Introduction to selected areas of geomatics

Lecture 5

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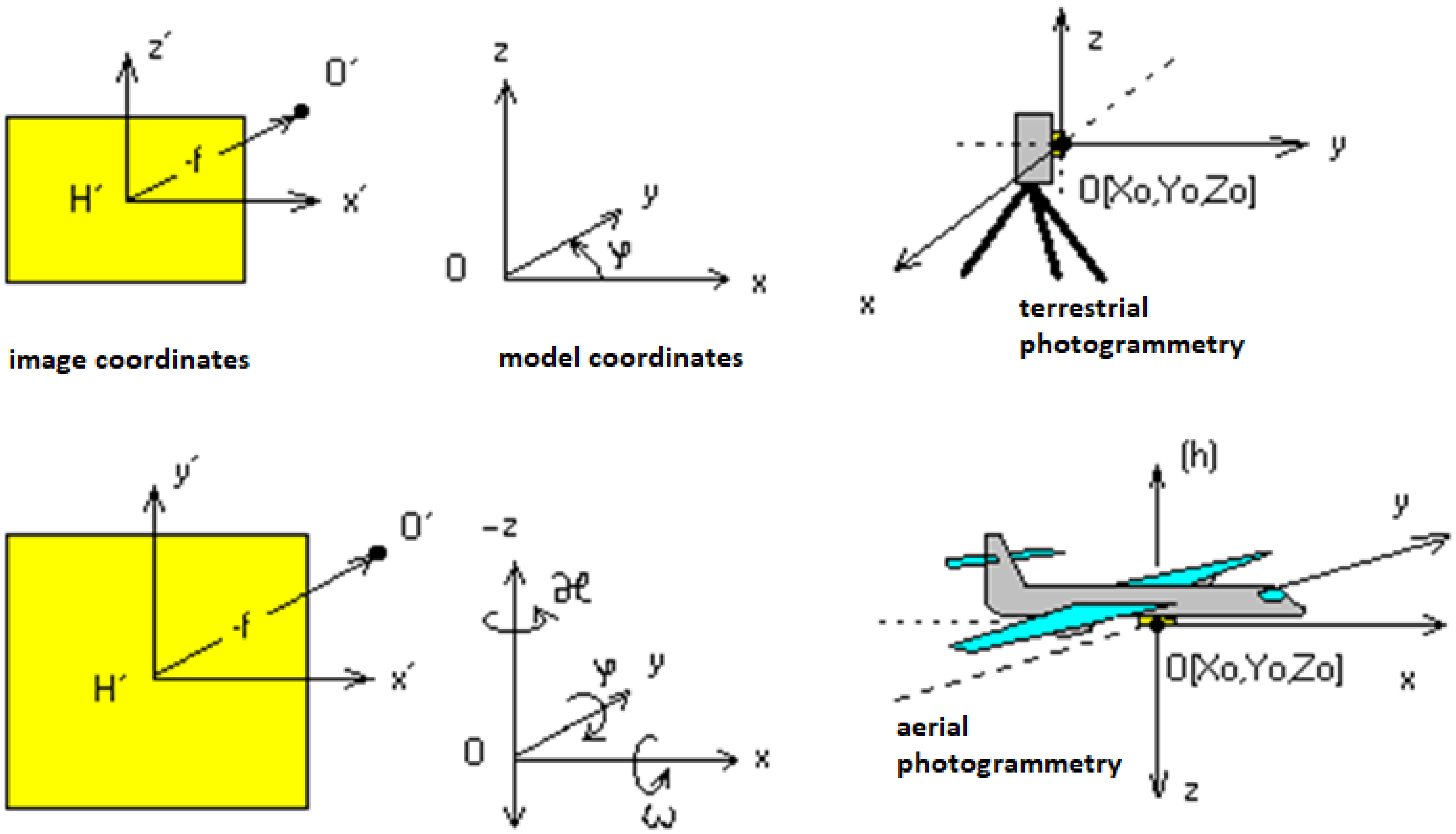
Coordinate systems and conversions

Elements of internal and external orientation



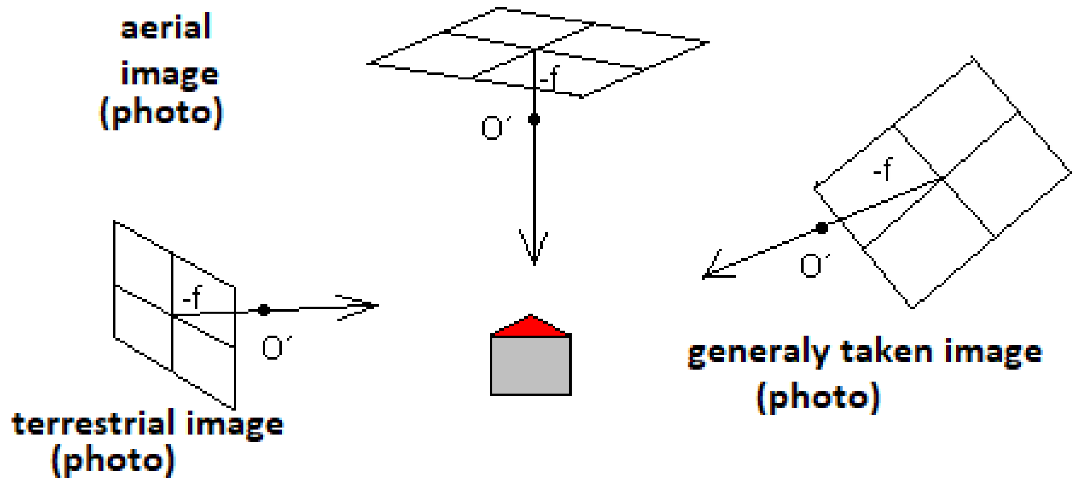
Internal and external orientation elements for terrestrial and aerial photogrammetry

