Fundamentals of photogrammetry

1 Introduction to photogrammetry

1.1 Inclusion of photogrammetry

Classical photogrammetry is one of the sciences that deal with obtaining a large amount of information about objects based on exact measurements. There are several definitions of photogrammetry, and they change over time depending on the use of the results and the technology used. To name a few:

Photogrammetry is the science, method, and technology that deals with the acquisition of further usable measurements, maps, digital terrain models, and other products that can be obtained from an image, most often a photographic record (International, ISPRS)

or

Photogrammetry is a discipline that deals with the extraction of geometric information from an image record

Traditional photogrammetry is based on the photographic record. The word photography has its origin in Greek (*Photos* - light and *Graphos* - drawing, record). Photographic recording can be done in the form of an analogue light-sensitive layer (classical photography) or digitally. Various devices with different output accuracy can be used to obtain the image - from ordinary amateur cameras to special photogrammetric cameras. The acquired image - a snapshot, is used to capture the surrounding reality. From the position of points on the measurement images it is possible to derive the shape, size and location of the object of measurement in space, to determine the relative spatial position of individual points, to evaluate the position and elevation, etc. The word *photogrammetry* was first used by Meydenbauer (1858) and he used a total of three Greek words for it: *Photos* - light, Gramma - record and *Metrie* - measurement.

Photogrammetry as a source of primary information

Localized information is a fundamental input to decision-making in the landscape and in human activities, and its importance is increasing. The speed with which we can acquire the necessary information plays an important role. Its timeliness, comprehensiveness, longevity and quality significantly influence its price. Thus, intensive development is being seen in the fields that primarily acquire data and in the fields that enable data analysis leading to optimal decisions. This is provided by the information systems known as GIS (*Geographic Information System*). GIS have different degrees of accuracy regarding what they are used for; therefore, their mapping parts also have corresponding accuracy. Geodesy and cartography, if we think of it as a discipline supplying primary information about the territory, has three main sources of positional information with different accuracies:

1) basic geodetic methods (mapping and surveying using total stations, GNSS, etc.): cm area and better

2) ***photogrammetry*** *(aerial, terrestrial and satellite): cm and dm area, excluding m*

3) remote sensing (satellite and airborne): area m, km

The photographic image - as a carrier of information - can be used in various ways. *Photogrammetry* is mainly concerned with precise measurements on images, *DPZ* with measurements as well as the detection of quantitative and qualitative aspects of the objects depicted. *Photointerpretation of images* is the graphic or descriptive expression of content, the expression of various features that speak to the quality of the depicted object, carried out by trained persons for a precisely defined purpose, without making extensive measurements on the images; it was carried out in this way, especially during the World Wars and in the 1960s in the military on analogue images, but today it has lost its importance in this form and has been replaced by automatic methods on digital images and artificial intelligence (AI). Nowadays, especially with the advancement of technology and computing, there is a slow merging of previously separate disciplines such as *remote sensing* (RS), *aerial and satellite image interpretation* and *photogrammetry* into one new science using advanced computing and digital technologies (digital image processing). It is becoming one of the most important suppliers of localised information for Geographical Information Systems (GIS).

1.2 Use of photogrammetry

The ideal photographic image is the central projection of the subject. In photogrammetry, in simplified form, it is a process of converting the central projection, which is not suitable for displaying facts on maps, into a rectangular projection used for maps. Some of the procedures of photogrammetry can in this respect be included in descriptive geometry, where similar problems were solved long before the invention of photography. However, modern methods of photogrammetry no longer suffice only with the knowledge of descriptive geometry. Image processing is considerably more complex, based on sophisticated mathematical principles and solved nowadays by using special software and digital images on computers. Previously, analogue procedures had to be used on very sophisticated and complex mechanical-optical plotters, later analytical plotters, but always measured on original analogue images. The main difference between photogrammetry and other measurement methods is that the collection and measurement of information is not carried out on the object of measurement itself, but on the measurement images. The images can be taken at short notice, thus showing the immediate state of the object, and the measurement can then be carried out independently in the laboratory, in a quiet and modern working environment. The images have an important documentary value, the measurements can be supplemented, verified and extended with new findings at any time. Periodically repeated imaging extends the knowledge of changes in the object of measurement with time and thus provides additional important information for other scientific disciplines. Photogrammetry in general reduces the time for collecting information and for processing it. Particularly for medium and small-scale mapping, very significant financial and time savings can be obtained compared to mapping with conventional surveying methods. In many cases, no other method than photogrammetry can be used - for example, mapping inaccessible or remote locations or large areas.

