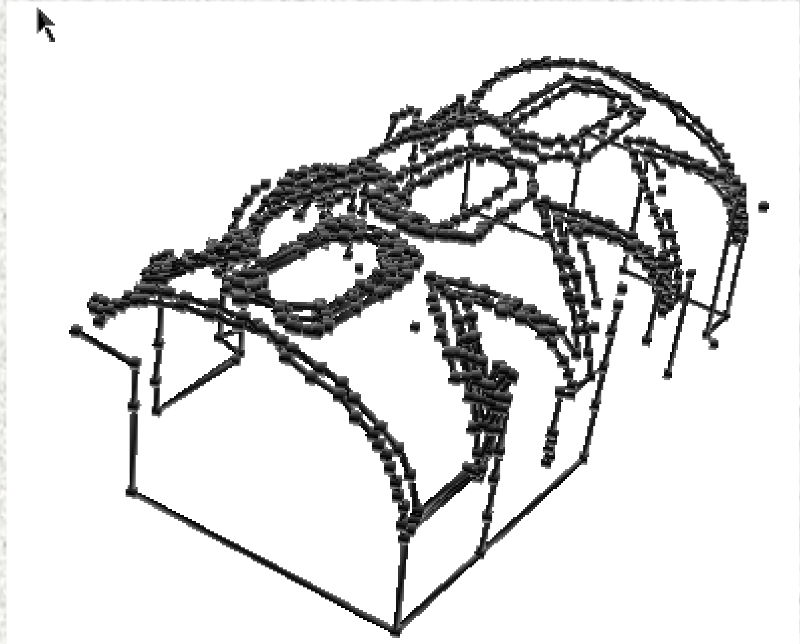




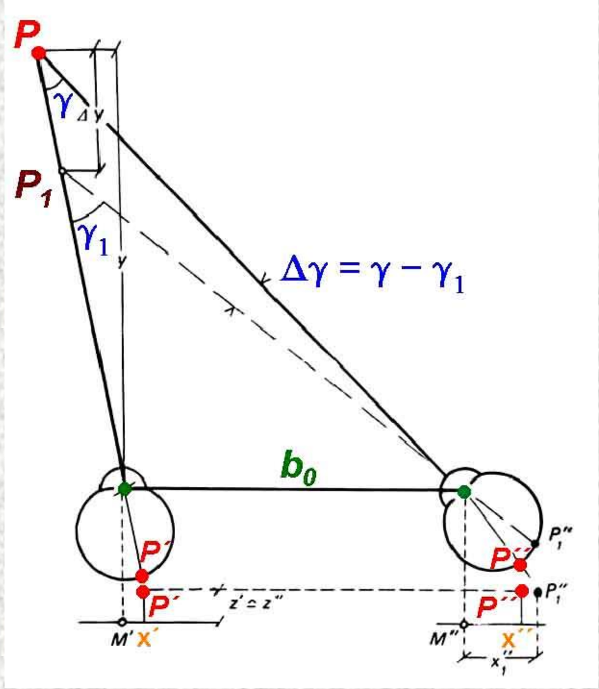


Documentation and modeling of the spatially distributed vault parts in the historical building by intersection photogrammetry (sw Photomodeler)



Stereophotogrammetry

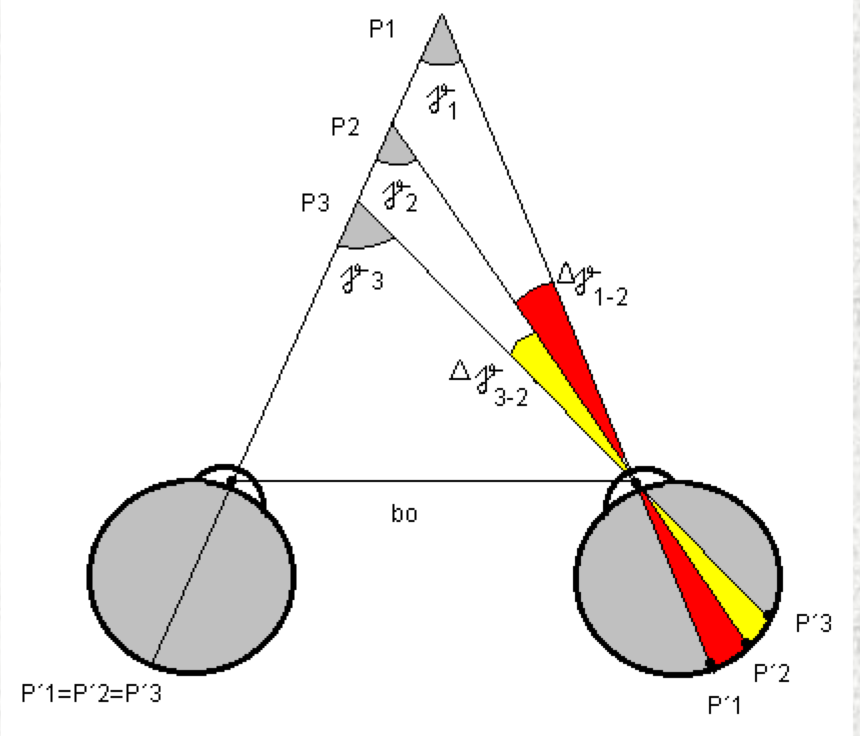


Stereophotogrammetric method was introduced at the beginning of the 20th century (dr.Pulfrich)

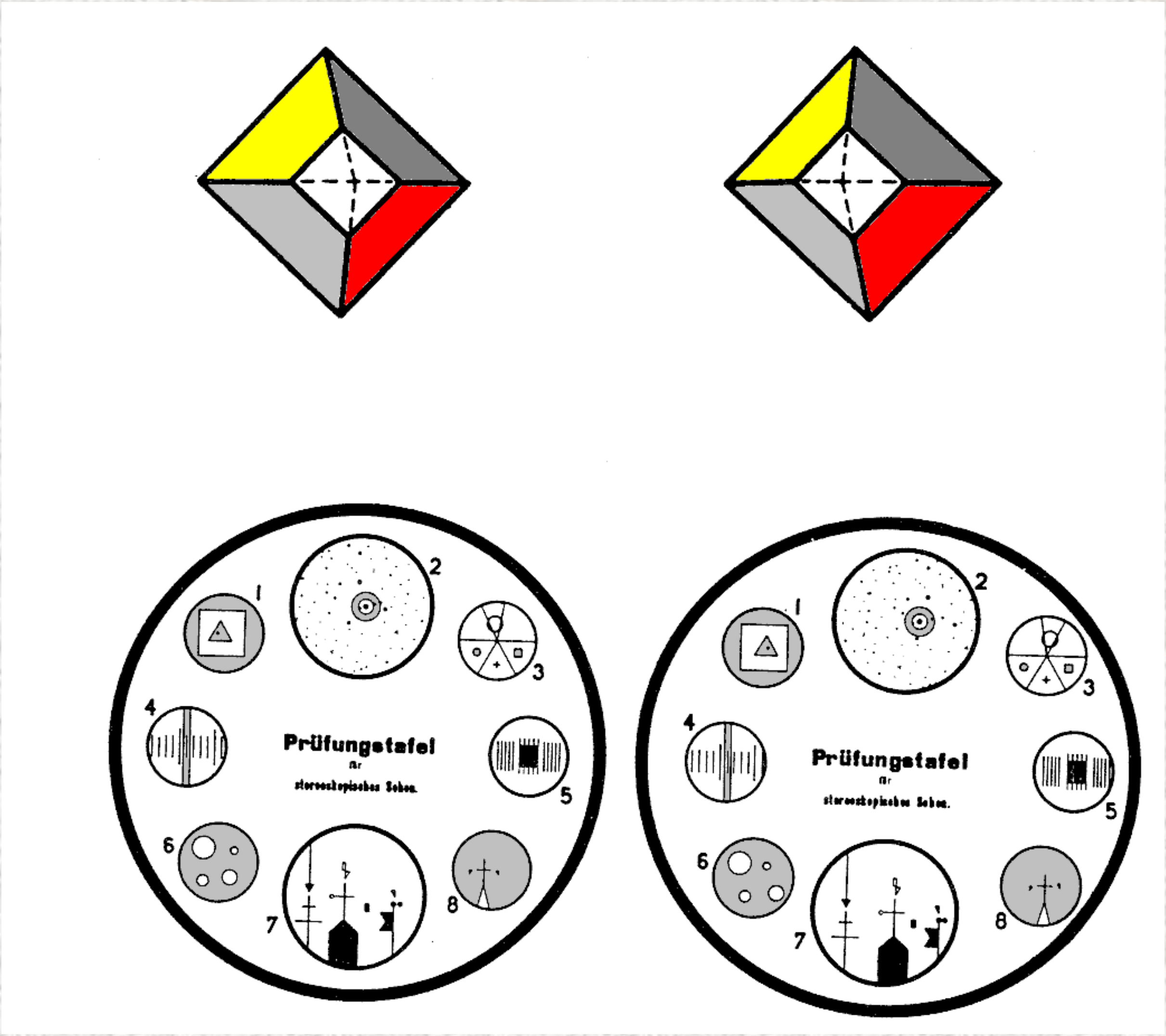
Evaluation based on stereoscopic perception → also for non-signalized points

Photogrammetric stereoscopic observation and evaluation:

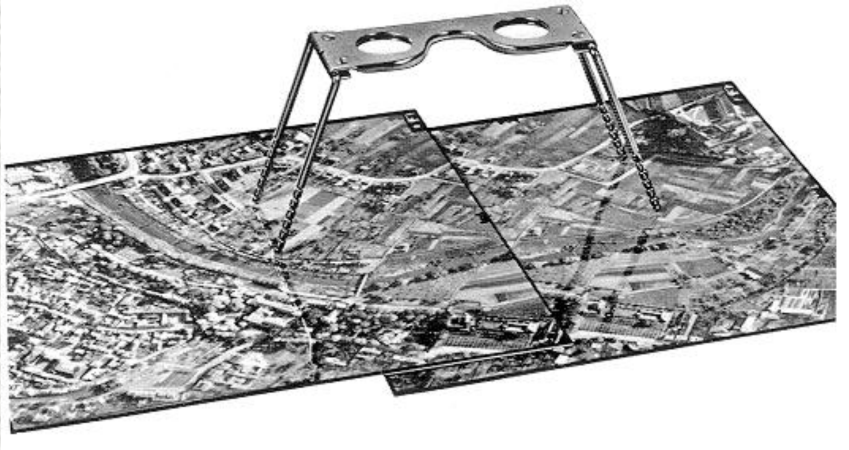
- artificial stereoscopic perception based on the natural perception of healthy eyes

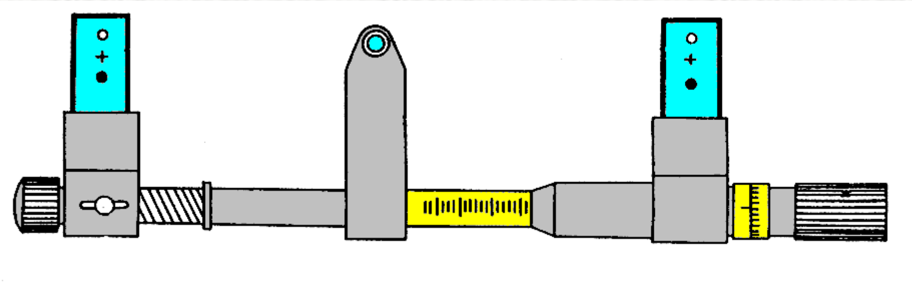


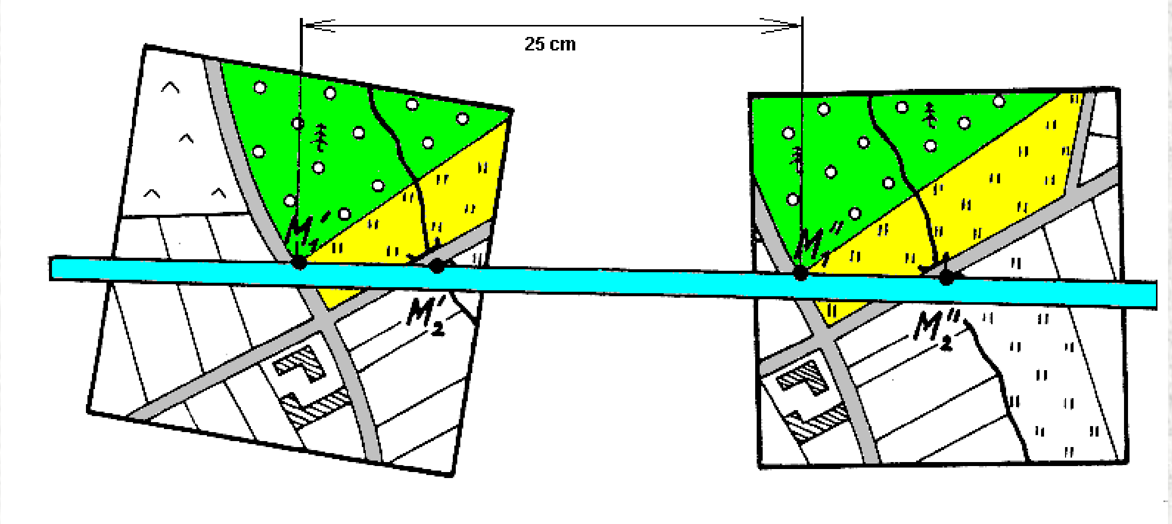
Stereoscopic tests



Stereoscopes

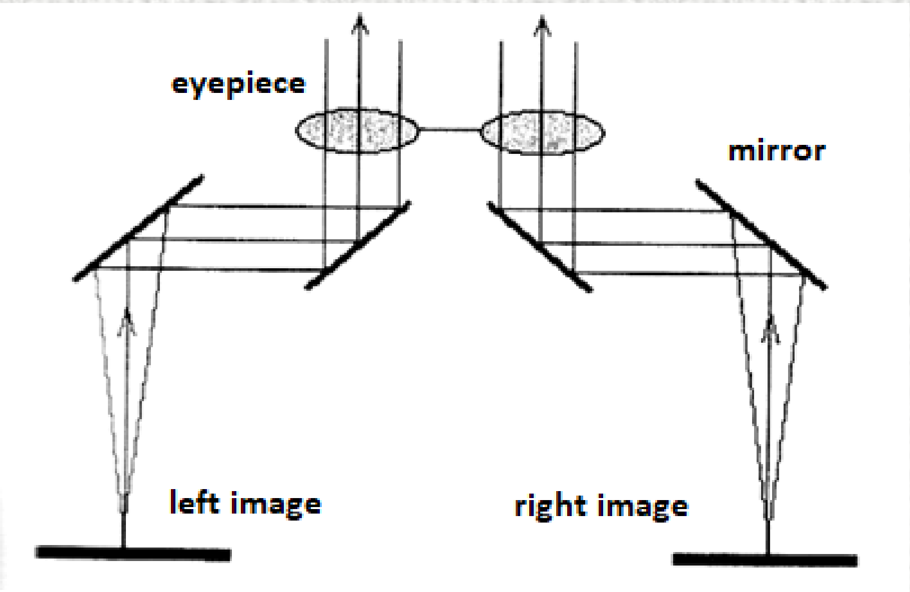


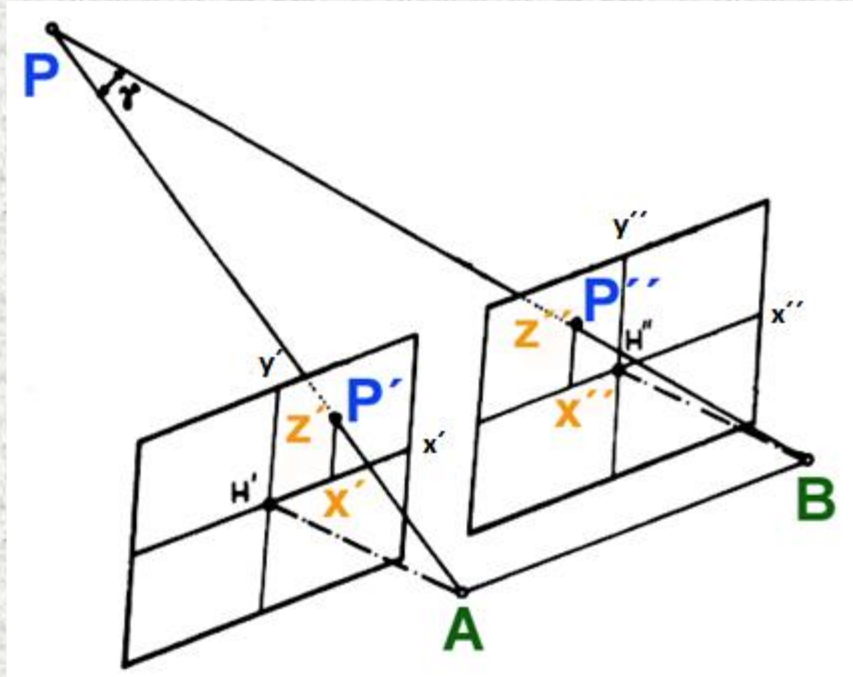




Stereophotogrammetry

Conditions for the formation of artificial stereoscopic perception: we observe both images separately at the same time





1. horizontal parallaxes *p* are non-zero, $\underline{p = x' - x'' = 0}$

2. vertical parallaxes *q* are zero $\underline{q = y' - y'' (= z' - z'') = 0}$

Stereophotogrammetry

Evaluation of image content based on stereoscopic perception, conversion of image coordinates to geodetic

older transfer procedure

$$x',y',(f) \Rightarrow x'_F, y'_F, z'_F \Rightarrow x, y, z \Rightarrow X, Y, Z$$

image fictitious sn. model geod. system

$\Rightarrow$ solved by a process of image orientations (step by step)