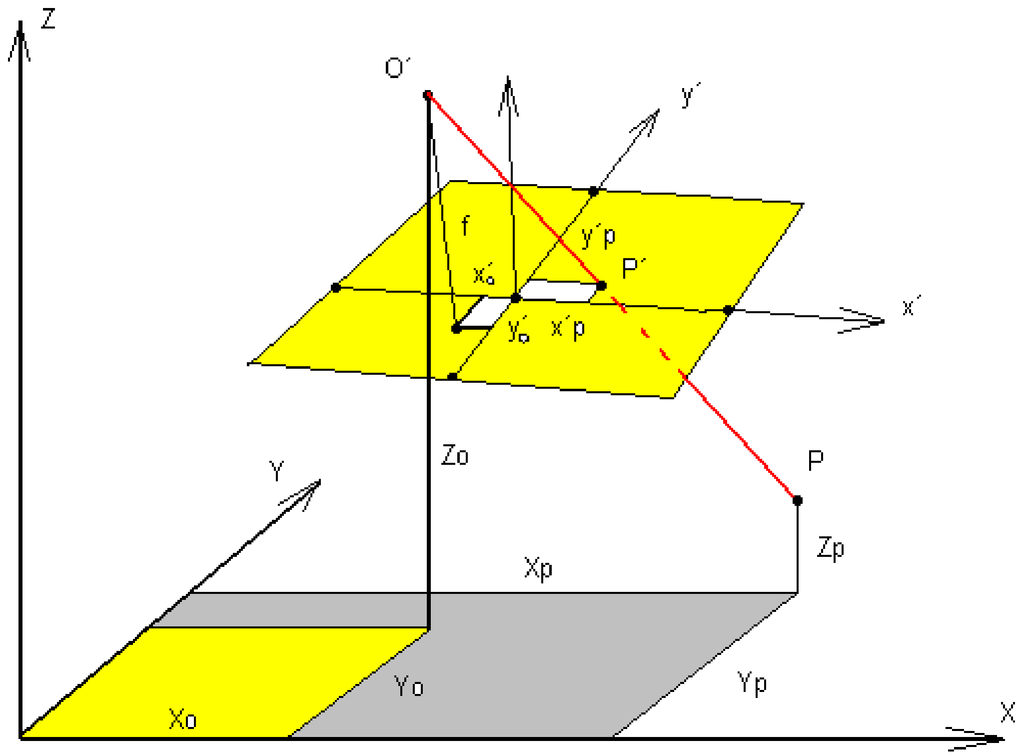
Coordinate systems

Image coordinate system label : x′ , y′ , (z′ = -f)
Model coordinate system label:: x , y , z
Geodetical system: X , Y , Z

Auxiliary systems
Fictional image coordinate system : x'_F , y'_F , z'_F
Vertical image coordinate system : x'_S , y'_S , z'_S

Classic terrestrial , aerial
and generally taken image





Coordinate systems in
aerial photogrammetry

Converting image information to geodetic systems

1) Restoration of internal orientation elements

2) External orientation

External orientations can be solved classically in two steps as:

1. **-relative orientation** (mutual orientation between the two stereo images, formation of an arbitrary spatially oriented stereo model)
2. **-absolute orientation** (rotation and displacement of the model into the geodetic reference system)

3. - in one step **using the Bundle** *Adjustment* **method** (*Bündelausgleichung*)

Rotation matrix

Resulting rotation matrix *R*

$$\mathbf{R}_{\omega\varphi} = \mathbf{R}_{\omega} \cdot \mathbf{R}_{\varphi} \qquad \mathbf{R}_{\omega\varphi\kappa} = \mathbf{R}_{\omega\varphi} \cdot \mathbf{R}_{\kappa}$$

$$\mathbf{R}_{\omega\varphi\kappa} = \begin{pmatrix} \cos\varphi\cos\kappa & -\cos\varphi\sin\kappa & \sin\varphi \\ \sin\omega\sin\varphi\cos\kappa + \cos\omega\sin\kappa & -\sin\omega\sin\varphi\sin\kappa + \cos\omega\cos\kappa & -\sin\omega\cos\varphi \\ -\cos\omega\sin\varphi\cos\kappa + \sin\omega\sin\kappa & \cos\omega\sin\varphi\sin\kappa + \sin\omega\cos\kappa & \cos\omega\cos\varphi \end{pmatrix}$$

$$\tan\omega = -\frac{r_{23}}{r_{33}}, \sin\varphi = r_{13}, \tan\kappa = -\frac{r_{12}}{r_{11}}$$

$$\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

*Here it is important to note that r_{13} = sin > 0 in the 1st and 2nd quadrants and further that r_{13} = sin < 0 in the 3rd and 4th quadrants. Thus, the rotation is not uniquely determined. The quadrants of the other two rotations , are uniquely determined given the expressions from which we calculate them. **Thus we get a total of two sets of rotations ,, to a single rotation matrix R**.*

Derivation of mathematical relations

Direct relationship between image and geodetic coordinates

collinear frame-model relationship

$$\frac{x' - x_0'}{-f} = \frac{x - x_0}{z - z_0}, \quad \frac{y' - y_0'}{-f} = \frac{y - y_0}{z - z_0}$$

$$\begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} = \mathbf{R} \cdot \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}, \qquad \mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

the model coordinate system can be converted to a geodetic system by rotations around three axes

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} = \mathbf{R}^T \cdot \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix}$$

<u>the direct relationship between image and geodetic coordinates, which is the basis of all contemporary photogrammetry</u>:

$$x' = x_0' - f\,\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

$$y' = y_0' - f\,\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

Photogrammetric series

Finally, these expressions need to be supplemented by the effect of translation

$$\Delta x' = -y' d\kappa' - \left(f + \frac{x'^2}{f} \right) d\varphi' + \frac{x' y'}{f} d\omega' + db'_x + \frac{x'}{f} db'_z$$

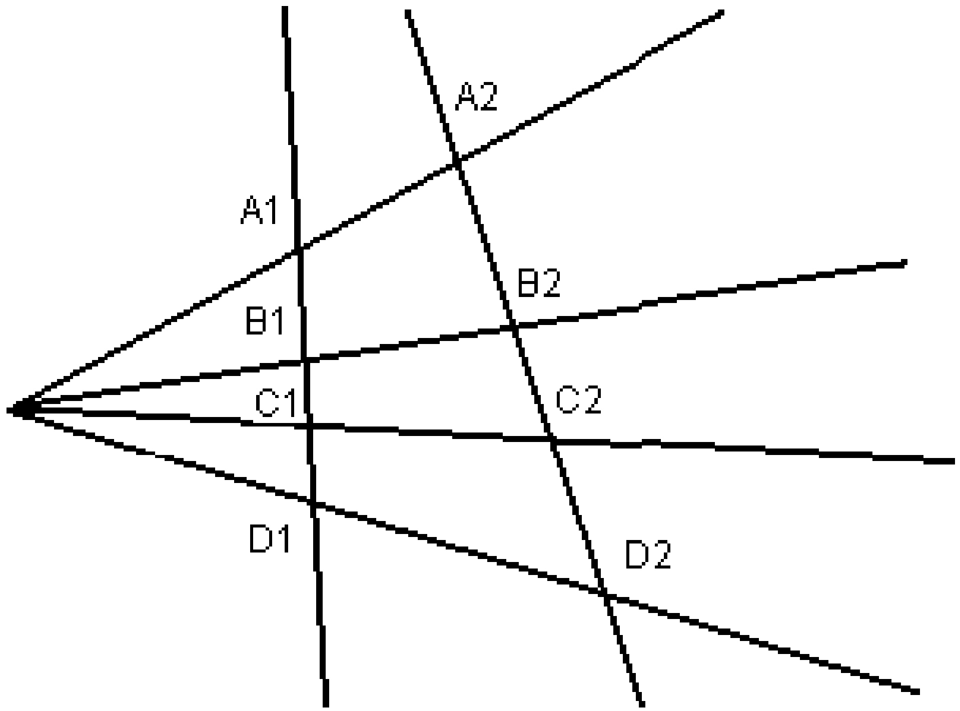
$$\Delta y' = x' d\kappa' - \frac{x' y'}{f} d\varphi' + \left(f + \frac{y'^2}{f} \right) d\omega' + db'_y + \frac{y'}{f} db'_z$$

The term is called "**complete photogrammetric series**", or historically Gruber series, and is used in simplified theoretical derivations. The meaning and use of series was quite fundamental, especially in the era of analogue photogrammetry.

Methods of photogrammetry

Single-shot photogrammetry

- relationship between two planes



Papp's theorem:

The point or ray quadrature binary ratio is preserved in the map and image planes.

$$\frac{\dfrac{A_1C_1}{B_1C_1}}{\dfrac{A_1D_1}{B_1D_1}} = \frac{\dfrac{A_2C_2}{B_1C_2}}{\dfrac{A_2D_2}{B_2D_2}}$$

Mathematical expression

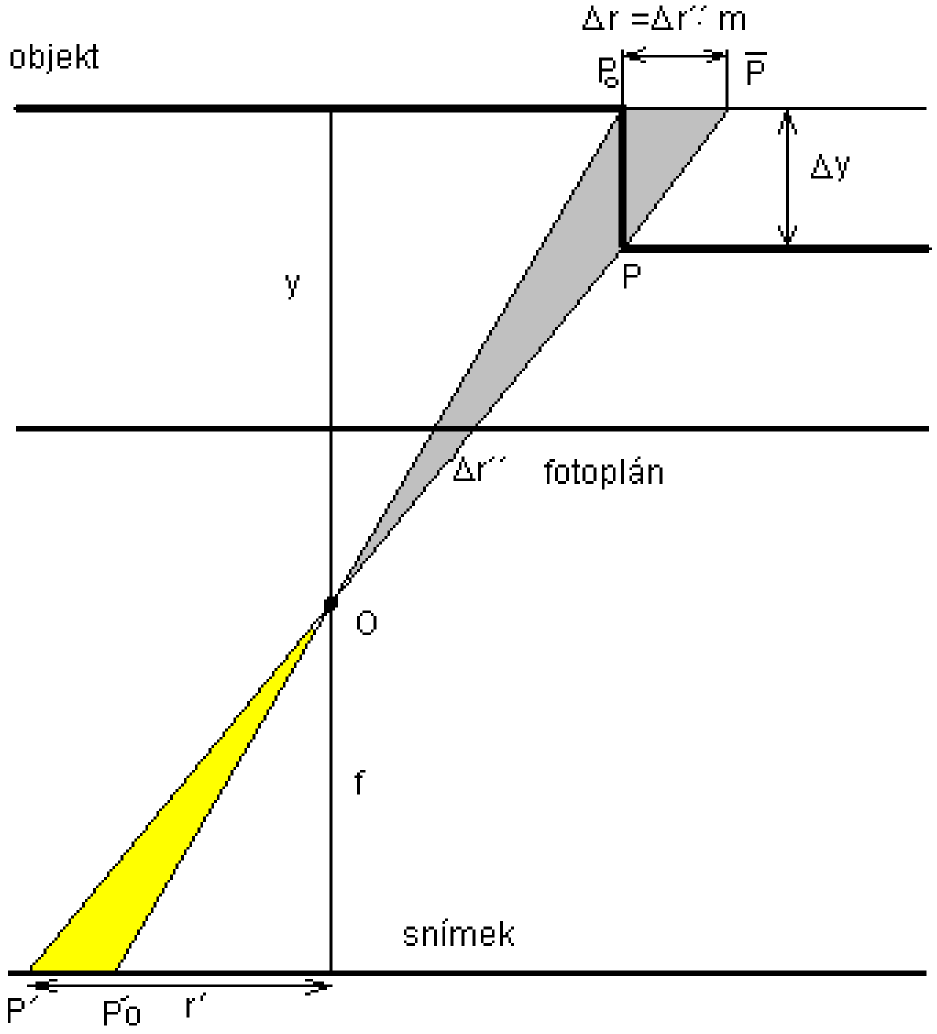
in fact, the **collinear transformation**

$$X = \frac{a_1x' + a_2y' + a_3}{c_1x' + c_2y' + 1}$$

$$Y = \frac{b_1x' + b_2y' + b_3}{c_1x' + c_2y' + 1}$$

Effect of depth zoning

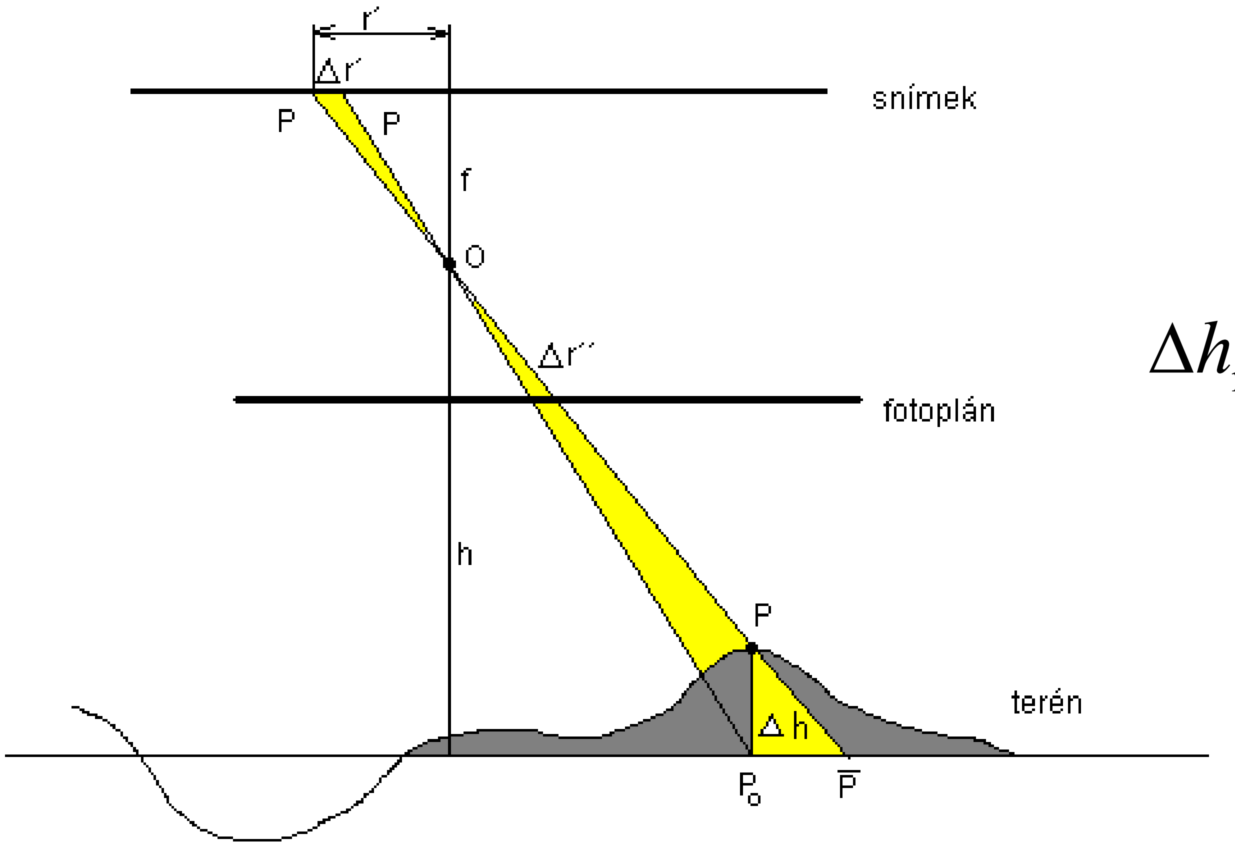
$$\Delta r = \Delta r'' \cdot m_F , \qquad \frac{\Delta r}{\Delta y} = \frac{r'}{f} , \qquad \Delta r'' = \frac{\Delta y \cdot r'}{f \cdot m_F}$$