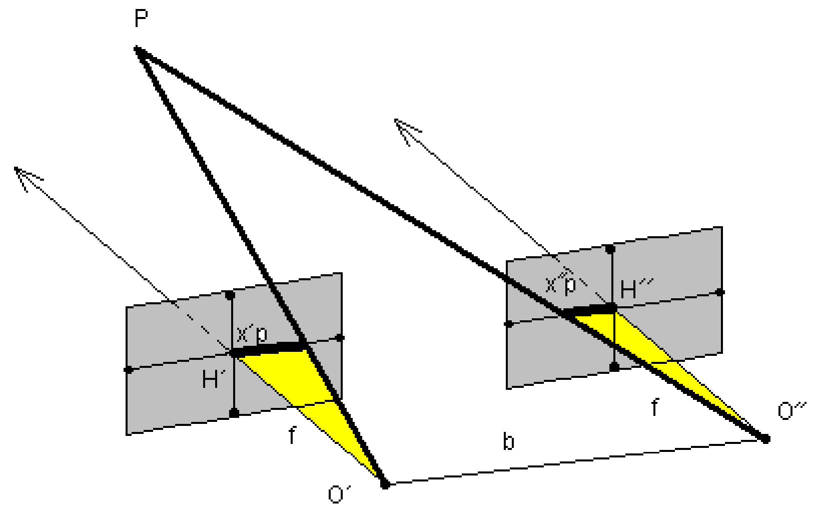
direct relationship

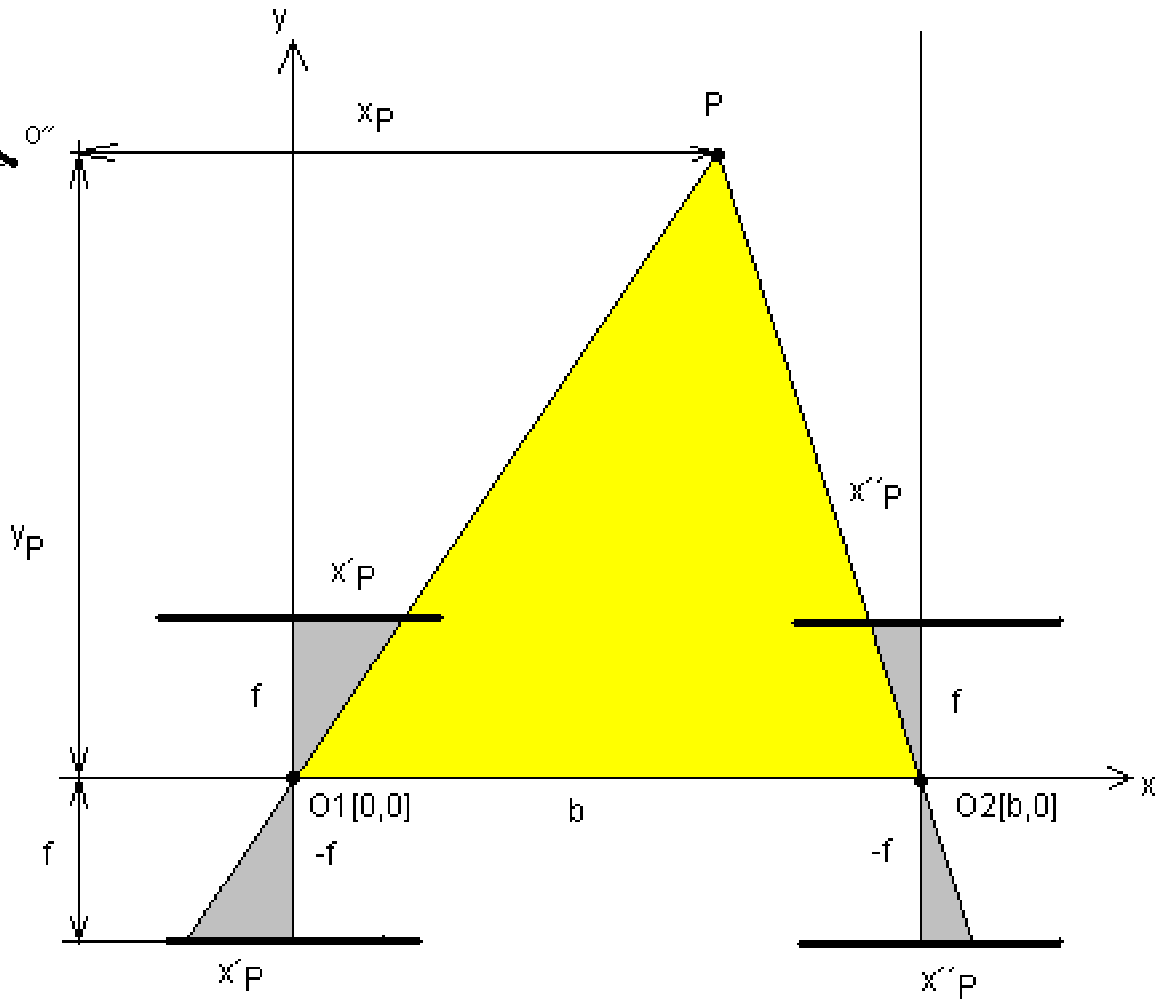
$$x',y',(f) \Rightarrow X, Y, Z$$

image geod. system

the basis of modern digital photogrammetry

Basic solution: terrestrial photogrammetry

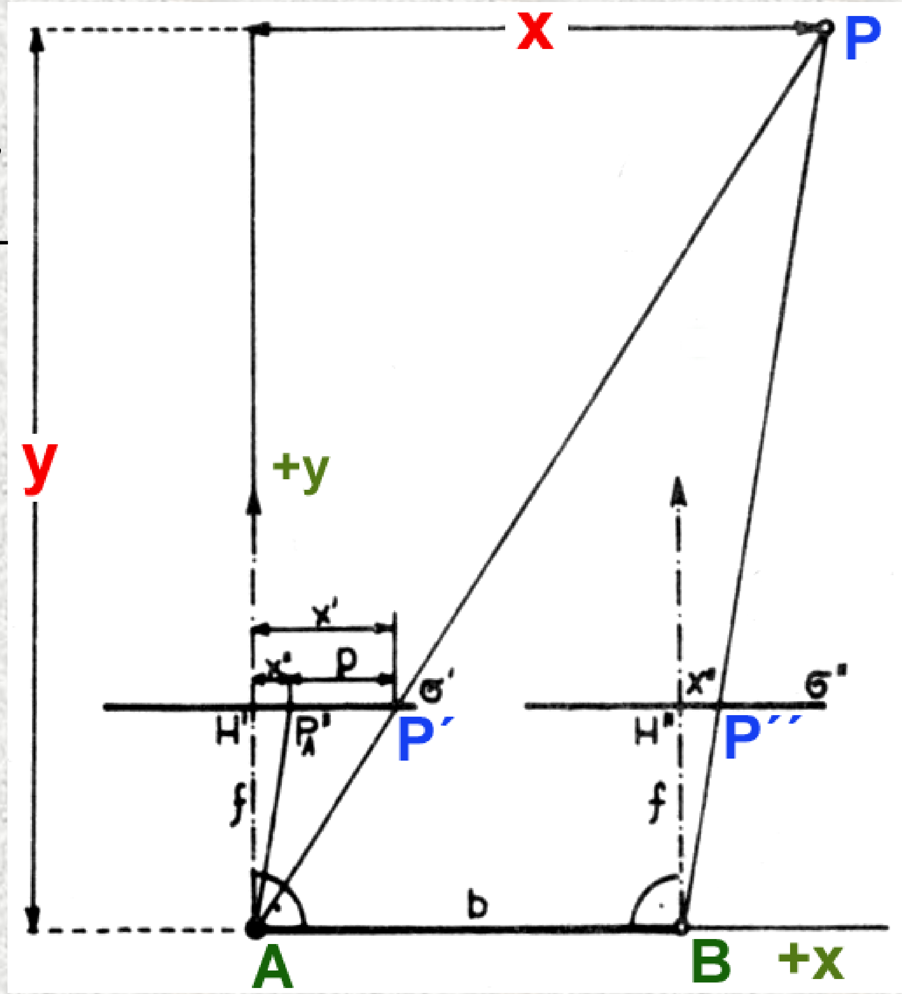




Terrestrial stereophotogrammetry - „normal case“ analoque solution

In terrestrial applications, standardization of the external orientation elements R=E can be ensured !

$$x = \frac{b \cdot x'}{p} \quad y = \frac{b \cdot f}{p} \quad z = \frac{b \cdot z'}{p}$$



b (AB) base

x y (z) model system

σ´, σ´´ image planes

x´, x´´,z´, z´´ image coordinates

p parallax

f camera constant

Accuracy of photogrammetry

$$\frac{y}{f} = \frac{b}{p}$$

$$x = \frac{b \cdot x'}{p} \qquad y = \frac{b \cdot f}{p} \qquad z = \frac{b \cdot z'}{p}$$

$$x = y\frac{x'}{f} \qquad z = y\frac{z'}{f}$$

$$dy = -\frac{b \cdot f}{p^2}dp, \quad p^2 = \left(\frac{b \cdot f}{y}\right)^2$$

$$dy = -\frac{y \cdot y}{b \cdot f}dp$$

$$y = \frac{b \cdot f}{p}, \qquad dy = \frac{f}{p}db + \frac{b}{p}df - \frac{bf}{p^2}dp$$

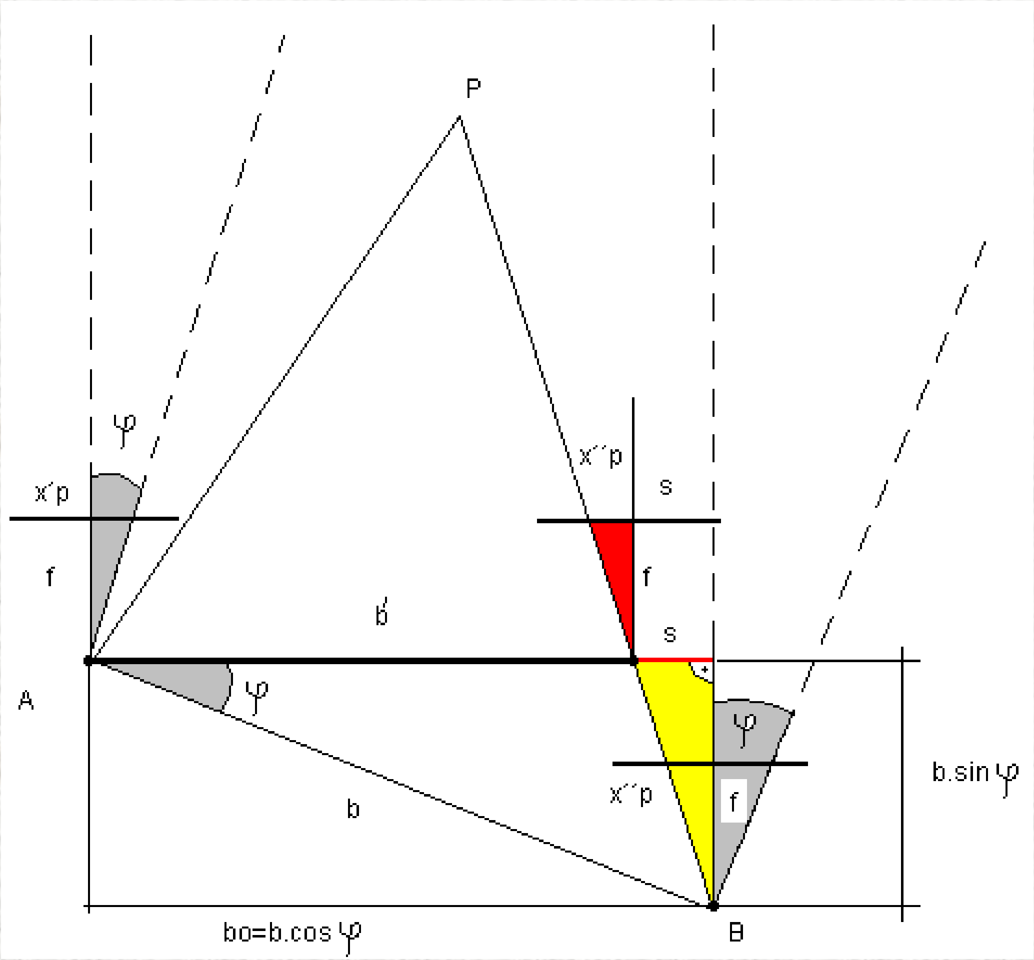
$$\boxed{m_y = \pm\frac{y \cdot y}{b \cdot f}m_P}$$

Rotated photogrammetric case, analoque solution

$$b' = b \cdot \left(\cos\varphi \pm \frac{x''_p}{f} \cdot \sin\varphi \right) \qquad \frac{s}{b \cdot \sin\varphi} = \frac{x''_p}{f}$$

$$b' = b_x \pm b_y \cdot \frac{x''_p}{f}$$

$$x = \frac{b' \cdot x'}{p} \qquad y = \frac{b' \cdot f}{p} \qquad z = \frac{b' \cdot z'}{p}$$



Inclined case, analoque solution

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{R}_\omega \cdot \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix}, \qquad \mathbf{R}_\omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{pmatrix}$$

Photogrammetric base

$$y = \frac{b \cdot f}{p} \qquad dy = -\frac{y \cdot y}{b \cdot f} dp$$

$$b = -\frac{y}{dy} \cdot \frac{y}{f} dp$$

where dp = 0.01mm = common mean error of horizontal parallax measurement, dy/y is the required accuracy of evaluation as relative error (e.g. 1/1000), f is the camera constant; p_{max} = 40-50mm

$$b_{\min} = y_{\max} \cdot \frac{y}{dy} \cdot \frac{dp}{f} \qquad b_{\min} = y_{\max} \cdot \frac{10mm}{f[mm]}$$

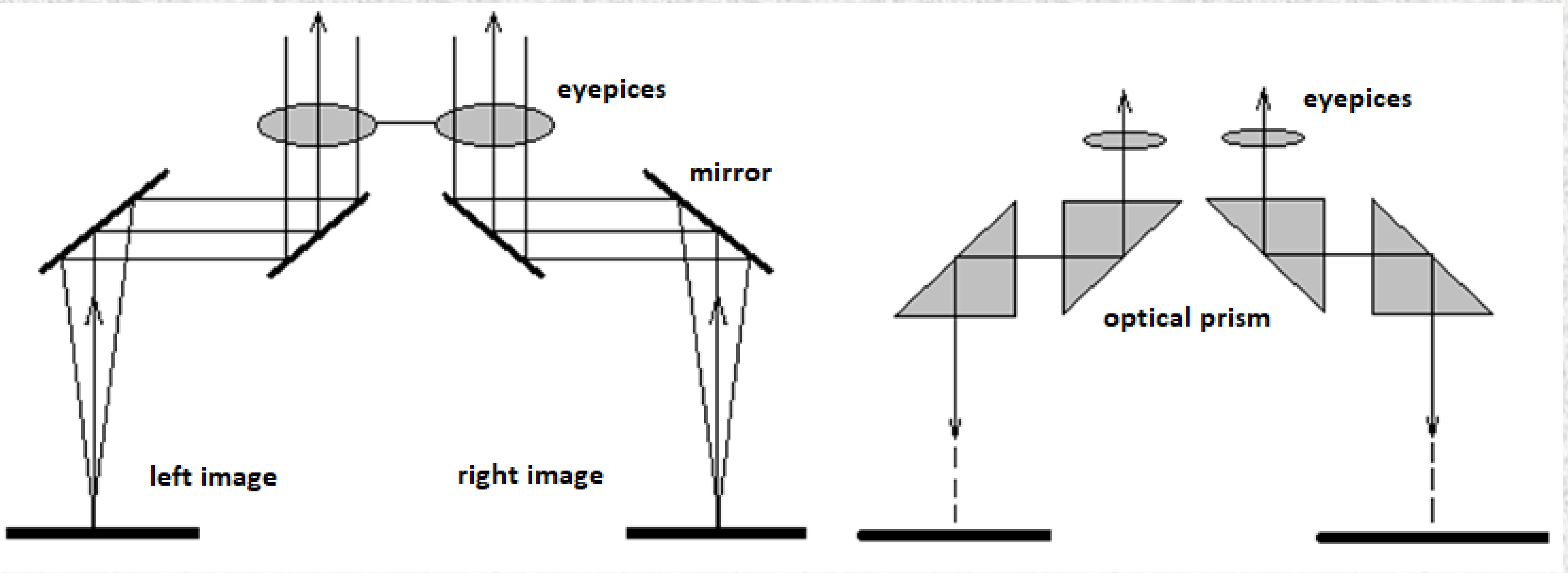
$$b_{\max} = y_{\min} \cdot \frac{p_{\max}}{f}$$

Photogrammetric bases for old cameras

Accuracy dy/y = 1/1000	*f [mm]*	*medium b*	*min. b*	*max. b*
PhoTheo 13x18	195	$b_{stř} = 1/10\ y_{stř}$	$b_{min} = 1/20\ y_{max}$	$b_{max} = (1/4 - 1/5)\ y_{min}$
RolleiMetric 6x6	40	1/2	1/4	4/5 - 1/1
RolleiMetric 6x6	80	1/3	1/8	2/3
UMK 10/1318	100	1/5	1/10	1/2
Nikon 100	17	1	1/2	2 - 3
Canon 20D	10	2	1	4 - 5
Canon 20D	22	1	1/2	2 - 3
Nikon 100	35	9/10	1/3	1-3/2
Canon 20D	85	1/3	1/9	1/2

Stereoscopic observation

stereoscopes



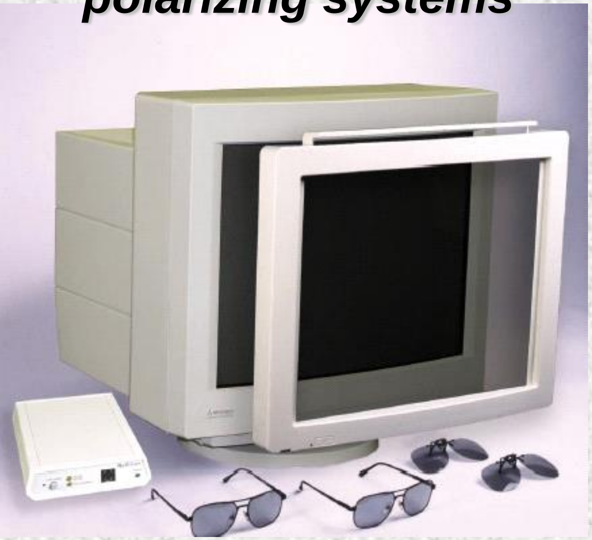
Hardware equipment for modern stereoscopy using a computer

stereoscope

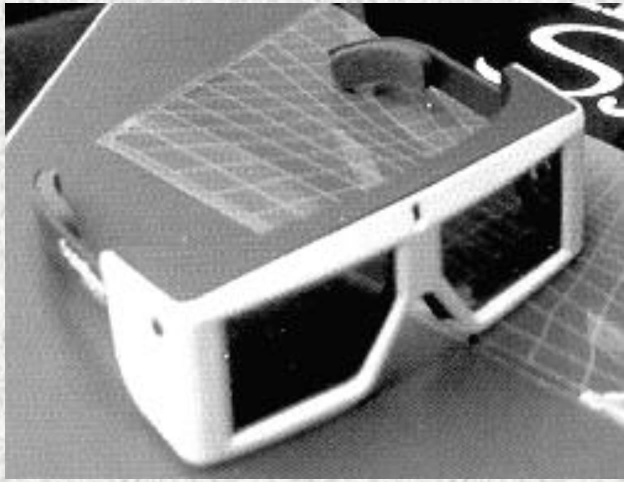
anaglyphs

Stereo mirror display

polarizing systems



crystal eyes



New solutions (LCD)

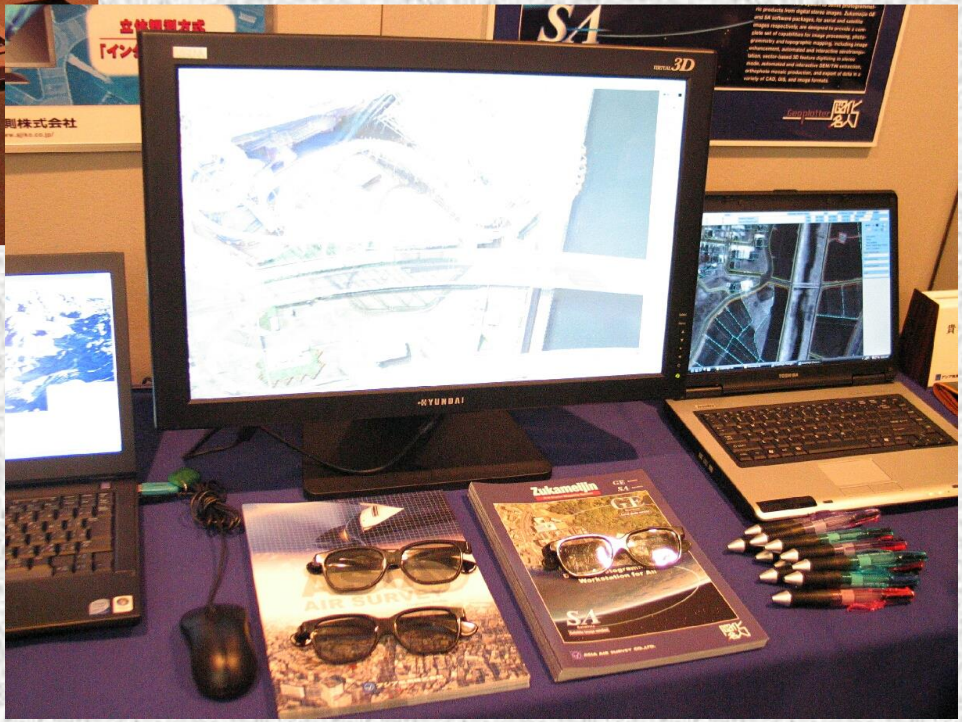


Image generation and capturing

Image generation

Photogrammetry is concerned with extracting measurement information from an image - this is captured by a **detector.**

Introduction:

$$M = S^E$$

where E is the number of elements, S is the number of possible states of one element and M is the total number of states (number of combinations). A unit of information is defined as the amount of information needed to write two different states of one element:

$$\log_2 M = E \cdot \log_2 S$$

where $\log_2 M$= amount of information [bit], (1byte=8bits). The basic unit of a digital image is the **pixel** (from the English *picture element*). **Principle : capturing radiation**

$$E = h \cdot \eta$$

Detectors:
-thermal -photonic
-integral -quantitative

$$Q = \Phi \cdot t \qquad \Phi = \frac{dQ}{dt}$$

where Q is the radiant energy, is the radiant flux and t is the time, h Planck const., η frequency

Image generation

Photographic material

In general, photographic material can be subdivided into:

positive material, negative material and inverse (slide, diapositive) material.

Photographic material is further divided into:

- •- coloured
- •- black and white (panchromatic, orthochromatic)
- •- inframaterial
- •- spectrozonal

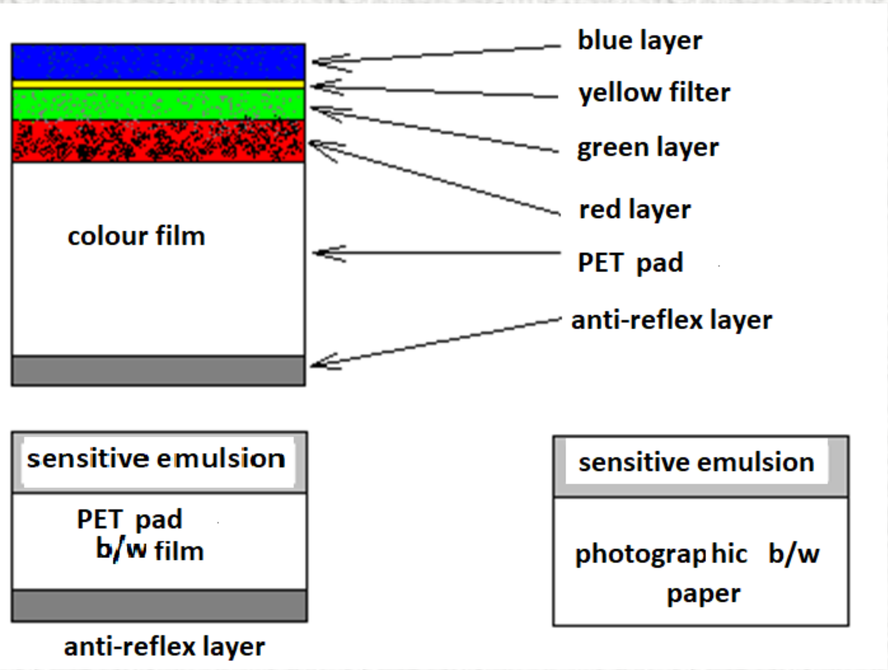


Image (photographs) parametres

- **general sensitivity**

$$100ASA = 21DIN$$

$$200ASA = 24DIN$$

$$400ASA = 27DIN$$

- **gradations**

• **indicates the relationship between the amount of light and the degree of blackening of the sensitive layer, or the blackening rate at constant illumination. The dependence of blackening on exposure is given by *the sensitometric curve*.**

- **Resolution (ReS)** /mm

$$ReS_{max} = \frac{1000 \cdot A}{2.4 \cdot \lambda \cdot f}$$

f/A	2.8	8.0	32.0
RS_{max} (line/mm)	298	83	26

Sensitometric curve

$$T = \frac{\Phi_{Passing}}{\Phi_{Emmited}}$$

where Φ is luminous flux, T is transmittance, 1/T is called transparency

$$D = \log\left(\frac{1}{T}\right) = -\log(T)$$

D is the density (optical density, degree of blackening).

$$E = \frac{\Phi}{S}$$

E is the illuminance [lux], S is the illuminated area.