$$\Delta y_{max} = \frac{f \cdot m_F \cdot \Delta r''_{max}}{r'}$$

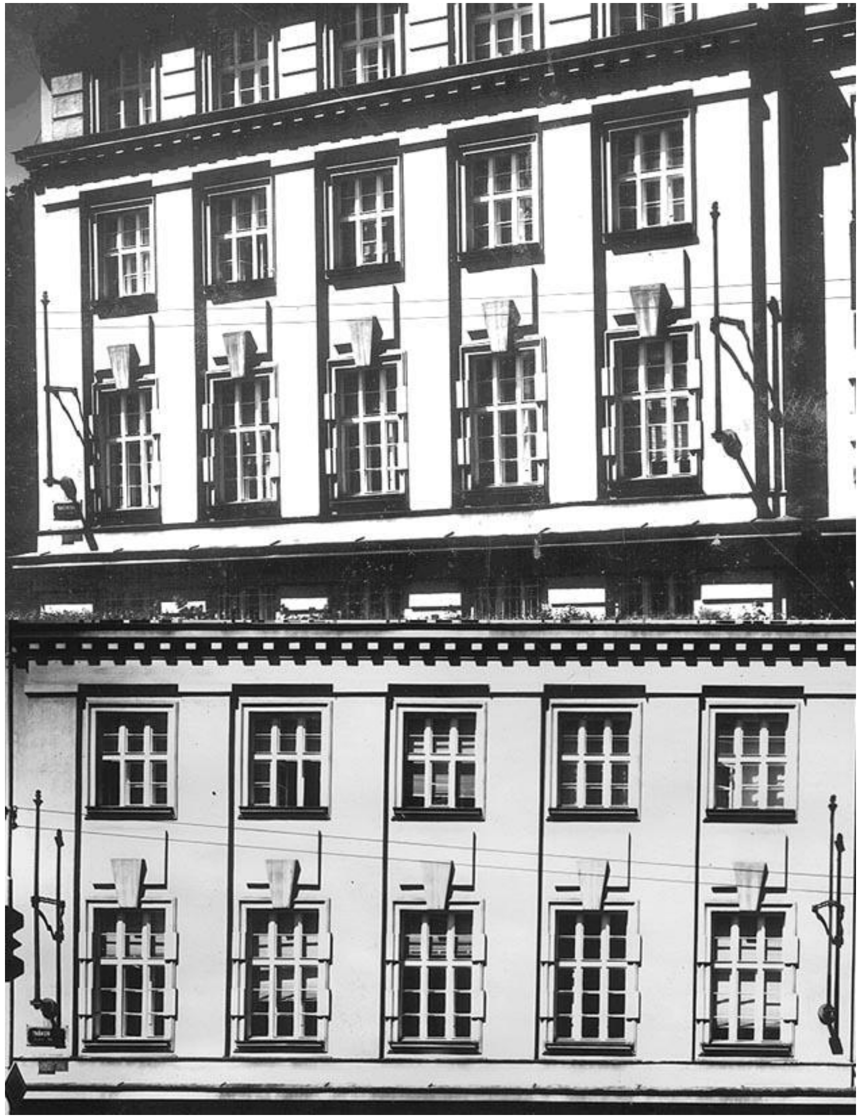
Effect of height zoning

$$\Delta r'' = \frac{\Delta h \cdot r'}{f \cdot m_F}$$



$$\Delta h_{\max} = \frac{f \cdot m_F \cdot \Delta r''_{\max}}{r'}$$

Digital redrawing



Original image and photo plan, taken by the measuring camera (left),
an image taken with an ordinary camera and its digitally redrawn form - an image vault,
due to uncorrected radial distortion is clearly visible (right)

Solution

$$X = \frac{a_1 x' + a_2 y' + a_3}{c_1 x' + c_2 y' + 1}$$

$$Y = \frac{b_1 x' + b_2 y' + b_3}{c_1 x' + c_2 y' + 1}$$

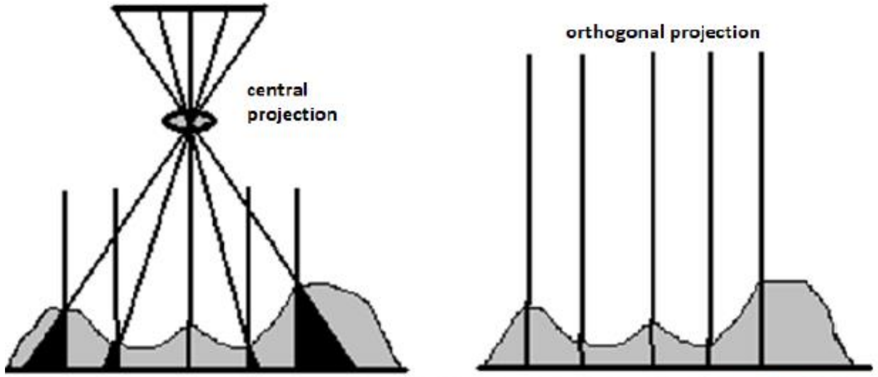
$$X = a_1 x' + a_2 y' + a_3 - c_1 x' X - c_2 y' X$$

$$Y = b_1 x' + b_2 y' + b_3 - c_1 x' Y - c_2 y' Y$$

$$\begin{pmatrix} x_1' & y_1' & 1 & 0 & 0 & 0 & -x_1' X_1 & -y_1' X_1 \\ 0 & 0 & 0 & x_1' & y_1' & 1 & -x_1' Y_1 & -y_1' Y_1 \\ x_2' & y_2' & 1 & 0 & 0 & 0 & -x_2' X_2 & -y_2' X_2 \\ 0 & 0 & 0 & x_2' & y_2' & 1 & -x_2' Y_2 & -y_2' Y_2 \\ x_3' & y_3' & 1 & 0 & 0 & 0 & -x_3' X_3 & -y_3' X_3 \\ 0 & 0 & 0 & x_3' & y_3' & 1 & -x_3' Y_3 & -y_3' Y_3 \\ x_4' & y_4' & 1 & 0 & 0 & 0 & -x_4' X_4 & -y_4' X_4 \\ 0 & 0 & 0 & x_4' & y_4' & 1 & -x_4' Y_4 & -y_4' Y_4 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ b_1 \\ b_2 \\ b_3 \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \\ X_4 \\ Y_4 \end{pmatrix}$$