$$H = E \cdot t$$

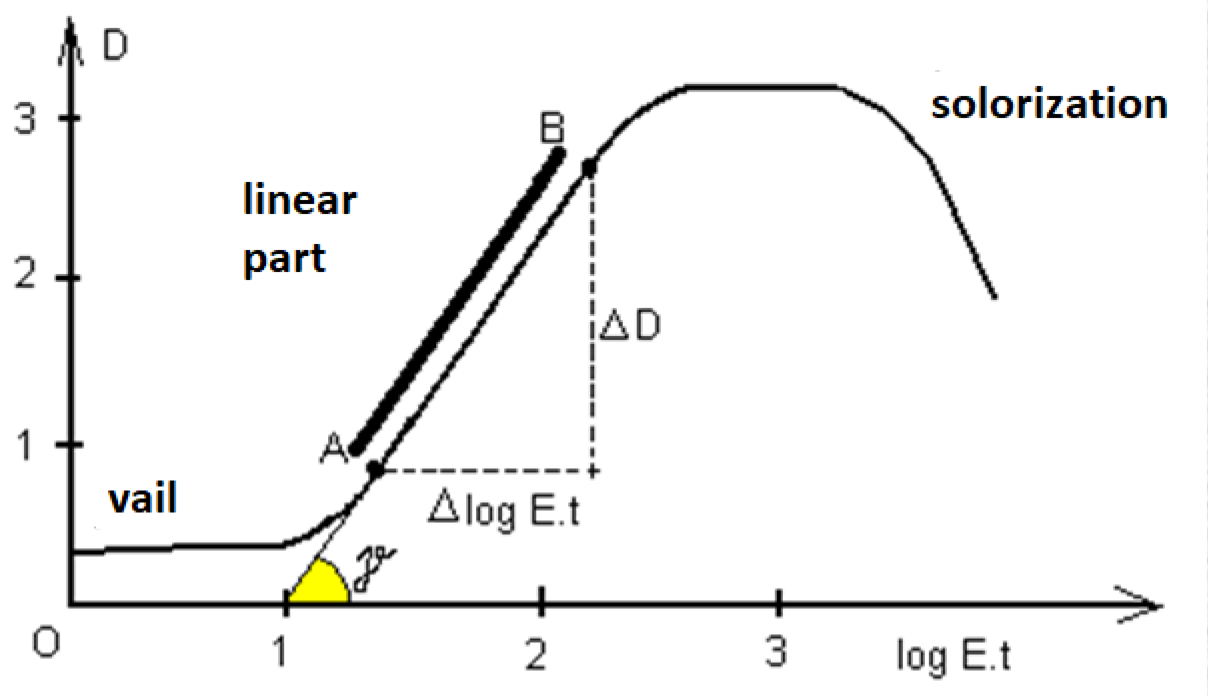
H is the exposure and t is the time (the exposure time).

$$G = \frac{\Delta D}{\Delta \log(H)}$$

Image (photographs) parametres

$$G = \mathrm{tg}(\gamma)$$

gradation (steepness) G
Gradation is a linear gradient parts of the sensitometric curve



G<1 ...(<45) ...soft working material
G=1 ...(=45) ...normal working material
G>1 ...(>45) ...steep working material (hard)

Image (photographs) parametres optical density measurement

Densitometer Meodenzi TRD01-Meopta (left), Zeiss Jena MD100 (right)

Image (photographs) process

Processing of photographic materials

Ordinary black and white photographic material is processed in the classical way:

a) material exposure

b) material development ; film strips in tanks, plates based in carriers in tubs; (developer used according to type 5-15 minutes)

c) intermittent bath (normally plain water, rinse)

d) settling (acid settler (fixer), 5-10 minutes)

e) washing (running water, 10-20 minutes)

(f) drying

The creation of digital image

A digital image is an image in digital form (expressed in numbers). It is created either primarily by digital capture devices or by scanning analogue images. A digital image consists of individual pixels (from the English *picture elements*) taking on certain values which are not arbitrary (determined by the technical possibilities of the computer and the coding).

Image size:

$$M = m \cdot n \cdot e \ [\mathrm{byte}]$$

The so-called image function describes the pixel value: $P[i,j] = \mathrm{f}(i,j)$

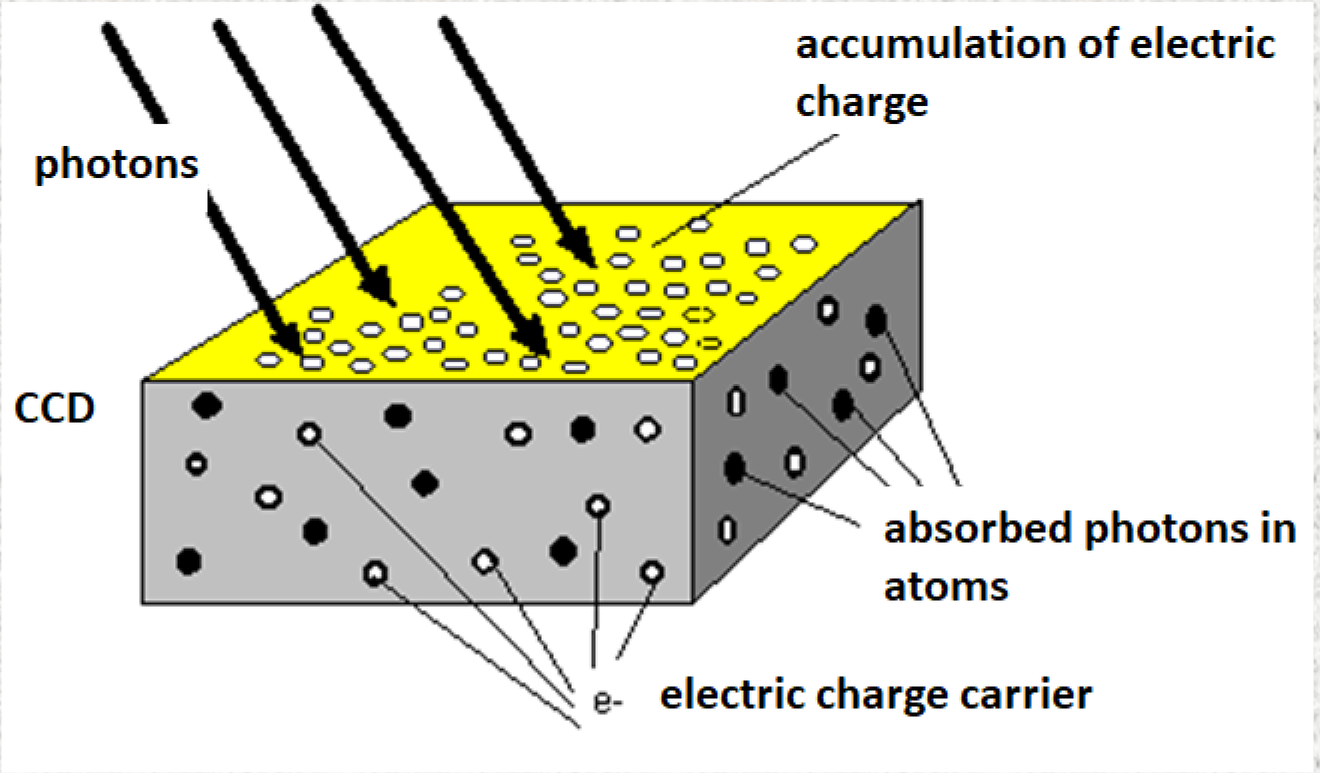
f(i,j)	f(i,j+1)	f(i,j+2)	f(i,j+3)	f(i,j+4)
f(i+1,j)	f(i+1,j+1)	f(i+1,j+2)	f(i+1,j+3)	f(i+1,j+4)
f(i+2,j)	f(i+2,j+1)	f(i+2,j+2)	f(i+2,j+3)	f(i+2,j+4)
.....				
.....				f(m,n)

Principe of used sensors

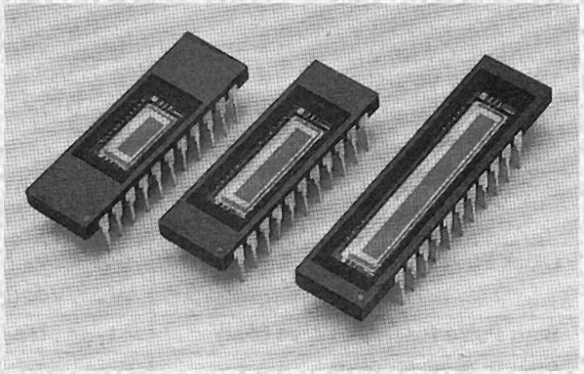
The most common type of detector is the CCD element. The name is derived from the name of the element in English "*Charge Coupled Device*".

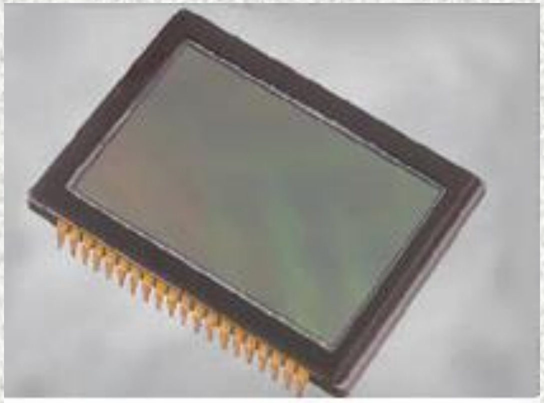
CMOS (*Complementary Metal Oxide Semiconductor*) is a transistor-based electronic componen Compared to a CCD, it is simpler to manufacture, smaller, up to 80% cheaper, and consumes les power than a CCD (only 1%!).

Photocell - the principle of its function is generally the same with CCD detectors, differing mainly in size



The creation of the digital image



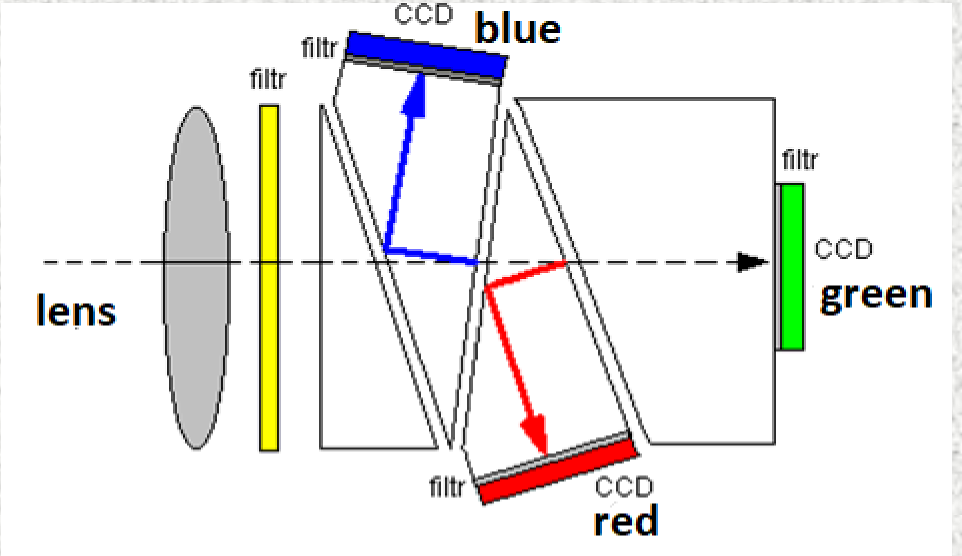


Linear and matrix CCD sensors

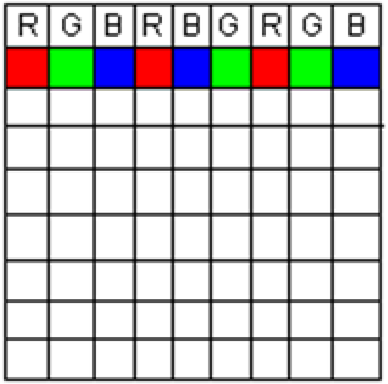
Creation of the colour digital image

Tree pass camera (Panasonic)

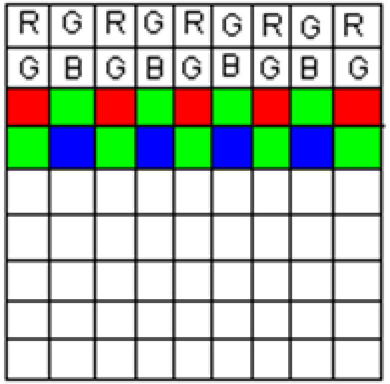
Three-sensor camera



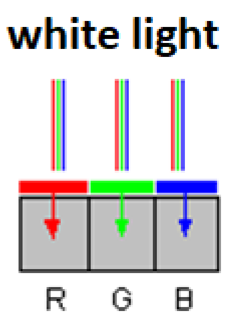
Single sensor (*one shot camera*)

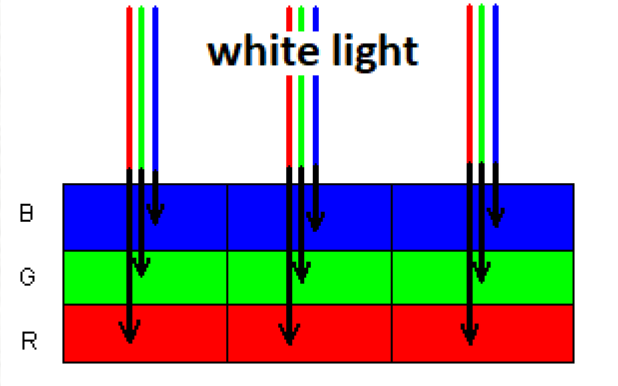


RGB linear mask



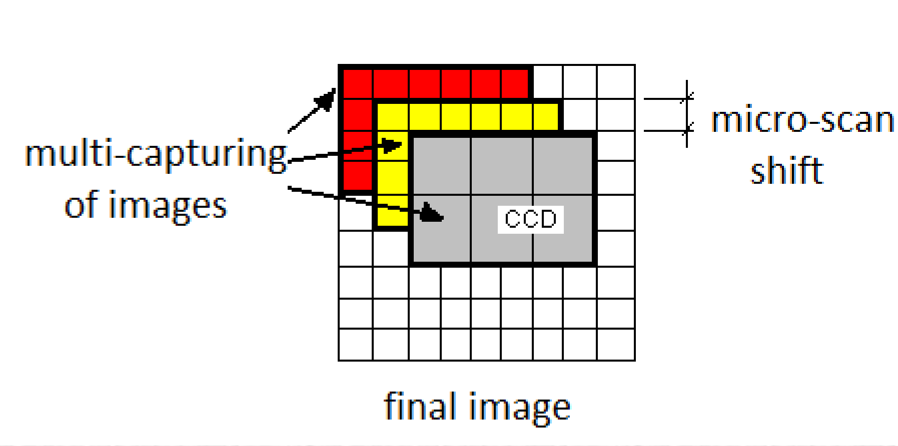
RGBG mosaic mask

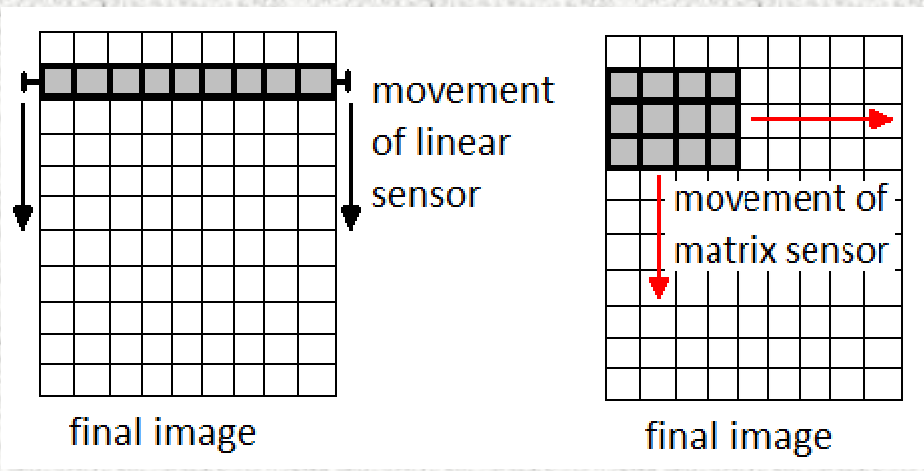




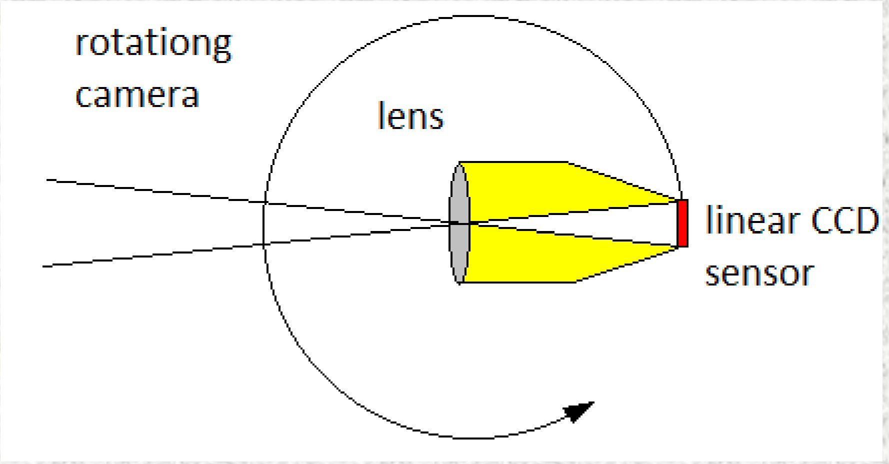
The special creation of the digital image

Micro-scanning



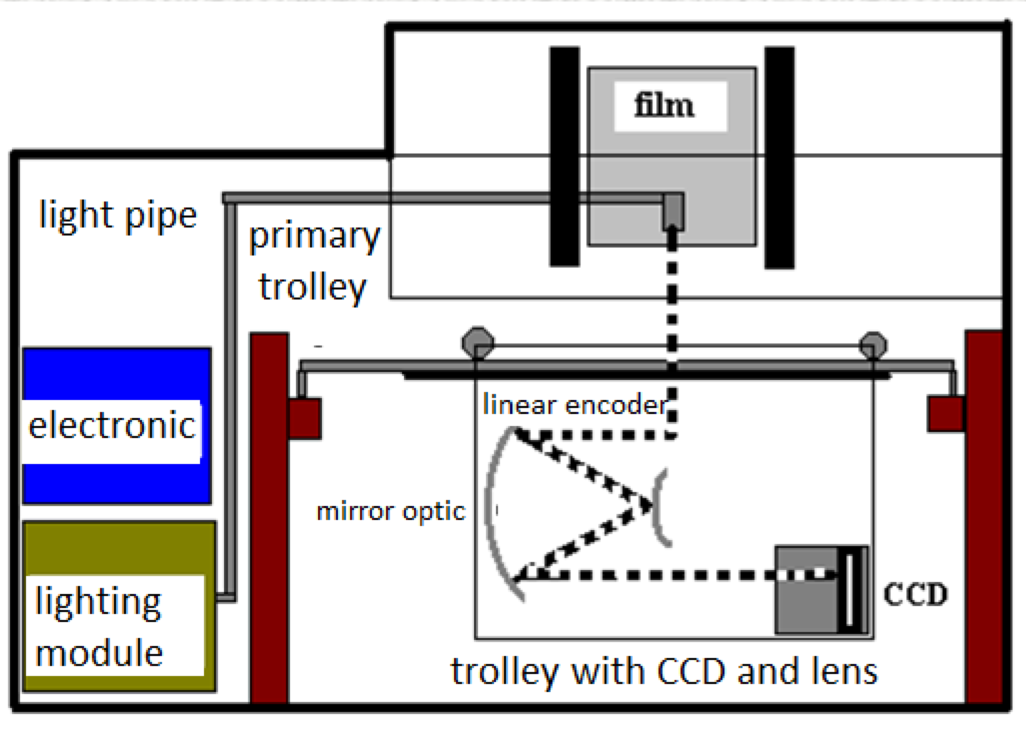


Macro-scanning



Panoramic camera

Secondary digital image creation: by film scanning

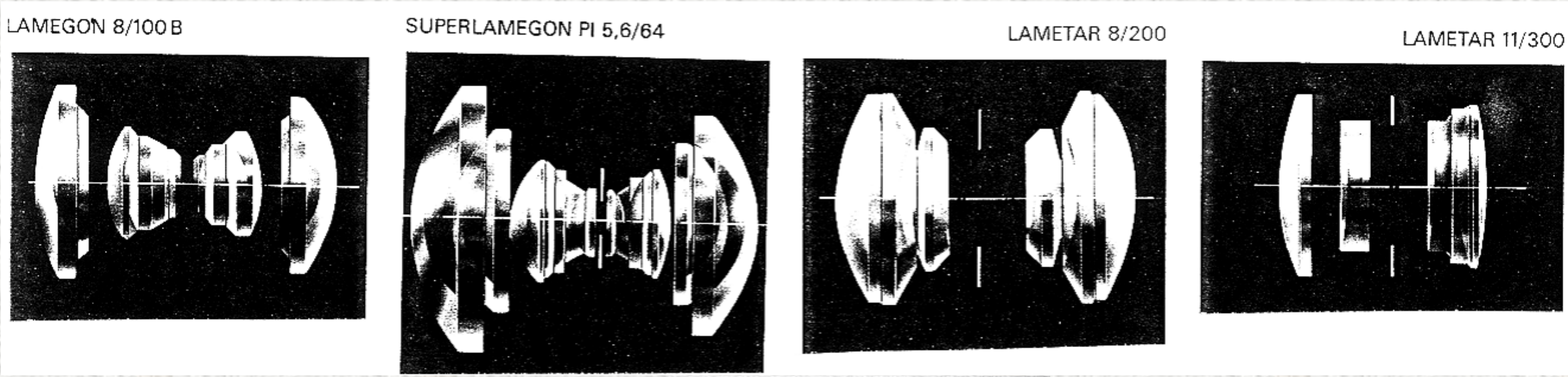


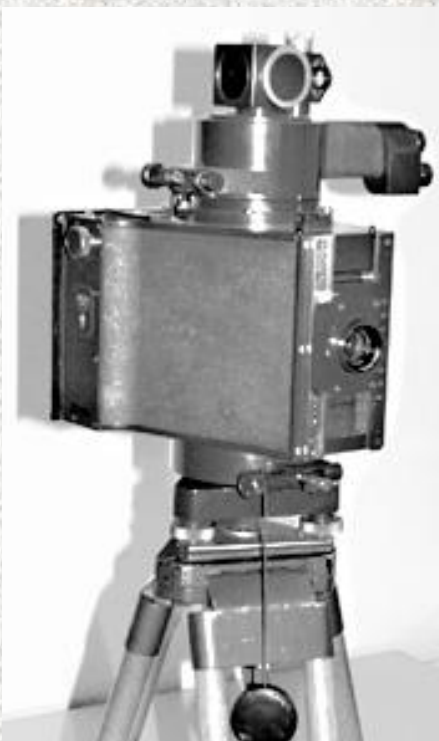
-typically nineties of 20th century till 2005

Scanner PhotoScan2001

Photogrammetric cameras

Photogrammetric terrestrial cameras





Photheo 13x18, UMK 201318 and Wild P31 camera

Terrestrial photogrammetric cameras

„Heavy“ cameras UMK

(glass plates or planfilm 13x18cm

Réseau camera RolleiMetric 6006

(roll film, 6x6cm)

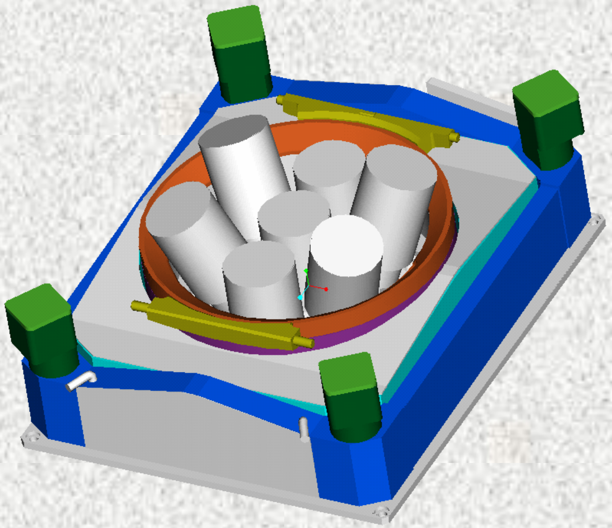
Photogrammetric analoque aerial cameras (till 2000)

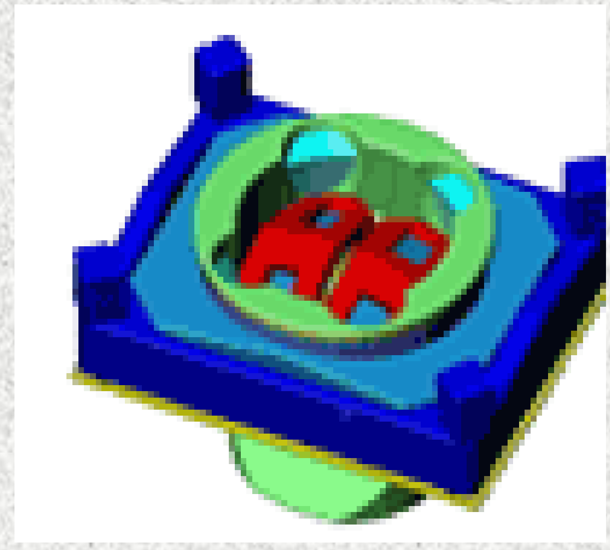
Leica RC 30 camera

Zeiss RMK-TOP camera and
TAS gyrostabilised platform

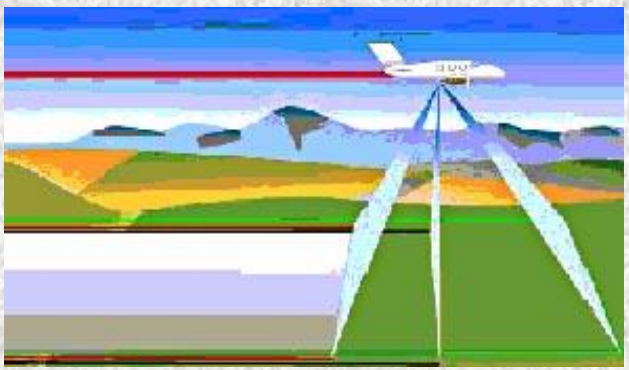
LMK camera (Zeiss Jena)

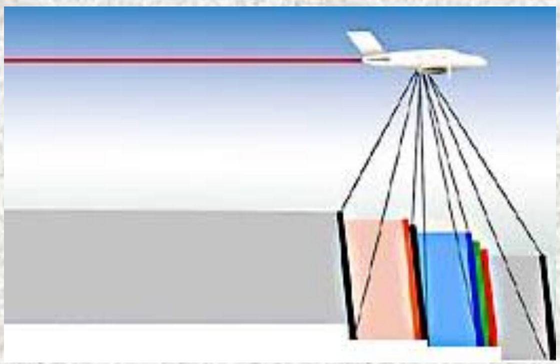
Digital aerial cameras (2000)





View of the DMC camera (top), original schematic of the gyrostabilized sensor part (left), current schematic (right)

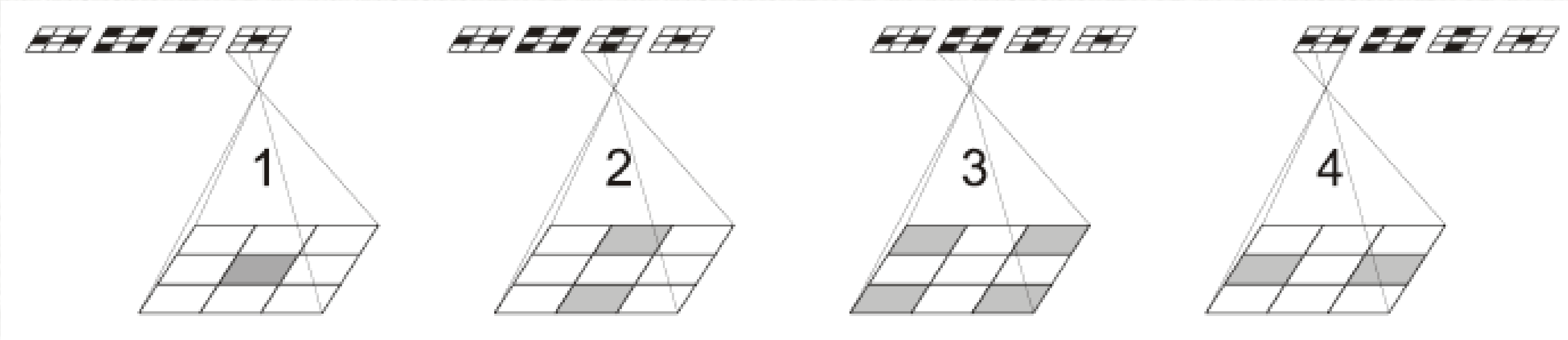




Sensor head ADS40 ADS40 surface scanning principle

Digital aerial camera UltraCam

UltraCam D camera (2005)



Professional Digital Cameras

Canon EOS and Sigma cameras, 2003, 3.1MPix

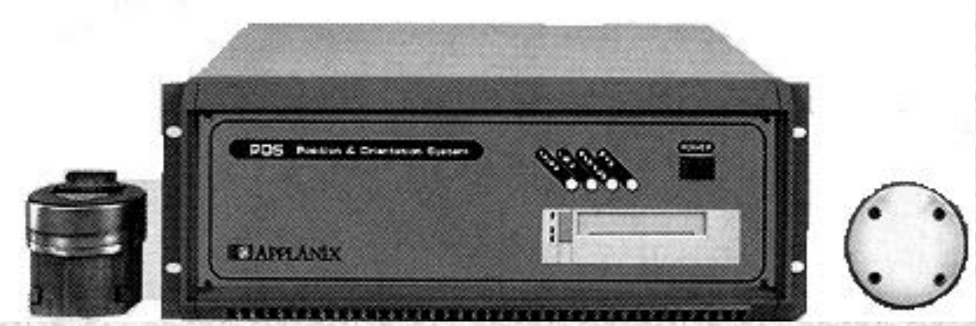
- ***Current resolution options 20-50MPix***

Canon EOS 5D Mark IV 2020, 30.4MPix (2020)

Aerial cameras - auxiliary equipment

Film *Motion Correction* (**FMC**) **: film movement at the moment of exposure, higher recording sharpness**

INS=IMU+GNSS (Inertial Navigation System, Inertial Measurement Unit*)* **equipment allows to determine the spatial orientation and position of the camera in time. The values of the exterior orientation and acceleration elements can be determined directly and stored with captured images**



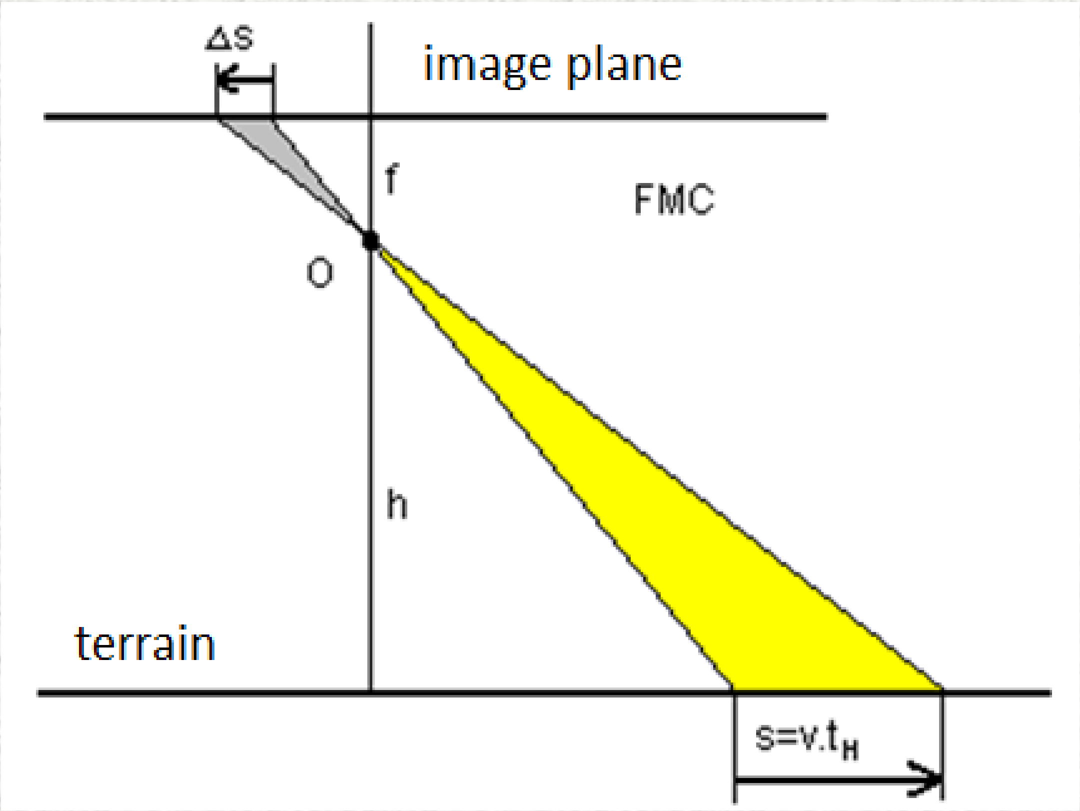
INS Applanix, 2000

etc.

Small and more precise INS Applanix, Trimble etc.2020

Others: altimeter, image overlay controller,

Image blurring

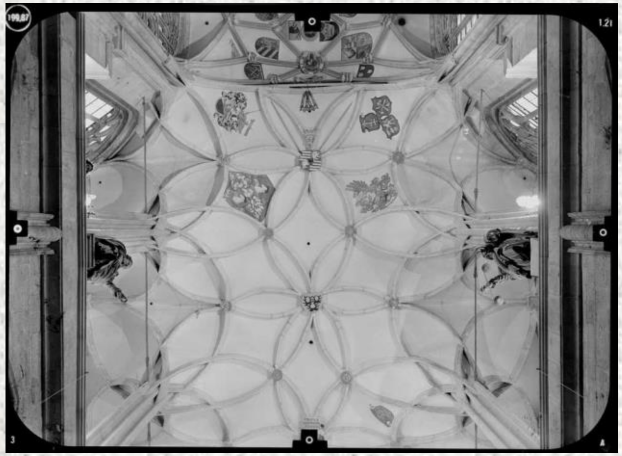


$$m_S = \frac{h}{f}, \Delta s = \frac{t_H \cdot v}{m_S}$$

Image blurring origin; example: m_{image} *=10 000,* t_H *=0.01s, v=360km.h*$^{-1}$ *: s=1/10000m=100m, i.e. a compensating shift at 10mm/s is necessary.*

Classical photogrammetric images

fiducial marks



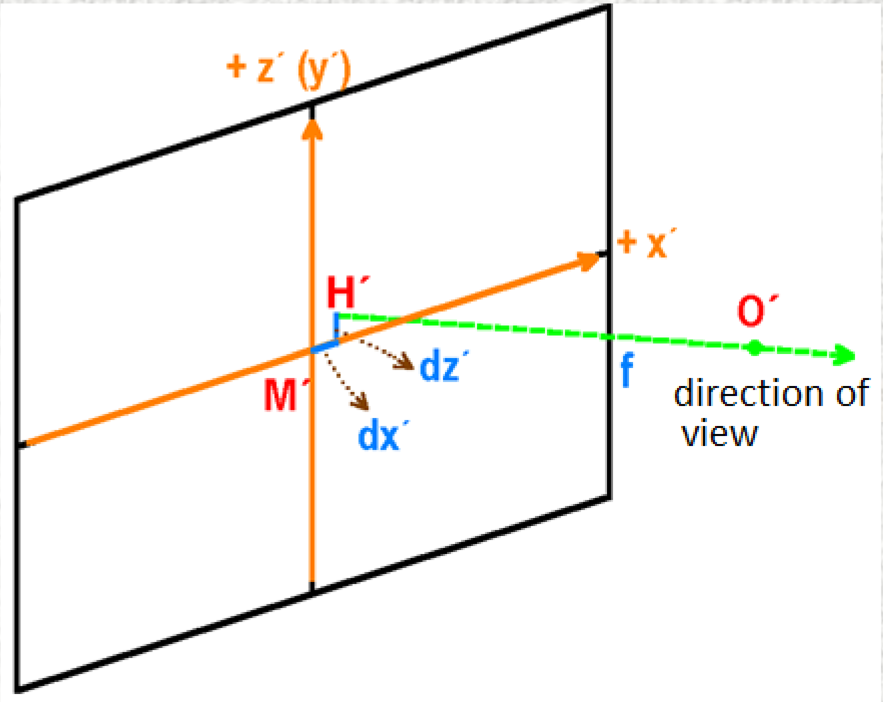
Satellite photogrammetry

Ikonos 1, 1999 (1m PAN, 4m MSS);
today's civilian satellites geometric resolution 0.31-0.5m PAN

Calibration of cameras

Metric analoque camera images

Known elements of interior orientation, known position of fiducial marks



Position of the image main point H´

- coordinates dx´, dy´ (dz´) with respect to M´ (to 0.01 mm)

camera constant *f*

- (to 0.01mm)

Knowledge of the distortion

- (if significant, i.e. greater than 0.01mm)

Calibration of cameras

The main steps of the validation and control for older conventional analogue terrestrial cameras are:

•checking *f*, *dx´*, *dz´* - by calculation or in the optical laboratory
•checking the perpendicularity of the vertical axis to the axis of the libel (level the place where the axis and the centre of the projection point)
•orientation device - the aiming axis of the telescope should be flush with the photo-camera's axis of view; the control is performed by aiming at a certain point and photographing it; for measuring its position on the image it should be shown in *H´* (beware of eccentricity!)
•vertical position of the frame (shooting two plumb lines)
•clamping device - checks whether the plate or film is actually pressed accurately against the marking frame; in case of non-adhesion, there are measurable differences in the position of the fiducial marks (tenths of mm)
•the plane of the image is to be parallel to the plane of the frame *(=0* should apply), a photograph of two plumb lines is taken and the distances between the hinges at the top and bottom are measured on the images
•rectification of calipers (a common method see geodesy)
•targetting device - like an ordinary theodolite (see surveying)

•For digital cameras, the whole process is limited to finding the interior orientation elements by calculating them from the calibration field and then usually to professional cleaning of the camera in the service (cleaning of the optics - lenses are often susceptible to dust particles due to their complexity, cleaning of the sensor from dust)

Methods for determining the elements of interior orientation

1. Laboratory methods
2. Measurement and numerical determination

a) determination of the elements of interior orientation without adjustment

(b) methods for determining the elements of interior orientation with adjustment

● Gruber method ● Hugershoff method ● Baeschlin method

c) simultaneous determination of the elements of interior orientation and exterior orientation by means of Direct Linear Transformation (*DLT*)

d) simultaneous determination of the interior and exterior orientation elements in analytical methods (e.g. complex method, see below)

Procedure for determining interior orientation elements for analogue cameras (not used now)

Taking a test image:

•Carefully horizon the test camera with the rectified libel and take an image of the space with well identifiable distant points, preferably distributed along the horizon.

Mesuring with theodolite:

•With the theodolite centered over the center of the entrance pupil of the tested photocamera it is necessary to measure angles or direction on preffered distant good visible points.

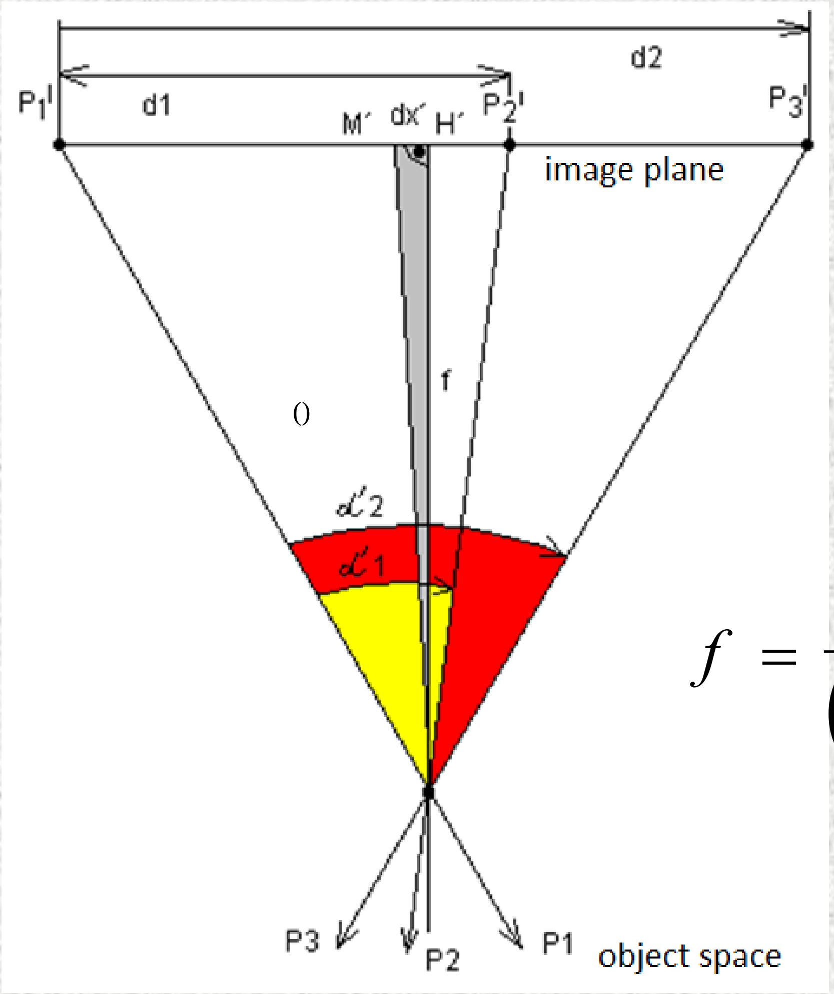
Measurement of image coordinates:

•For the selected points, the image coordinates are measured on the comparator (read to at least 0.01 mm depending on the type of instrument used).

Calculation:

•The actual calculation can be done quickly without adjustment, the methods with adjustment are done on a personal computer (a special software was made for example on the CTU FCE at the Lab. of photogrammetry in nineties of 20th century.