$$\mathbf{A} \cdot \mathbf{a} = \mathbf{X}$$

$$\mathbf{a} = \mathbf{A}^{-1} \cdot \mathbf{X}$$

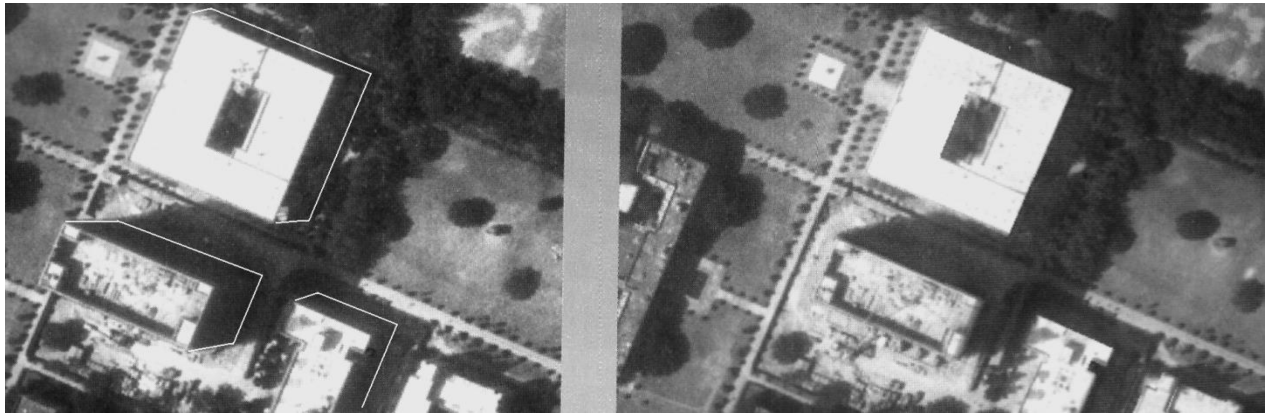


Digital orthophoto

Converting the central projection to orthogonal - to orthophoto - allows you to work with the image information as a map and insert it as a data layer in GIS.

Digital orthophoto

Digital orthophoto is a photogrammetric product - image conversion with central projection to orthogonal projection

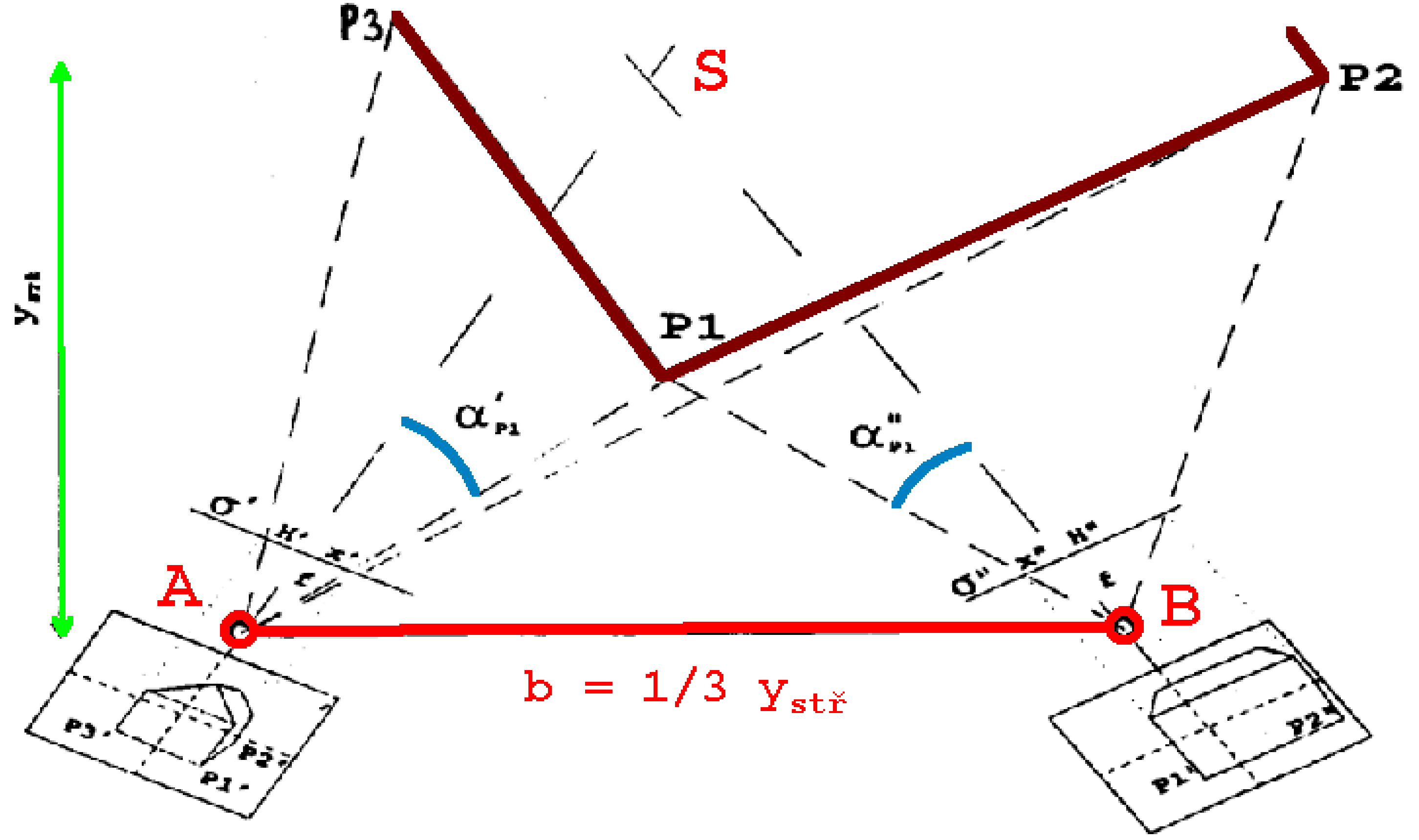


For creation:
Required image with known elements of internal and external orientation and DMT

Ortophoto and true ortophoto

The problem of mosaicking
- seamless orthophoto

Multi-frame photogrammetry
- intersection photogrammetry



The oldest photogrammetric method
- based on intersection of rays
- today only in digital form