Determination of elements of interior orientation without adjustment (old method)



$$d_1 = x_2' - x_1', d_2 = x_3' - x_1'$$

$$\alpha_1 = \psi_2 - \psi_1, \quad \alpha_2 = \psi_3 - \psi_1$$

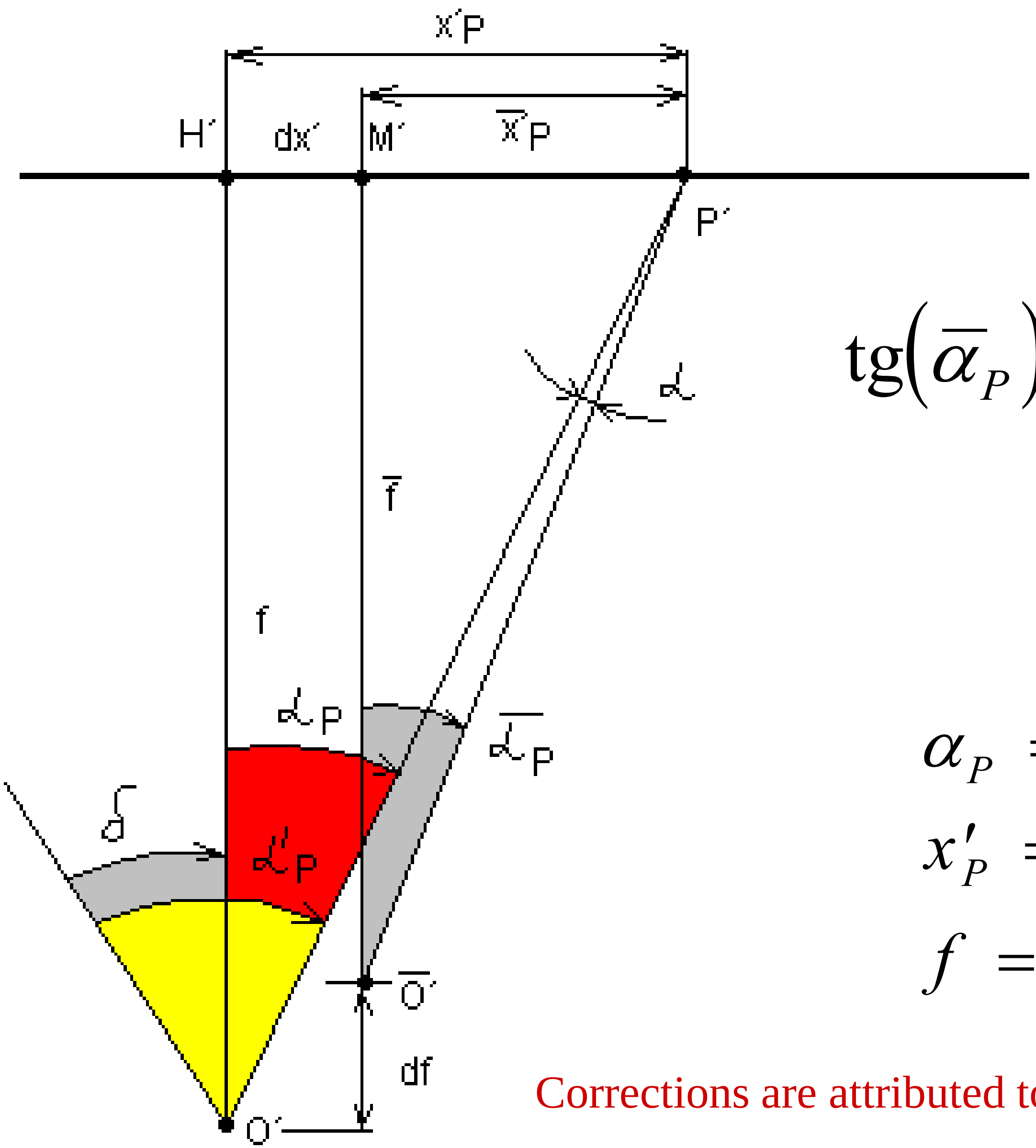
$$\overline{H'P_1'} = f \cdot \frac{d_2 \cdot \operatorname{cotg}\alpha_2 - d_1 \cdot \operatorname{cotg}\alpha_1}{(d_2 - d_1)}, \overline{M'P_1'} = x'_{P_1}$$

$$f = \frac{d_1 d_2 (d_2 - d_1) \cdot (\operatorname{cotg}\alpha_1 - \operatorname{cotg}\alpha_2)}{(d_2 \cdot \operatorname{cotg}\alpha_2 - d_1 \cdot \operatorname{cotg}\alpha_1)^2 + (d_2 - d_1)}$$

$$d_x' = \overline{H'P_1'} - \overline{M'P_1'}$$

Accurate methods of numerical determination of interior orientation elements (old technology)

Gruber method



$$\mathrm{tg}\left(\bar{\alpha}_P\right) = \frac{\overline{x'_P}}{\bar{f}}, \quad \mathrm{tg}\left(\alpha_P\right) = \frac{x'_P}{f}$$

$$\alpha_P = \alpha'_P - \delta = \bar{\alpha}_P - d\alpha$$

$$x'_P = \overline{x'_P} + dx'$$

$$f = \bar{f} + df$$

Corrections are attributed to the measured directions

Gruber method

$$\mathrm{tg}(\alpha_P) = \mathrm{tg}(\bar{\alpha}_P + d\alpha) = \frac{x'_P}{f} = \frac{\overline{x'_P} + dx'}{\bar{f} + df}$$

$$\mathrm{tg}(\bar{\alpha}_P) + \frac{1}{\cos^2(\bar{\alpha}_P)} d\alpha = \frac{\overline{x'_P}}{\bar{f}} + \frac{1}{\bar{f}} dx' - \frac{\overline{x'_P}}{\bar{f}^2} df$$

$$\frac{1}{\cos^2(\bar{\alpha}_P)} = 1 + \mathrm{tg}^2(\bar{\alpha}_P) = 1 + \frac{\overline{x'_P}^2}{\bar{f}^2} = \frac{\bar{f}^2 + \overline{x'_P}^2}{\bar{f}^2}$$

$$\frac{\overline{x'_P}}{\bar{f}} + \frac{1}{\cos^2(\bar{\alpha}_P)} d\alpha = \frac{\overline{x'_P}}{\bar{f}} + \frac{1}{\bar{f}} dx' - \frac{\overline{x'_P}}{\bar{f}^2} df \qquad / \cdot \frac{\bar{f}^2}{\left(\overline{x'_P}^2 + \bar{f}^2\right)}$$

$$d\alpha = \frac{\bar{f}}{\left(\overline{x'_P}^2 + \bar{f}^2\right)} dx' - \frac{\overline{x'_P}}{\left(\overline{x'_P}^2 + \bar{f}^2\right)} df$$

$$\alpha_P = \alpha'_P - \delta = \bar{\alpha}_P - d\alpha$$

$$d\alpha = \alpha'_P - \delta - \bar{\alpha}_P$$

$$v_{\alpha_P} = \delta + \frac{\bar{f}}{\left(\overline{x'_P}^2 + \bar{f}^2\right)} dx' - \frac{\overline{x'_P}}{\left(\overline{x'_P}^2 + \bar{f}^2\right)} df + (\bar{\alpha}_P - \alpha'_P)$$

Hugershoff method

attributes corrections to the measured image coordinates:

$$\mathrm{tg}\left(\bar{\alpha}_P - \delta\right) = \frac{\overline{x'_P} + v_{x'} + dx'}{\bar{f} + df}$$

$$\mathrm{tg}\left(\alpha'_P\right) - \frac{1}{\cos^2\left(\alpha'_P\right)}\delta = \frac{\overline{x'_P}}{\bar{f}} + \frac{1}{\bar{f}}\left(v_{x'} + dx'\right) - \frac{\overline{x'_P}}{\bar{f}^2}df$$

the following applies:

$$\mathrm{tg}\left(\bar{\alpha}_P\right) = \frac{\overline{x'_P}}{\bar{f}} \qquad \mathrm{tg}\left(\alpha'_P\right) - \mathrm{tg}\left(\bar{\alpha}_P\right) \cong \frac{1}{\cos\left(\bar{\alpha}_P\right)}\left(\alpha'_P - \bar{\alpha}_P\right) \qquad \frac{1}{\cos^2\left(\alpha'_P\right)} \cong \frac{1}{\cos^2\left(\bar{\alpha}_P\right)}$$

$$\left(\alpha'_P - \bar{\alpha}_P - \delta\right)\frac{1}{\cos^2\left(\bar{\alpha}_P\right)} = \frac{1}{\bar{f}}\left(v_{x'} + dx'\right) - \frac{\overline{x'_P}}{\bar{f}^2}df$$

$$\left(\alpha'_P - \bar{\alpha}_P - \delta\right) = \frac{\bar{f}}{\bar{f}^2 + \overline{x'}^2_P}\cdot v_{x'} + \frac{\bar{f}}{\bar{f}^2 + \overline{x'}^2_P}\cdot dx' - \frac{\overline{x'_P}}{\bar{f}^2 + \overline{x'}^2_P}df$$

$$v_{x'} = -dx' + \frac{\overline{x'_P}}{\bar{f}}df - \frac{\bar{f}^2 + \overline{x'}^2_P}{\bar{f}}\cdot\delta - \frac{\bar{f}^2 + \overline{x'}^2_P}{\bar{f}}\left(\alpha'_P - \bar{\alpha}_P\right)$$

Direct Linear Transformation (DLT) still used

$$x' = x'_0 - f\,\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

$$z' = z'_0 - f\,\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

$$x' = \frac{\hat{a}_1 X + \hat{a}_2 Y + \hat{a}_3 Z + \hat{a}_4}{\hat{c}_1 X + \hat{c}_2 Y + \hat{c}_3 Z + \hat{c}_4},\ \hat{a}_1 = x'_0 r_{13} - f \cdot r_{11}, \hat{a}_2 = x'_0 r_{23} - f \cdot r_{21}, \hat{a}_3 = \ldots$$

$$z' = \frac{\hat{b}_1 X + \hat{b}_2 Y + \hat{b}_3 Z + \hat{b}_4}{\hat{c}_1 X + \hat{c}_2 Y + \hat{c}_3 Z + \hat{c}_4}$$

$$x' = \frac{a_1 X + a_2 Y + a_3 Z + a_4}{c_1 X + c_2 Y + c_3 Z + 1}$$

$$z' = \frac{b_1 X + b_2 Y + b_3 Z + b_4}{c_1 X + c_2 Y + c_3 Z + 1}$$

$$f_x = \sqrt{\left(\left(a_1^2 + a_2^2 + a_3^2\right)d^2 - x_0'^2\right)}$$
$$f_z = \sqrt{\left(\left(b_1^2 + b_2^2 + b_3^2\right)d^2 - z_0'^2\right)}$$

$$d^2 = \frac{1}{c_1^2 + c_2^2 + c_3^2}$$

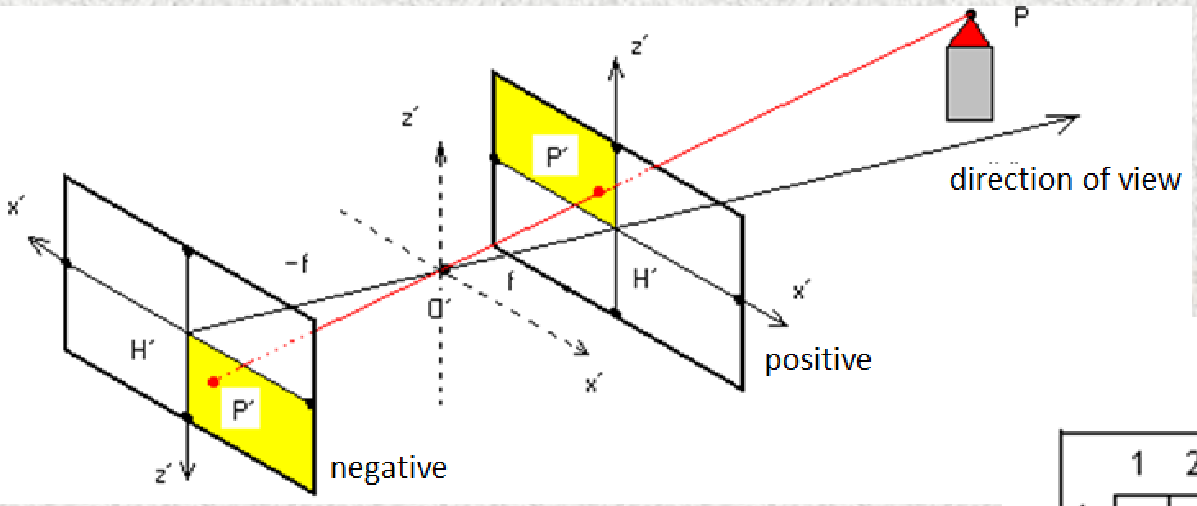
$$x'_0 = dx' = \left(a_1 c_1 + a_2 c_2 + a_3 c_3\right) \cdot d^2$$
$$z'_0 = dz' = \left(b_1 c_1 + b_2 c_2 + b_3 c_3\right) \cdot d^2$$

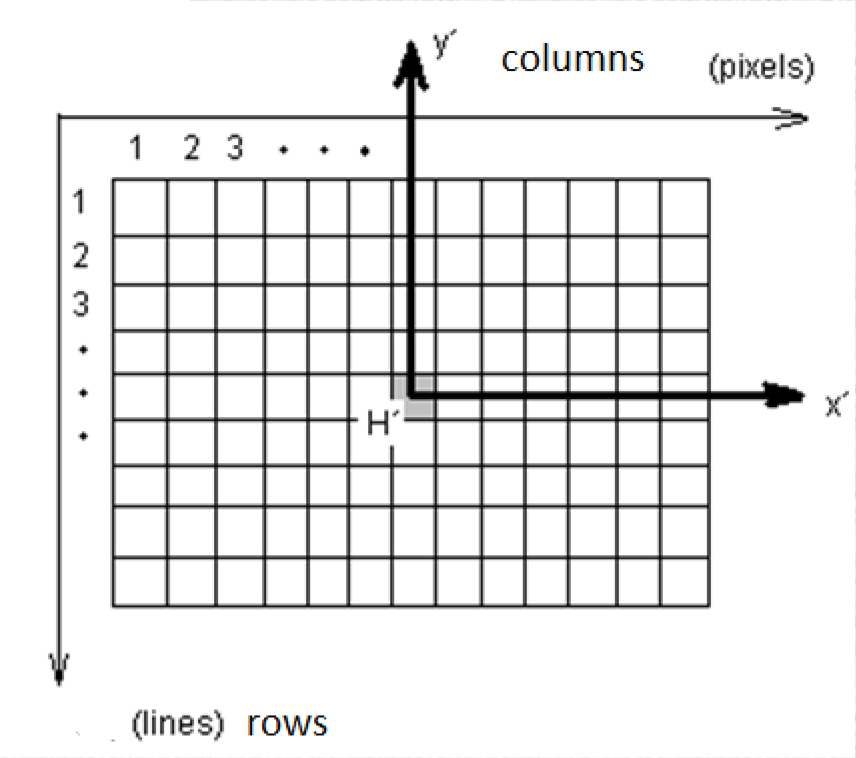
$$f = \frac{f_x + f_z}{2}$$

$$v_{x'} = a_1 X + a_2 Y + a_3 Z + a_4 - c_1 X x' - c_2 Y x' - c_3 Z x' - x'$$
$$v_{z'} = b_1 X + b_2 Y + b_3 Z + b_4 - c_1 X z' - c_2 Y z' - c_3 Z z' - z'$$

Digital image

Coordinates are in pixels (columns, rows)





Instead of fiducial marks: corner pixels

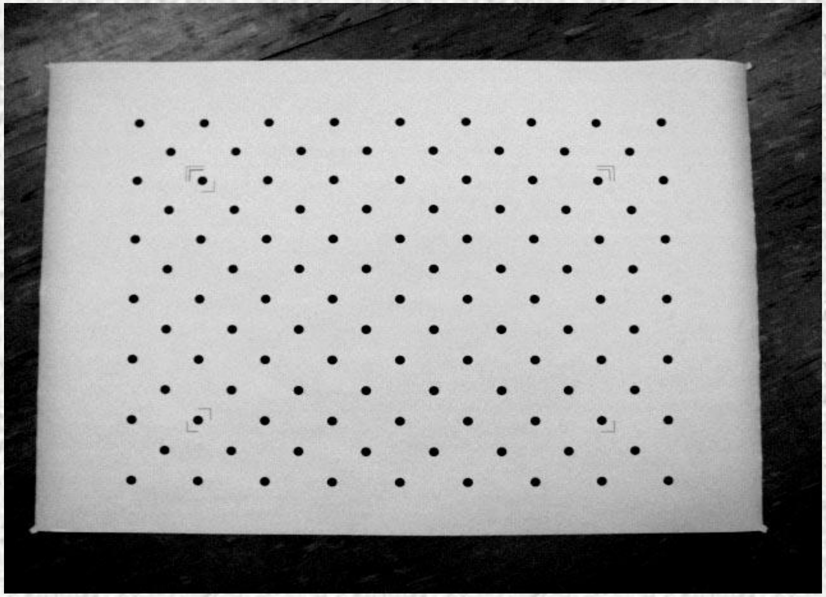
Calibration of the camera

If we do not know the elements of the interior orientation, we need to calculate them by measuring

Calibration methods:

-the most common calibration is using a calibration field / plate and using a software,

-in a specialized optical laboratory.



Aerial photogrammetry

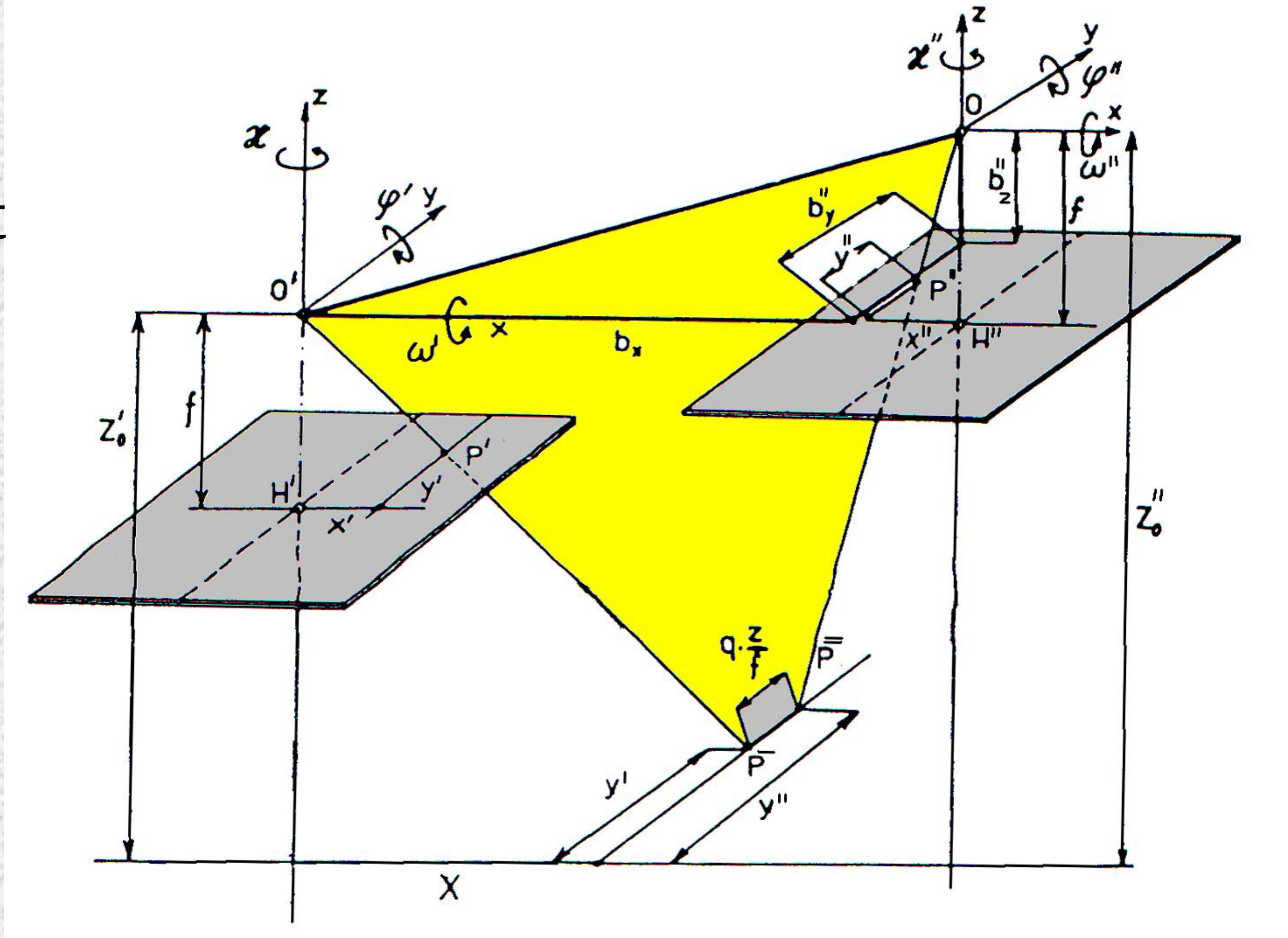
Aerial stereophotogrammetry

$$X = X_o + (Z - Z_o) \cdot \frac{r_{11} \cdot (x' - x'_o) + r_{12} \cdot (y' - y') - r_{13} \cdot f}{r_{31} \cdot (x' - x'_o) + r_{32} \cdot (y' - y') - r_{33} \cdot f}$$

$$Y = \quad Y_o + (Z - Z_o) \cdot \frac{r_{21} \cdot (x' - x'_o) + r_{22} \cdot (y' - y') - r_{23} \cdot f}{r_{31} \cdot (x' - x'_o) + r_{32} \cdot (y' - y') - r_{33} \cdot f}$$

In aerial applications, standardisation of exterior orientation elements cannot be ensured

$R \neq E$!