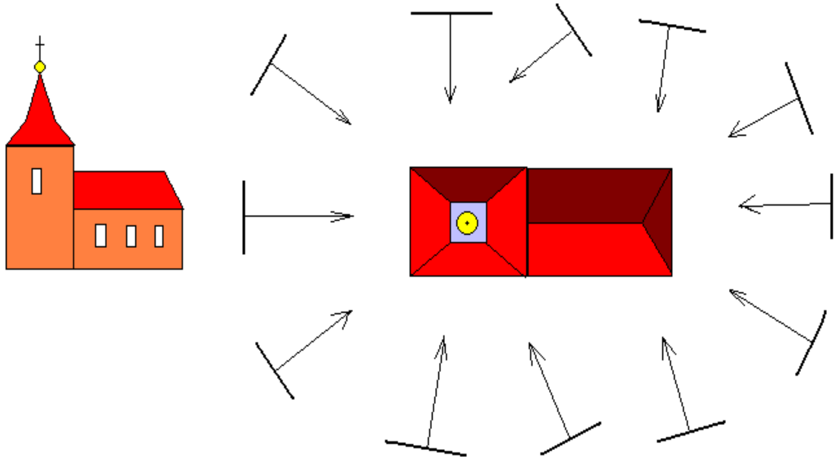
Today's solution

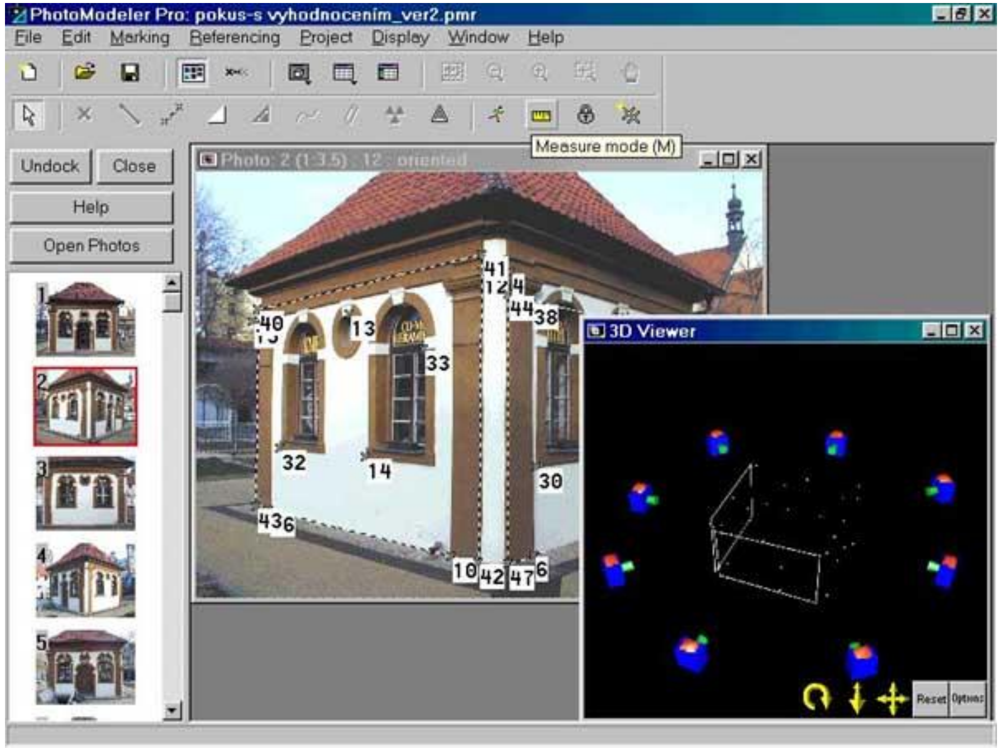
$$X = X_0 + (Z - Z_0)\frac{r_{11}(x' - x'_0) + r_{12}(y' - y'_0) - r_{13}f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33}f}$$

$$Y = Y_0 + (Z - Z_0)\frac{r_{21}(x' - x'_0) + r_{22}(y' - y'_0) - r_{23}f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33}f}$$

$$\begin{pmatrix} x' - x'_0 + \Delta x' \\ y' - y'_0 + \Delta y' \\ -f \end{pmatrix} = m \cdot \boldsymbol{R}^T \cdot \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix}$$

intersection photogrammetry





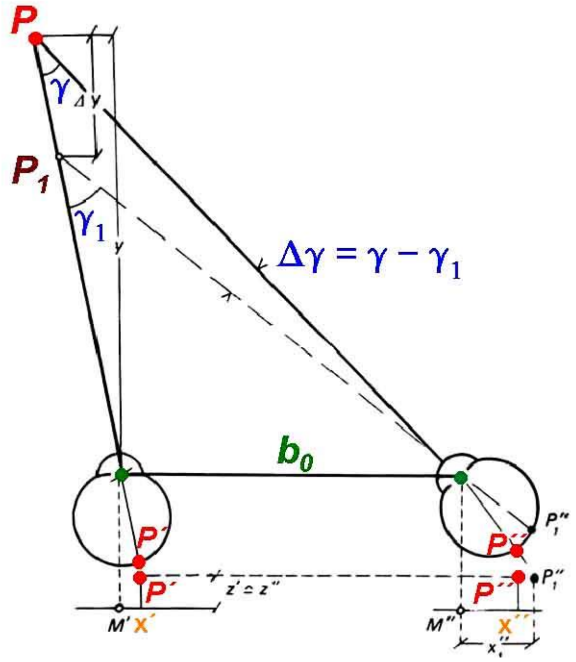
Stereophotogrammetry

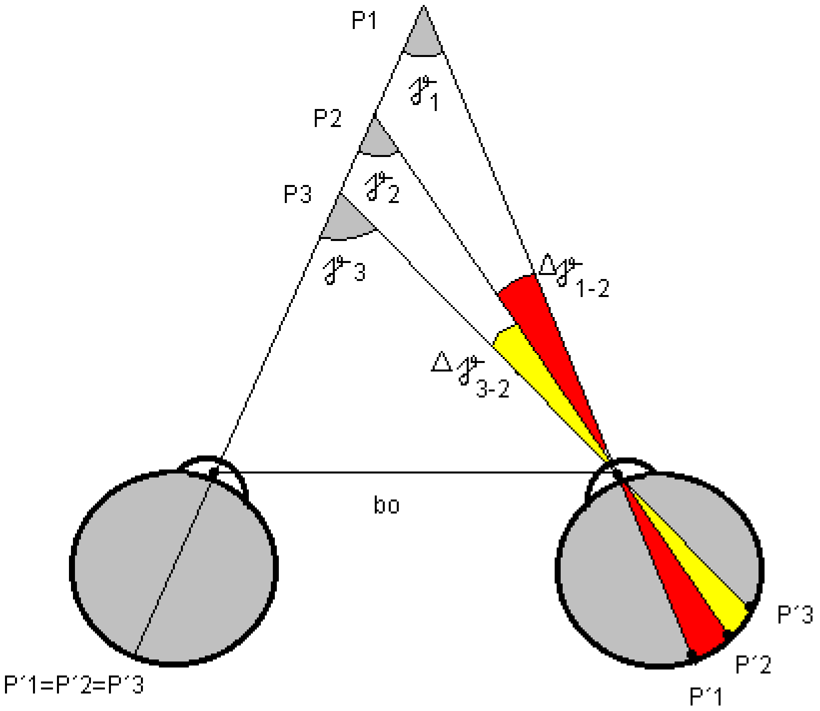
Stereophotogrammetric method introduced at the beginning of the 20th century (Pulfrich)

Evaluation based on stereoscopic perception also for non-signalized points

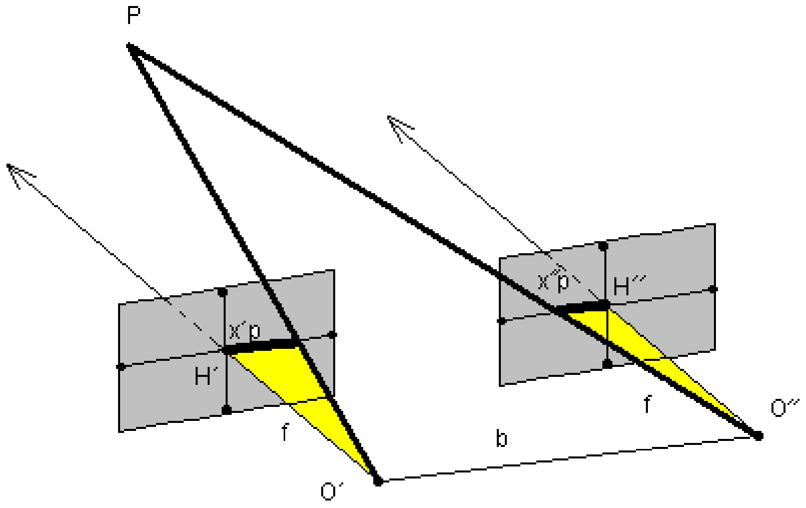
Photogrammetric stereoscopic observation and evaluation

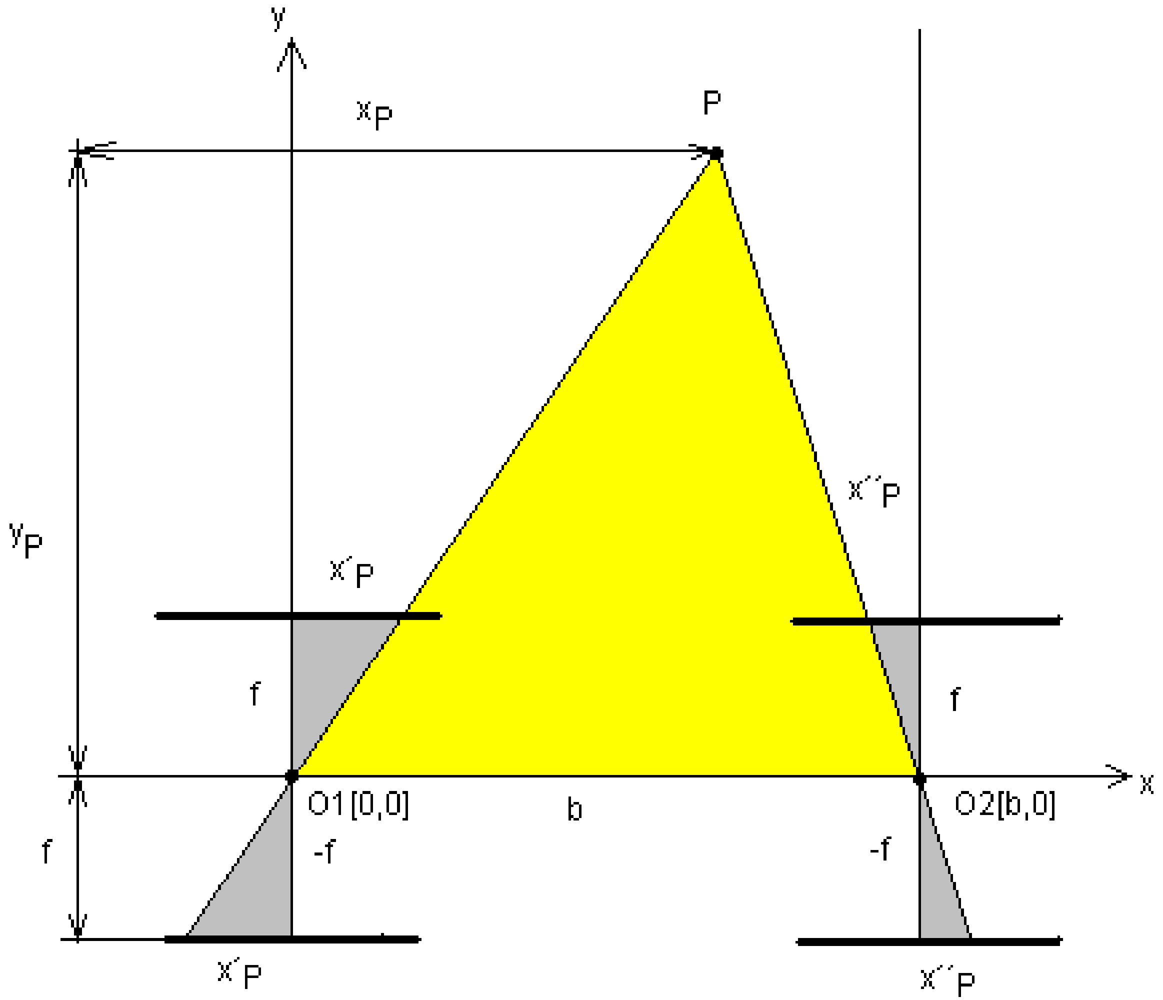
- artificial stereoscopic perception based on the natural perception of healthy eyes





Solution





Terrestrial stereophotogrammetry - normal case

In terrestrial applications, standardization of the external orientation elements can be ensured **R=E**

$$x = \frac{b \cdot x'}{p} \quad y = \frac{b \cdot f}{p} \quad z = \frac{b \cdot z'}{p}$$

b (AB) base

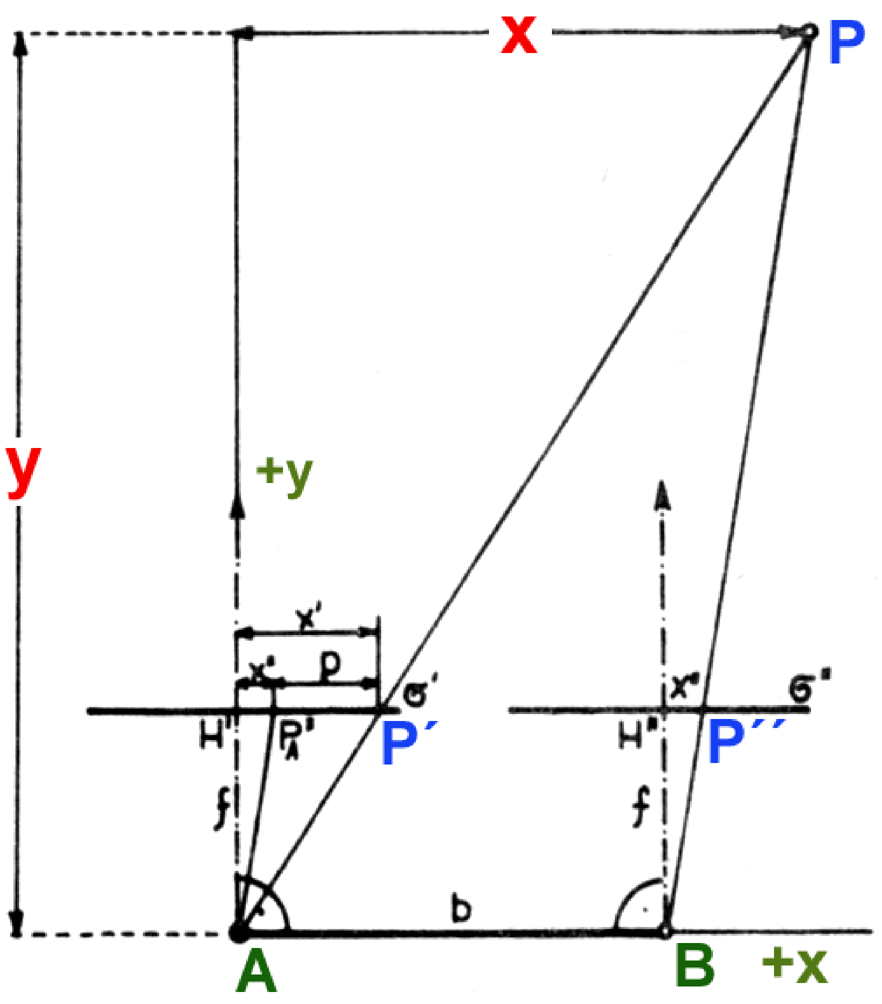
x y (z) model system

´, ´´ image planes

x´, x´´,z´, z´´ image coordinates

p parallax

f camera constant (focal length)