Photogrammetric technologies for image content evaluation

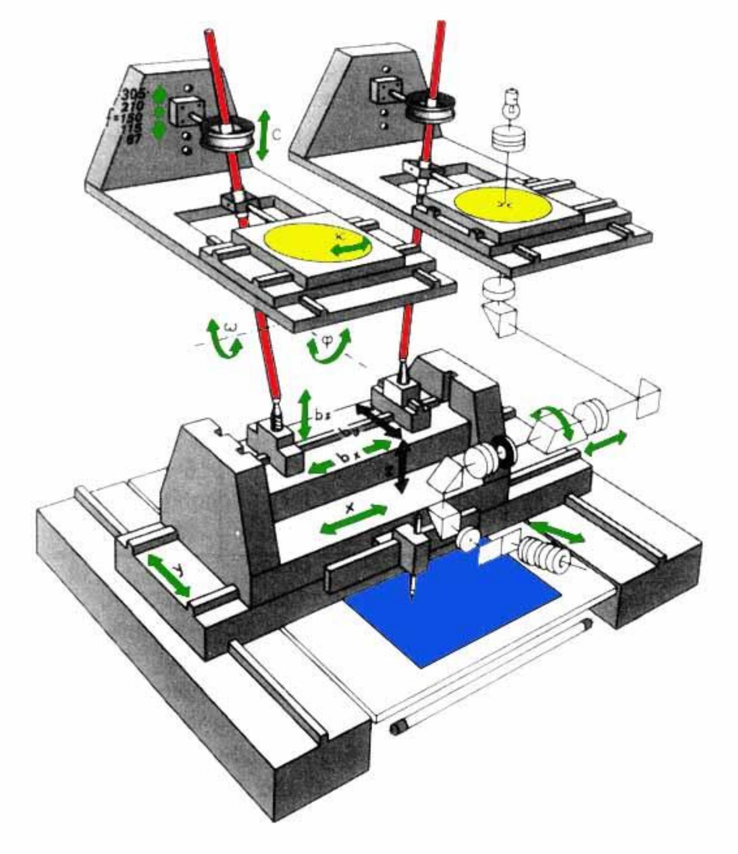
Analog technology
(obsolete, not used)

Numerical (analytical) technology
- using analogue machines and partial counting steps (classical)

Analytical technology
-cannot be solved without a comp
- present

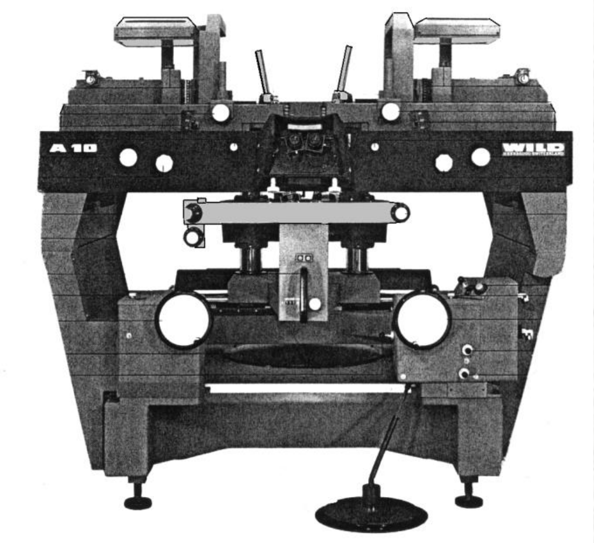
Spatial evaluation using streptophotogrammetric devices

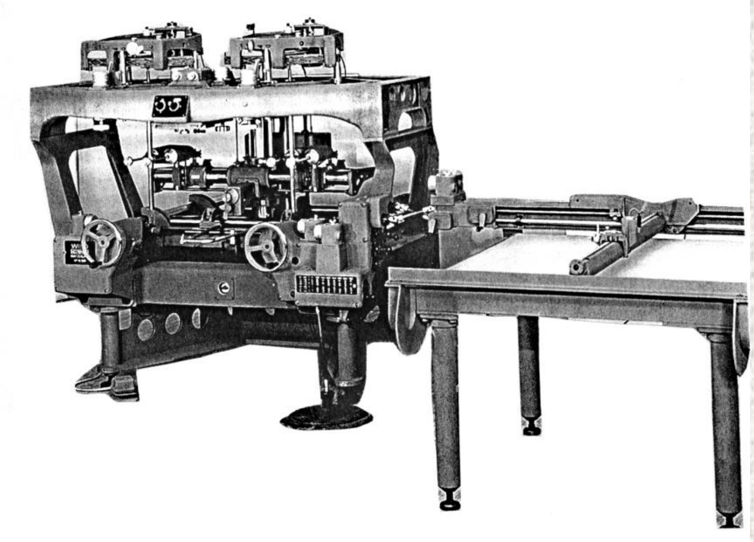
Analog plotters



-produced until 1986 (Wild) and 1990 (Zeis Jena)
- a complex and precise mechanical device, allowing the restoration of the elements of the external orientation
- a realistic stereoscopic model is created by tilting and shifting the images
- latest models with computer support
- model coordinates are controlled
- currently obsolete

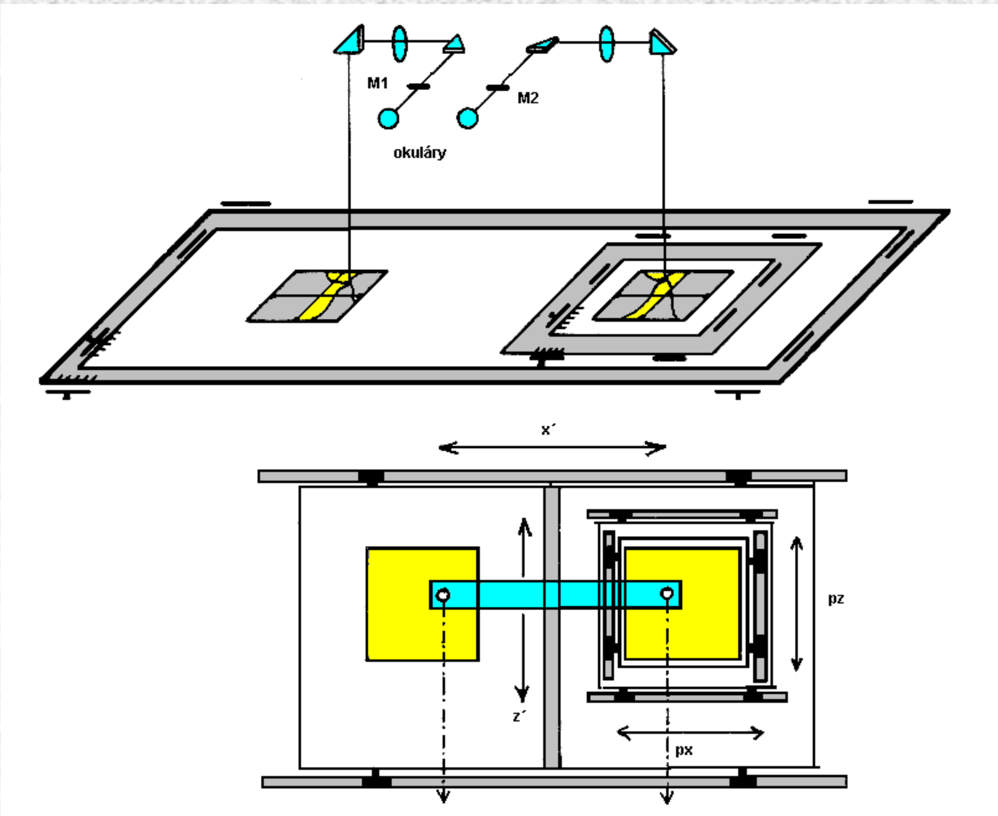
Analog plotters



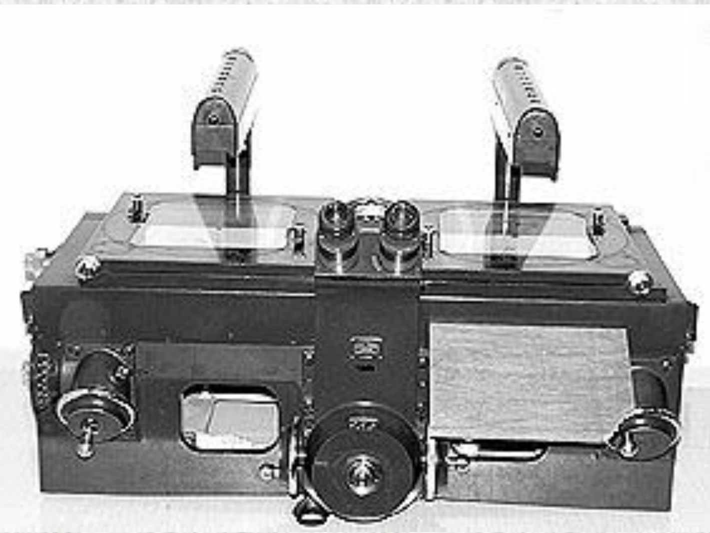


Stereometrograph, Topocart (Zeiss Jena) A-10 and A-7 (Wild)

Analogue comparators



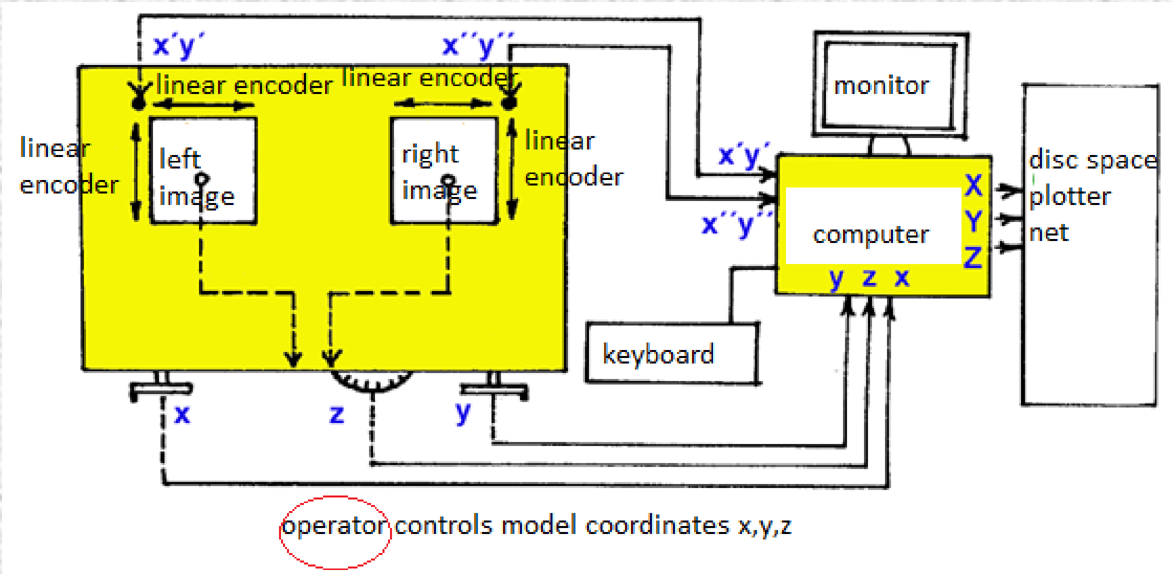
Principle (Steko 1818)



Steko 18x18,
sixties of 20th century

Stecometer,
Zeiss Jena, 1980, high precision 2 μm

Analytical plotters

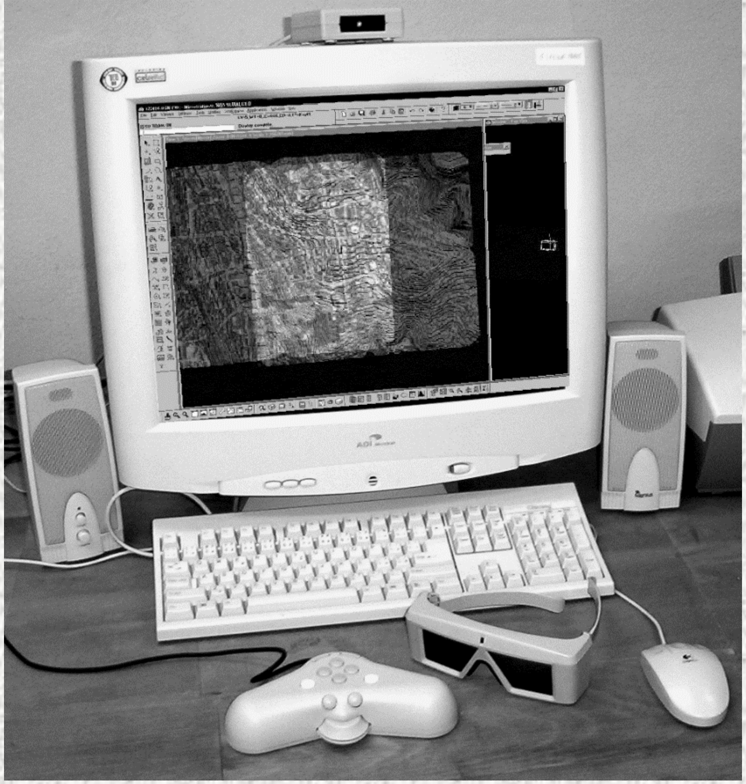


- measuring on real images, computer required
- no real model is created
- the most accurate fm method
- model coordinates are controlled by an operator, are automatically converted to image coordinates on the computer and the mobile image carriers are set images to the calculated image coordinates

BC-1 (Wild, 1985) SD 2000 (Leica, 1995)

Digital stations - nineties of 20th century

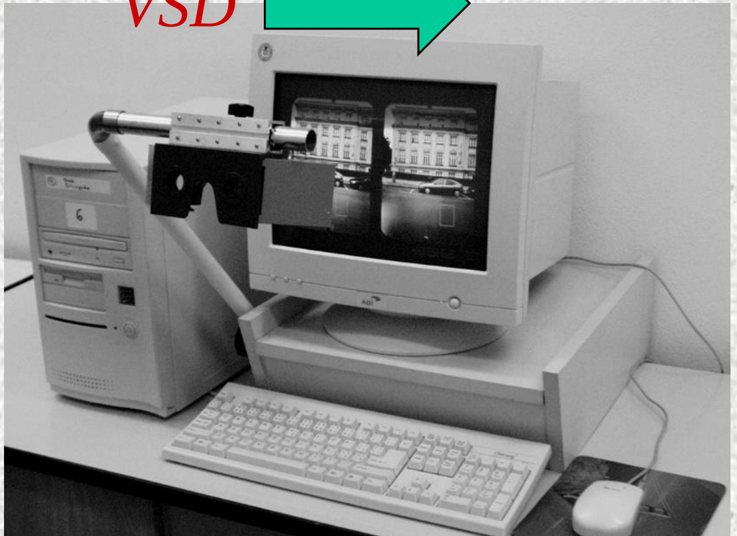
Imagestation Unix



Imagestation SSK

Helava (Leica)

VSD



Aerial photogrammetry

a)Aerial imaging- flight project

$$m_S = \frac{h}{f} \qquad m_Z = \frac{h}{b} m_S \cdot m_P, \qquad m_{XY} = m_S \cdot m_{x'y'}$$

b)Imaging flight design (analogue cameras, digital cameras, add-on devices)

c)Ground works (GCP´s, signalling)

d)Aerotriangulation (the goal of today's aerotriangulation is mainly the creation of new GCP´s directly from the images in optimal positions for later image content detailed processing , the adjustment of the whole block, which allows the continuity of the evaluation of models in sequence and also the accurate calculation of the elements of the external orientation of each image).

e)Classification and local investigation

f)Evaluation of the image content (planimetry, elevation – DTM, DEM, thematic content)

- analogue, empirically(partially till nineties of 20th century)
- analogically using numerical or semi-analytical methods (partially till nineties of 20th century)
- by analytical methods (till nineties of 20th century)
- digitally (since nineties of 20th century)

Aerial photogrammetry

Image orientation:

•***Interior orientation (elements of interior orientation, x'_0, y'_0, f; if necessary distortion parametres)***

•***Exterior orientation (coordinates of the centre of the input pupile X_0 , Y_0 , Z_0 , then inclinations ω, φ, κ)***

For stereo-evaluation of photogrammetric images, the exterior orientation is defined as:

1)direct transformation relation based on „complex solution“ – **Bundle adjustment**

2)older step-by-step procedure called:

*a) **relative orientation** (relative orientation between the two stereo images, forming an arbitrary spatially oriented virtual stereo model)*
*b) **absolute orientation** (scaling, rotation and shifting of the model to the geodetic reference system)*

Aerial stereophotogrammetry

Evaluation of stereo images with known exterior orientation parameters

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

$$X = X_0 + (Z - Z_0)\frac{r_{11}(x' - x'_0) + r_{12}(y' - y'_0) - r_{13} f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33} f}$$

$$Y = Y_0 + (Z - Z_0)\frac{r_{21}(x' - x'_0) + r_{22}(y' - y'_0) - r_{23} f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33} f}$$

$$X = X_{01} + (Z - Z_{01})k_{x1}$$
$$Y = Y_{01} + (Z - Z_{01})k_{y1}$$

$$X = X_{02} + (Z - Z_{02})k_{x2}$$
$$Y = Y_{02} + (Z - Z_{02})k_{y2}$$

$$Z = \frac{X_{02} - Z_{02}k_{x2} + Z_{01}k_{x1} - X_{01}}{k_{x1} - k_{x2}}$$

Aerial stereophotogrammetry

Evaluation of stereo images with unknown exterior orientation parameters

In principle, the determination of the external orientation elements can be divided into 3 methods:

<u>1) Empirical (not used today)</u>

a) **relative orientation** (the basis is the gradual manual removal of vertical parallaxes on landmarks with direct introduction of empirical corrections)

- relative orientation of the independent pair

- relative orientation when attaching an image

b) **absolute orientation** (shift, rotate, scale, height adjustment, tilt of the model)

<u>2) Numerical (classical, still used)</u>

a) **Relative orientation** (the core is the measurement of vertical parallaxes at a minimum of five orientation points with a stereocomparator or using an analoque / analytical plotter, followed by the calculation of unknown elements):

- relative orientation of the independent stereo-image pair (change of orientation of both images)

- relative orientation of conected image (change the orientation of only one image)

b) **absolute orientation** (calculation of elements using linearized relations with subsequent adjustment)

Aerial stereophotogrammetry

Evaluation of stereo images with unknown external orientation parameters

3) Analytical

Use of a direct relationship between the measured image coordinates and the geodetic coordinates of the reference system; the core is the measurement of image coordinates.

- **complex solution** (the entire orientation is solved in one step, including calculation of detailed points, bundle adjustment), necessary to use a powerful computer, **today the most used solution**,

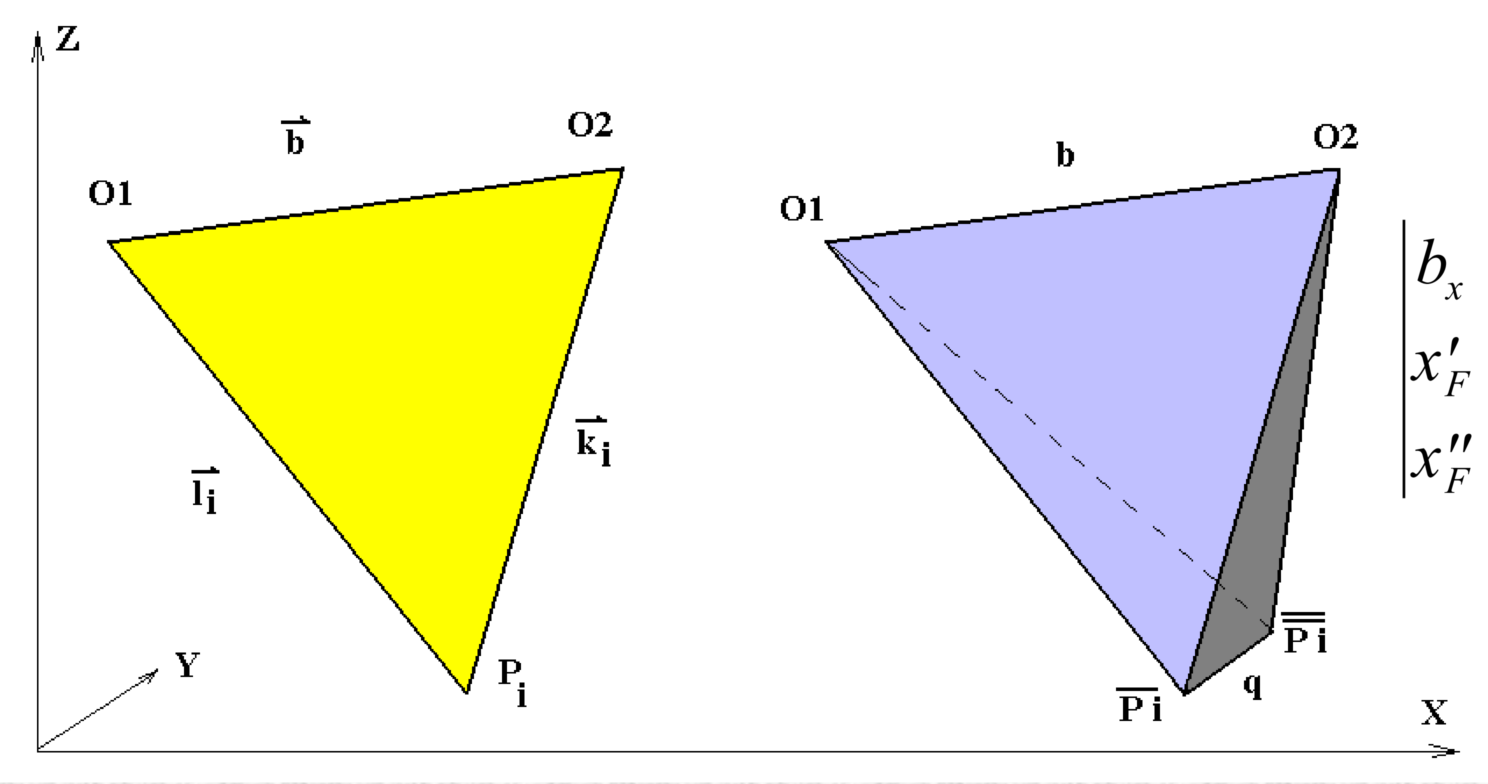
- **sequential solution** (solution is divided into steps, block adjustment).

Aerial photogrammetry

When determining the elements of the relative orientation numerically, we can rely on various conditions, the satisfaction of which guarantees the solution of the relative orientation. The most well-known methods include the **complanarity condition** and the **zero vertical parallax condition**. In general, in relative orientation we can identify **five** unknown elements for which at least five equations must be constructed.

Condition of complanarity

$$\left(\mathbf{b}, \mathbf{k_i}, \mathbf{l_i}\right) = 0, \quad i = 1, \ldots, 5$$



$$\begin{vmatrix} b_x & b_y & b_z \\ x'_F & y'_F & z'_F \\ x''_F & y''_F & z''_F \end{vmatrix} = 0$$

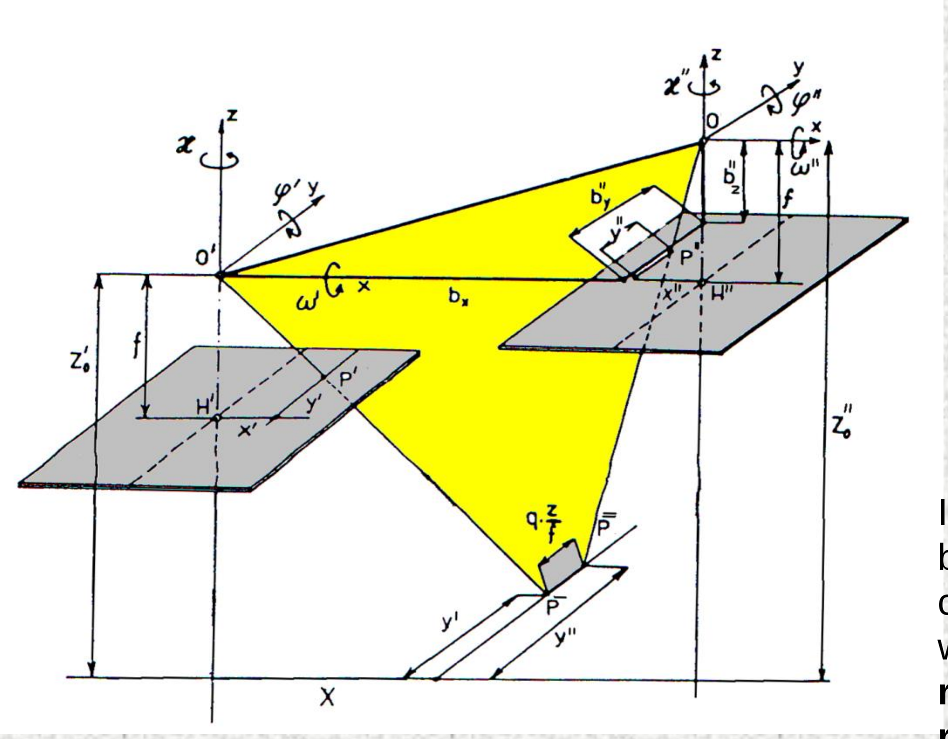
Aerial photogrammetry

Zero vertical parallax condition

$$x'_s = -f\,\frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f}, \quad y'_s = -f\,\frac{r_{21}x' + r_{22}y' - r_{23}f}{r_{31}x' + r_{32}y' - r_{33}f}$$

$$\Delta x' = x'_s - x' = -y'd\kappa' - \left(f + \frac{x'^2}{f}\right)d\varphi' + \frac{x'y'}{f}d\omega' + db'_x + \frac{x'}{f}db'_z$$

$$\Delta y' = y'_s - y' = x'd\kappa' - \frac{x'y'}{f}d\varphi' + \left(f + \frac{y'^2}{f}\right)d\omega' + db'_y + \frac{y'}{f}db'_z$$



$$\boxed{\begin{aligned} y'_s - y''_s &= y' + \Delta y' - y'' - \Delta y'' \\ 0 \quad &= q + \Delta y' - \Delta y'' \end{aligned}}$$

$$0 = q + x'd\kappa' - \frac{x'y'}{f}d\varphi' + \left(f + \frac{y'^2}{f}\right)d\omega' + db'_y + \frac{y'}{f}db'_z -$$

$$- x''d\kappa'' + \frac{x''y''}{f}d\varphi'' - \left(f + \frac{y''^2}{f}\right)d\omega'' - db''_y - \frac{y''}{f}db''_z$$

In this equation, there are 8 orientation elements in total, but only 5 of them are independent; we can therefore choose 3 elements preferably as zero. Depending on which we choose as independent, we speak of the **relative orientation of the independent pair** or the **relative orientation**. **of conected image**

Aerial photogrammetry

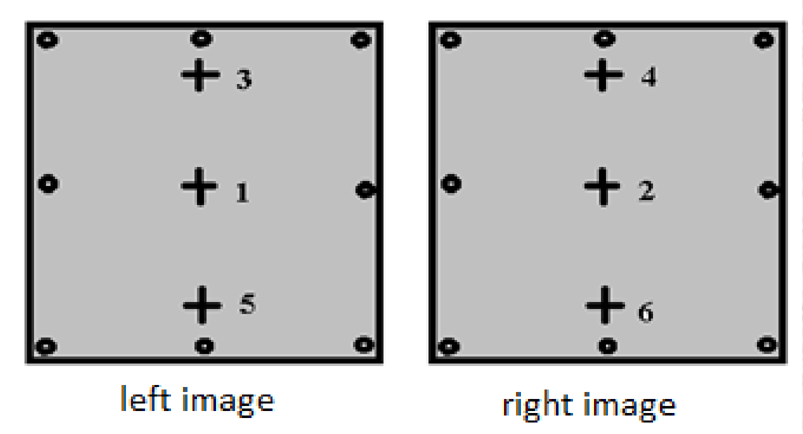
Relative orientation of the independent pair

The rotations $d\kappa'$, $d\varphi'$, $d\kappa''$, $d\varphi''$, $\Delta\omega = d\omega' - d\omega''$ are chosen as the unknowns with an assumption $y' \cong y''$

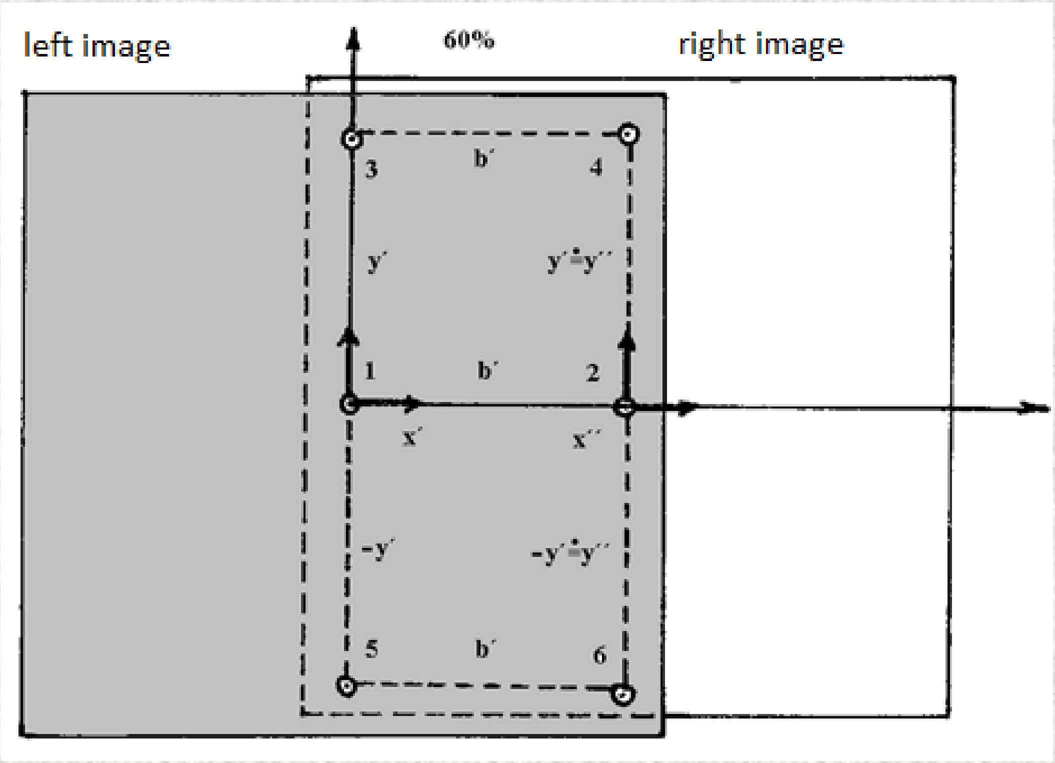
$$0 = q + x'd\kappa' - x'd\kappa'' - \frac{x'y'}{f}d\varphi' + \frac{x'y'}{f}d\varphi'' + \left(f + \frac{y'^2}{f}\right)\Delta\omega$$

$$\left(f + \frac{y'^2}{f}\right)d\omega' - \left(f + \frac{y''^2}{f}\right)d\omega'' \cong \left(f + \frac{y'^2}{f}\right)\left(d\omega' - d\omega''\right) = \left(f + \frac{y'^2}{f}\right)\Delta\omega$$

orinthal point	left picture x	left frame y	right picture x	right image y
1	0	0	-b	0
2	+b	0	0	0
3	0	+y	-b	+y
4	+b	+y	0	+y
5	0	-y	-b	-y
6	+b	-y	0	-y



Aerial photogrammetry



$$0 = q_1 + b' d\kappa'' + f\Delta\omega$$

$$0 = q_2 + b' d\kappa' + f\Delta\omega$$

$$0 = q_3 + b' d\kappa'' - \frac{b' y'}{f} d\varphi'' + \left(f + \frac{y'^2}{f} \right) \Delta\omega$$

$$0 = q_4 + b' d\kappa' - \frac{b' y'}{f} d\varphi' + \left(f + \frac{y'^2}{f} \right) \Delta\omega$$

$$0 = q_5 + b' d\kappa'' + \frac{b' y'}{f} d\varphi'' + \left(f + \frac{y'^2}{f} \right) \Delta\omega$$

$$0 = q_6 + b' d\kappa' + \frac{b' y'}{f} d\varphi' + \left(f + \frac{y'^2}{f} \right) \Delta\omega$$

Aerial photogrammetry

Relative orientation for connecting image

The rotations $d\kappa''$, $d\varphi''$, $d\omega''$, *and translations* db''_y, db''_z are chosen as the unknowns.

$$0 = q - x''d\kappa'' + \frac{x''y''}{f}d\varphi'' - \left(f + \frac{y''^2}{f}\right)d\omega'' - db''_y - \frac{y''}{f}db''_z$$

$$0 = q_1 + b'd\kappa'' - fd\omega'' - db''_y$$

$$0 = q_2 - fd\omega'' - db''_y$$

$$0 = q_3 + b'd\kappa'' - \frac{b'y''}{f}d\varphi'' - fd\omega'' - \frac{y''^2}{f}d\omega'' - db''_y - \frac{y''}{f}db''_z$$

$$0 = q_4 - fd\omega'' - \frac{y''^2}{f}d\omega'' - db''_y - \frac{y''}{f}db''_z$$

$$0 = q_5 + b'd\kappa'' + \frac{b'y''}{f}d\varphi'' - fd\omega'' - \frac{y''^2}{f}d\omega'' - db''_y + \frac{y''}{f}db''_z$$

$$0 = q_6 - fd\omega'' - \frac{y''^2}{f}d\omega'' - db''_y + \frac{y''}{f}db''_z$$

Aerial photogrammetry

Absolute orientation

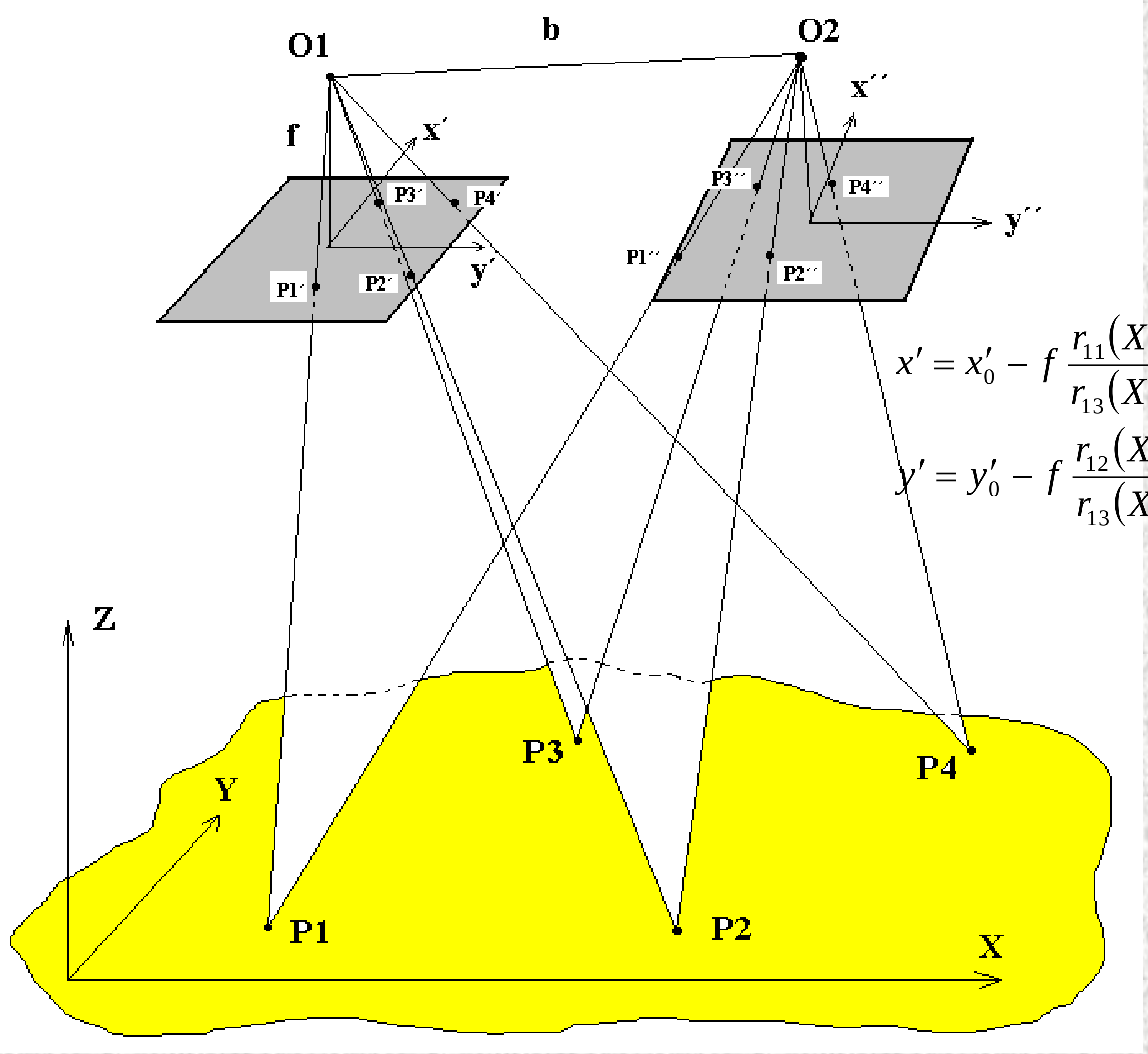
$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} + m.\mathbf{R}.\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

where X,Y,Z are geodetic coordinates, X_0 , Y_0 , Z_0 are the geodetic coordinates of the origin of the model coordinate system x,y,z , m is the scale of the model and $\boldsymbol{R}$ is the spatial rotation matrix of the **model** coordinate system in the geodetic coordinate system, containing the three angles Ω,Φ,K. The relation is a spatial similarity transformation with unknowns X_0 ,Y_0 ,Z_0 , m, Ω ,Φ, K.

$$M = \begin{pmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & m_z \end{pmatrix}$$

Analytical methods

Complex solution („Schmid“ solution)



$$x' = x_0' - f\,\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} = x_0' - f\,\frac{\mathbf{R_1} \cdot \mathbf{X}}{\mathbf{R_3} \cdot \mathbf{X}}$$

$$y' = y_0' - f\,\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} = y_0' - f\,\frac{\mathbf{R_2} \cdot \mathbf{X}}{\mathbf{R_3} \cdot \mathbf{X}}$$

Analytical methods

Complex solution

$$x'_{i1} = F_x\left(f, x'_0 = d'x, X_{01}, Y_{01}, Z_{01}, \omega_1, \varphi_1, \kappa_1, X_i, Y_i, Z_i\right)$$
$$y'_{i1} = F_y\left(f, y'_0 = d'y, X_{01}, Y_{01}, Z_{01}, \omega_1, \varphi_1, \kappa_1, X_i, Y_i, Z_i\right)$$
$$x''_{i2} = F_x\left(f, x''_0 = d'x, X_{02}, Y_{02}, Z_{02}, \omega_2, \varphi_2, \kappa_2, X_i, Y_i, Z_i\right)$$
$$y''_{i2} = F_y\left(f, y''_0 = d'y, X_{02}, Y_{02}, Z_{02}, \omega_2, \varphi_2, \kappa_2, X_i, Y_i, Z_i\right)$$

$$f(x_1, \ldots, x_n) = f(x_1^0, \ldots, x_n^0) + \left(\frac{\partial f}{\partial x_1}\right)^0 dx_1 + \ldots + \left(\frac{\partial f}{\partial x_n}\right)^0 dx_n$$

$$\begin{aligned} v_{xij} = {} & \left(\frac{\partial x'}{\partial X_{0j}}\right)^0 dX_{0j} + \left(\frac{\partial x'}{\partial Y_{0j}}\right)^0 dY_{0j} + \left(\frac{\partial x'}{\partial Z_{0j}}\right)^0 dZ_{0j} + \\ & + \left(\frac{\partial x'}{\partial \omega_j}\right)^0 d\omega_j + \left(\frac{\partial x'}{\partial \varphi_j}\right)^0 d\varphi_j + \left(\frac{\partial x'}{\partial \kappa_j}\right)^0 d\kappa_j + \\ & + \left(\frac{\partial x'}{\partial X_i}\right)^0 dX_i + \left(\frac{\partial x'}{\partial Y_i}\right)^0 dY_i + \left(\frac{\partial x'}{\partial Z_i}\right)^0 dZ_i - \left(x'_{ij} - {x'_{ij}}^0\right) \end{aligned}$$

Analytical methods

Complex solution

$$\mathbf{v} = \mathbf{A}_1 \cdot \mathbf{x}_1 + \mathbf{A}_2 \cdot \mathbf{x}_2 - \mathbf{l} \quad \leftrightarrow \quad (\mathbf{v} = \mathbf{A} \cdot \mathbf{x} - \mathbf{l})$$

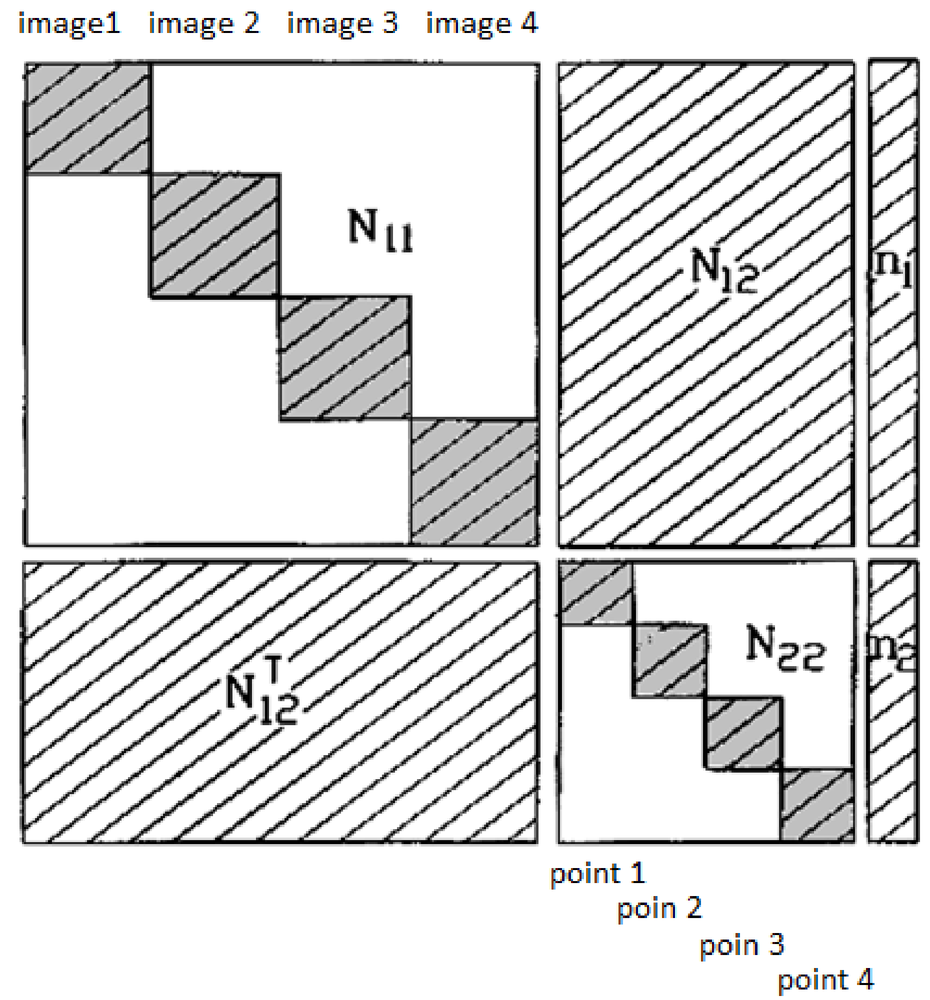
$$\mathbf{x} = \left(\mathbf{A}^T \mathbf{A}\right)^{-1} \cdot \mathbf{A}^T \mathbf{l}$$

$$\mathbf{A}^T \mathbf{A} = \mathbf{N} \qquad \mathbf{A}^T \cdot \mathbf{l} = \mathbf{n}$$

$$\mathbf{v} = \mathbf{A} \cdot \mathbf{x} - \mathbf{l}$$

$$\mathbf{N} \cdot \mathbf{x} = \mathbf{n}$$

$$\begin{pmatrix} \mathbf{N}_{11} & \mathbf{N}_{12} \\ \mathbf{N}^T{}_{12} & \mathbf{N}_{22} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{pmatrix}$$



$$\left(\mathbf{N}_{11} - \mathbf{N}_{12} \cdot \mathbf{N}_{22}{}^{-1} \cdot \mathbf{N}^T{}_{12}\right)\mathbf{x}_1 = \mathbf{n}_1 - \mathbf{N}_{12} \cdot \mathbf{N}_{22}{}^{-1} \cdot \mathbf{n}_2$$

Analytical methods

Sequential solution (not used today)

$$x, y, z\ (= -f),\ x, y, z(= -f) \rightarrow \textbf{rel.or.} \rightarrow x_F\ , y_F\ , z_F\ , x_F\ , y_F\ , z_F \rightarrow$$
$$\textbf{scale adjustment} \rightarrow x,y,z \rightarrow \textbf{abs.or.} \rightarrow X,Y,Z$$