Accuracy of photogrammetry

$$\frac{y}{f} = \frac{b}{p}$$

$$x = \frac{b \cdot x'}{p} \qquad y = \frac{b \cdot f}{p} \qquad z = \frac{b \cdot z'}{p}$$

$$x = y\frac{x'}{f} \qquad z = y\frac{z'}{f}$$

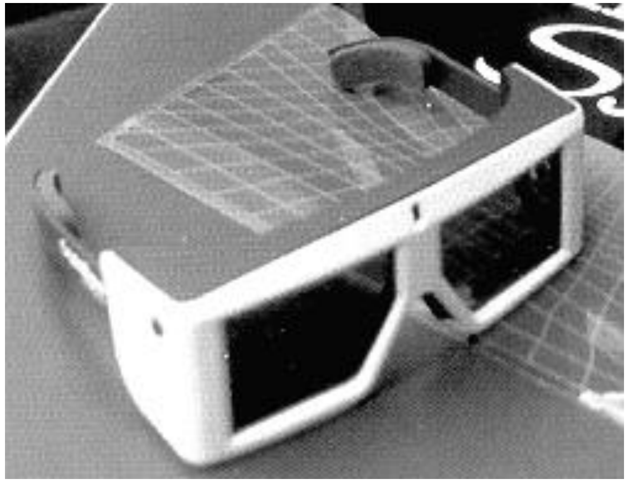
$$y = \frac{b \cdot f}{p}, \qquad dy = \frac{f}{p}db + \frac{b}{p}df - \frac{bf}{p^2}dp$$

$$dy = -\frac{b \cdot f}{p^2}dp, \quad p^2 = \left(\frac{b \cdot f}{y}\right)^2$$

$$dy = -\frac{y \cdot y}{b \cdot f}dp$$

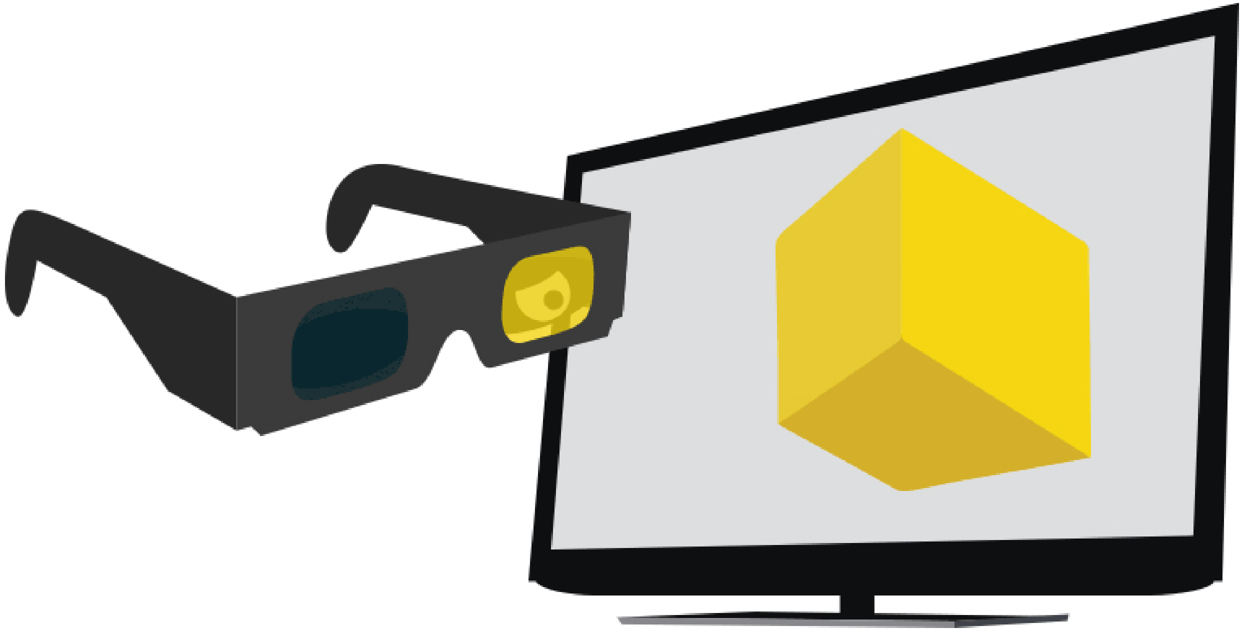
$$m_y = \pm\frac{y \cdot y}{b \cdot f}m_P$$

Device for stereoscopic perception



polarization system solutions

Active shutter 3D system



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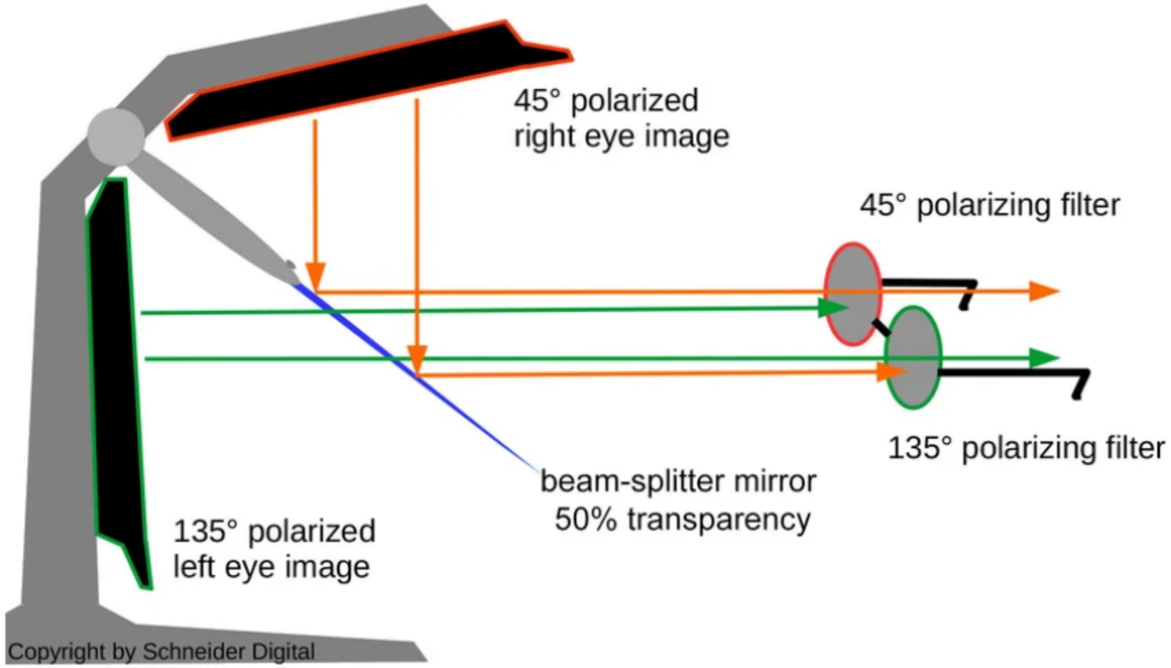
Pluraview stereo monitor



https://www.youtube.com/watch?v=cJ9MAntxPqw

Thank you for your attention.

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