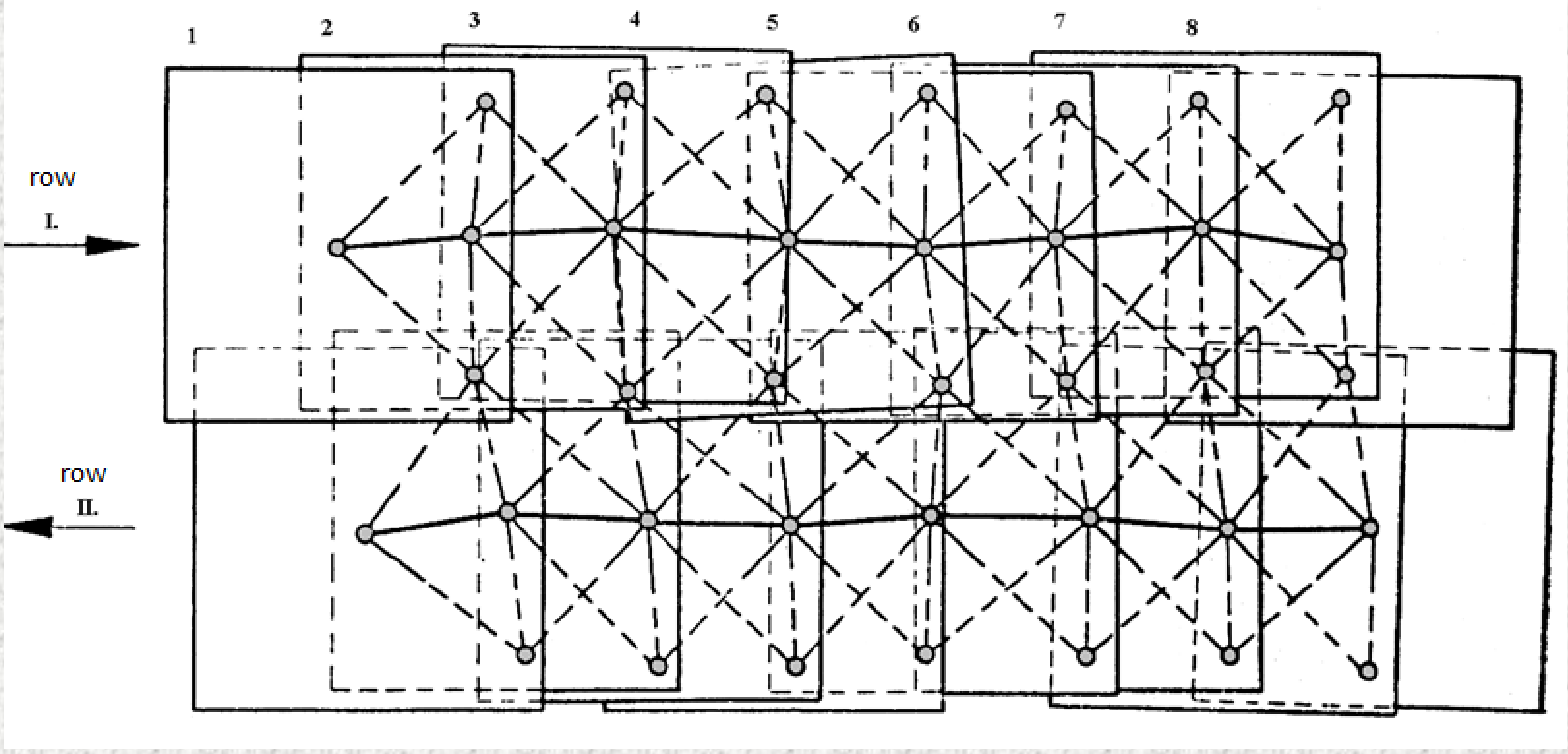
Image triangulation

The goal of aerotriangulationin general is:

- ***obtaining new GCP points for detailed image context evaluation***
- ***adjustment of a set of images to ensure continuous evaluation***
- ***<u>accurate calculation of exterior orientation elements for all images</u>***

- *Historically, there are radial, analoque,semianalytical and analytical technology of aerotriangulation*
- *Today only automated (digital) aerotriangulation is used (**AAT**)*

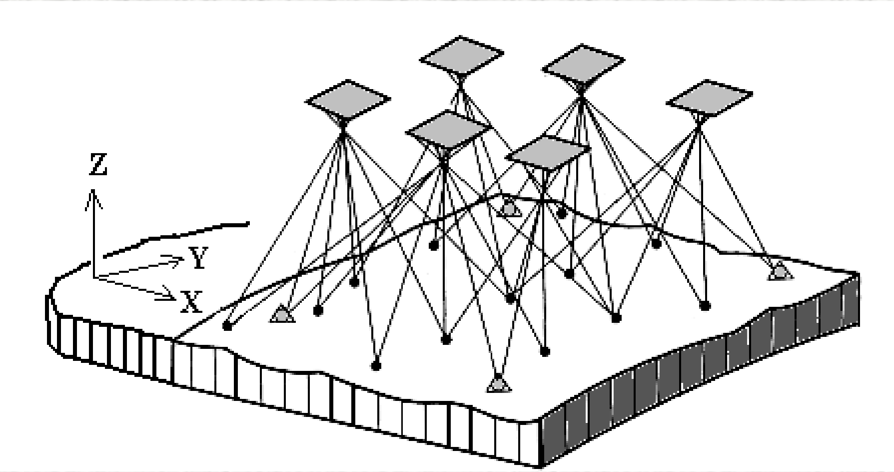
Image triangulation historical radial triangulation



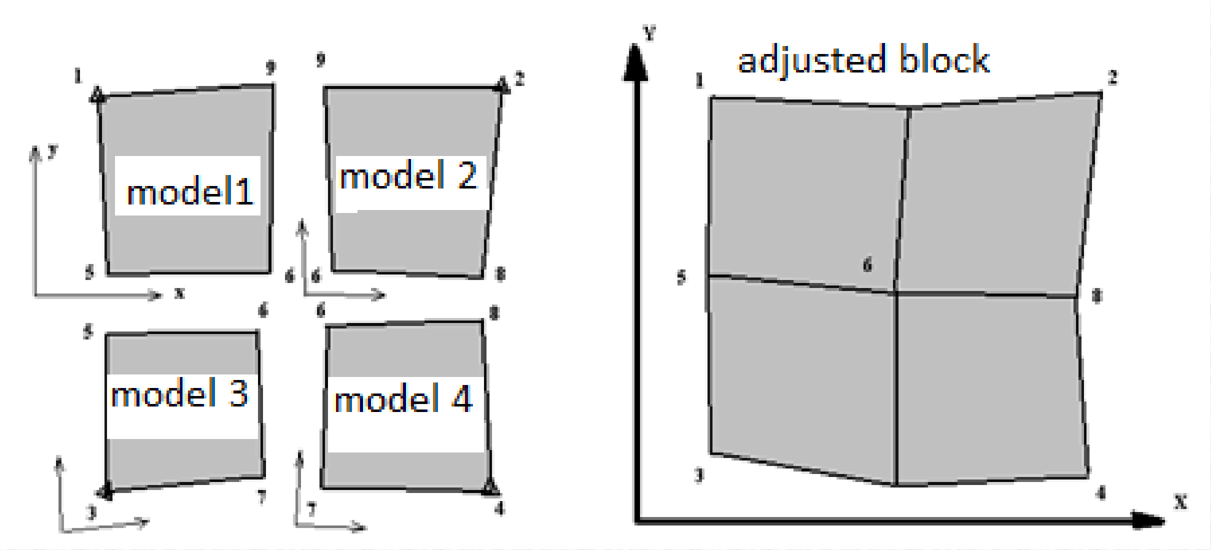
Full triangular radial mesh and normal image block in radial triangulation

Image triangulation

1)Block bundle adjustment (complex solution)



integrated INS



2)Block adjustment (Sequential solution)

-positional
-spatial
-elevation

INS (*Inertial Navigation System*) or GNSS/IMU (*Inertial Measurement Unit*).

Digital photogrammetry

Theory of image correlation

$$\rho\,(A,B)=\frac{\operatorname{cov}(A,B)}{\rho(A)\cdot\rho(B)}$$

If we want to calculate the correlation coefficient for two equally sized digital images (or their cuts), we will use to calculate the pixel value $p(A)_{i,j}$ for image **A** and $p(B)_{i,j}$ for image **B**. We obtain the expression:

$$r(A,B)=\frac{C(A,B)}{\sqrt{C(A)\cdot C(B)}}$$

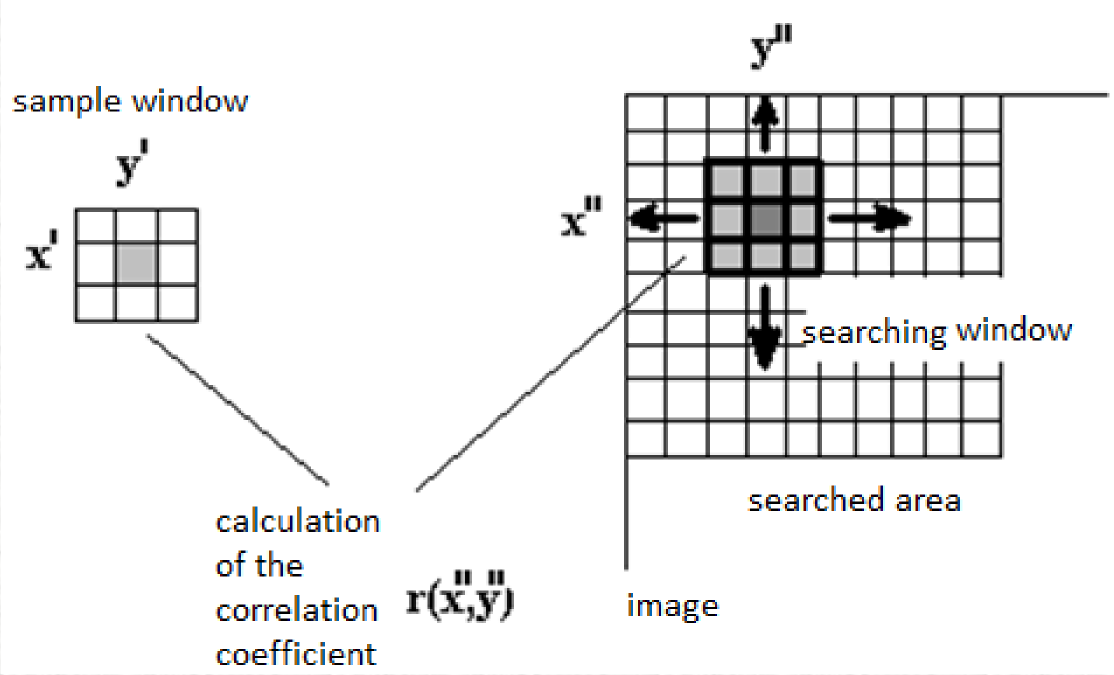
where the individual expressions are (n is the number of pixels in the side of the square window):

$$C(A,B)=\frac{1}{n^2-1}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p(A)_{i,j}-\overline{p}(A)\right)\cdot\left(p(B)_{i,j}-\overline{p}(B)\right)$$

$$C(A)=\frac{1}{n^2-1}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p(A)_{i,j}-\overline{p}(A)\right)^2,\quad \overline{p}(A)=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p(A)_{i,j}\right)$$

$$C(B)=\frac{1}{n^2-1}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p(B)_{i,j}-\overline{p}(B)\right)^2,\quad \overline{p}(B)=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p(B)_{i,j}\right)$$

Digital photogrammetry



We are looking for correlation coeficient a maximum with subpixel precision. Since it reaches its maximum only in a limited region, we can replace the discrete correlation function by a continuous function and describe it by, for example, a second degree polynomial:

$$r = \bar{r} + v = a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2$$

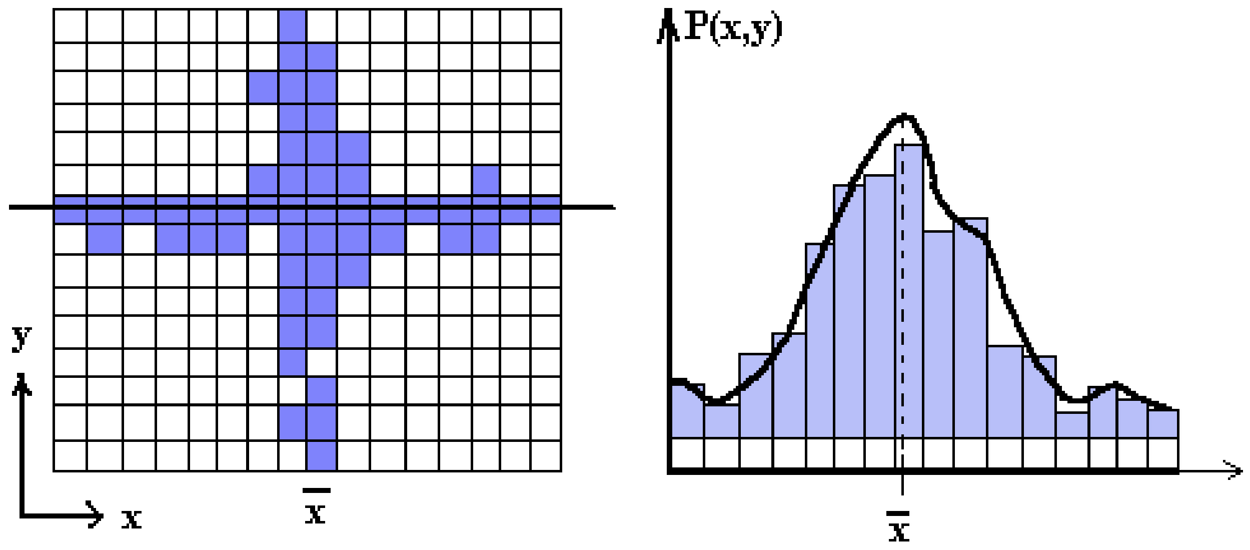
where x_i , y_i is the position of the search window for the calculated value of the correlation coefficient r_{i}; if we have a matrix of 3x3 correlation coefficients, we get 9 values. We need a total of 6 to calculate the coefficients a_i

Digital photogrammetry

Subpixel transformation

$$p_B(x) = p_A(x + a_1)$$

$$p_B(y) = p_A(y + a_2)$$



a detail of displayed cross (for example fiducial mark)

its probable centre

$$v_x + p_B(x) = p_A(x + a_1)a_3 + a_5$$

$$v_y + p_B(y) = p_A(y + a_2)a_4 + a_6$$

$$v_x + p_B(x) = p_A \cdot a_3(x) + p'_A \cdot a_3 a_1(x) + a_5$$

$$v_y + p_B(y) = p_A \cdot a_4(y) + p'_A \cdot a_4 a_2(y) + a_6$$

Digital photogrammetry

Digital orthophoto

What is necessary for an orthophoto creating?

-images,

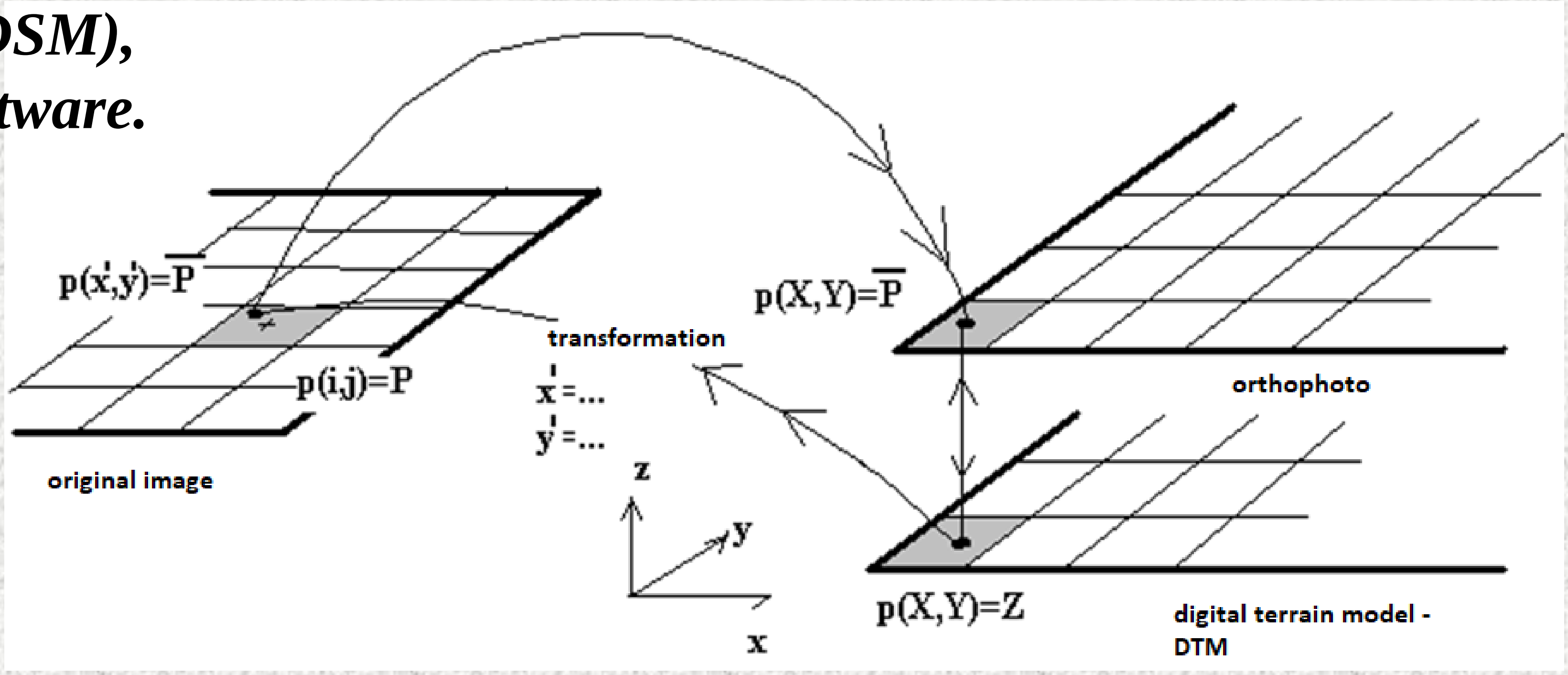
-known interior and exterior orientation,

-digital terrain model (DRM, DEM or DSM),

-software.

$$x' = x'_0 - f\,\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

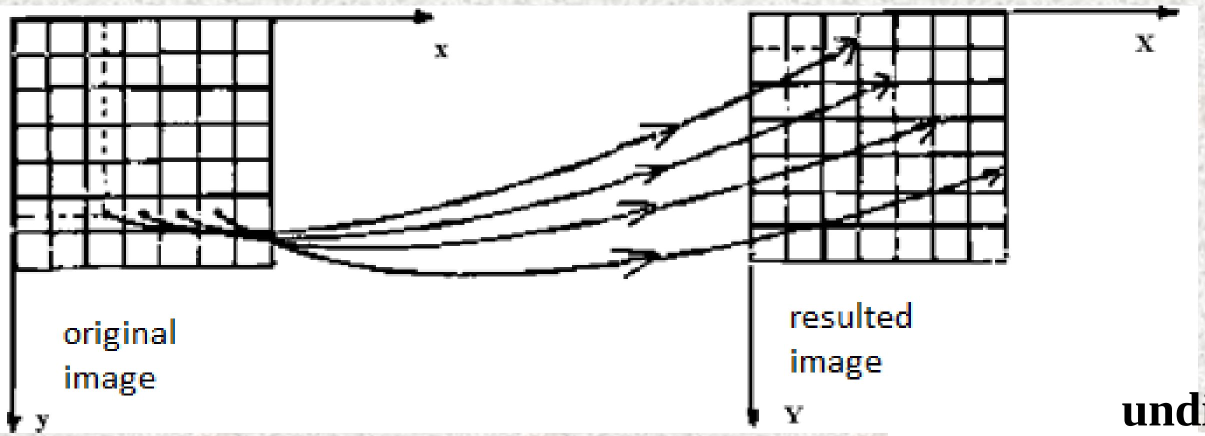
$$y' = y'_0 - f\,\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$



Geometric transformation

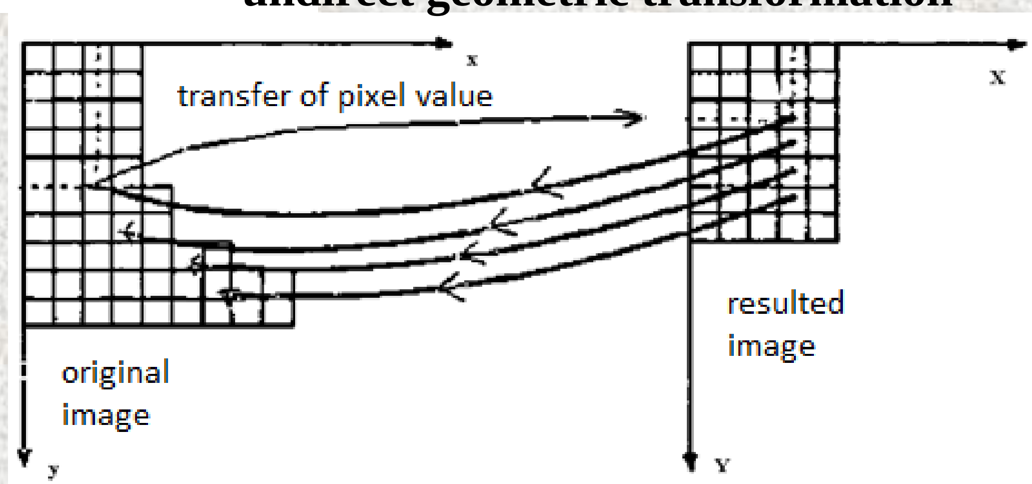
The aim of geometric transformation is either to remove image distortion caused by the instability of geometric conditions during measurement and to convert the data into a suitable projection (especially in RS) or to create a new image (e.g. orthophoto) based on transformation relations. Geometric correction can be performed in three different ways:

- ***data transformation based on precisely known carrier trajectory parameters***
- ***direct geometric transformation based on GCP ´s or vectors***
- ***undirect geometric transformation based on GCP ´s or vectors (most used)***



undirect geometric transformation

direct geometric transformation



Data from the original image matrix to the corrected matrix can be transferred

- ***Nearest Neighbour Method***
- ***Bilinear Transformation***
- ***Bicubic Convolution***

End