Photogrammetry has found application not only in the field of *geodesy and cartography*, where it has long been used as an important mapping method, but also in various other areas of human activity. Among the most used are:

(a) construction: documentation purposes, documentation for reconstruction, measurement of deformations of buildings and their parts
(b) conservation: documentation of heritage
(c) monitoring the progress of construction or extraction of raw materials, inventory of landfills
(d) agriculture: monitoring of seeding plans, slope gradients and exposure
(e) forestry: stand maps, logging progress, calamities
(f) water management: determination of digital terrain model, catchment modelling, extent of flooding
(g) engineering: surveying precision engineering products, monitoring the accuracy of assembly of large parts, e.g. in shipbuilding
(h) medicine: documentation, monitoring of rehabilitation results, plastic surgery, dental applications, movement studies, ergonomics
(i) anthropology
(j) Police: criminalistics, documentation of serious traffic accidents
(k) ecology: landfill monitoring, pollution
(l) urban planning, 3D urban models
design, shape determination and modelling

1.3 Historical overview

OVERVIEW OF THE DEVELOPMENT OF PHOTOGRAMMETRY

The theoretical beginnings of photogrammetry date back to a time long before the invention of photography. Considering that images are central projections of measurement objects and photogrammetry deals with their measurement, the beginning of photogrammetry can be dated back to 1032, when the Arab scholar Al Hassan bin Al Haithm (965-1039) first described the *"camera obscura"*. Today, for example, this discovery is depicted on an Iraqi banknote (Figure 1).

Fig.1.1: The principle of centre projection

During the Renaissance, Leonardo da Vinci (1452-1519) described the "*pinhole camera*" for the construction of central projections. However, pinhole cameras were not much used because of their low luminosity. The camera, equipped with a conjugate lens and a concave mirror, was described by G.B. della Porta in 1588 (Foundations of Cinematography). The construction of the light camera, which was perfected under John Kepler as "*camera clara, camera lucida*", laid the first real foundation for photogrammetry. In 1605 Galileo Galilei invented the telescope, in 1657 Schott Kasper constructed the first portable camera (*box camera*). In 1727, J.H.Schultze investigated the photochemical effect of light, which led to the invention of light-sensitive AgNO3 in 1750(G.B.Beccaria) and the invention of the higher-quality AgCl in 1777 (C.H.Scheele). The reconstruction of the obtained perspective images was the subject of theoretical papers by Taylor (1715) and J.H.Lambert (1759). The first practical reconstructions of perspective images were carried out by M.A.Cappeler (1726) in the Alps, and later by Beatemps-Beaupré (1791-1792) for the acquisition of a plan of the coast of the island of Santa Gruz. However, these methods required hand-drawn images, considerable painting experience and could not be widely applied. They fell almost into oblivion. They were revived again after the invention of photography by Niepce and Daquerre (1839), the name of the photograph coming from J. Herschel in the same year. Already two years after the invention of photography, the Slovak scientist Prof. J.M. Petzval constructed a modern lens, introduced exact computational methods into geometrical optics and thus contributed to the development of photogrammetry. In 1851 W.H.Talbot was the first to use photographic paper and F.S.Archer invented colloidal wet plates. The first aerial photographs were taken by the famous French photographer G.F.Tournachon (called Nadar) in 1858. The first phototeodolite was designed by A.Laussedat (1859). From 1860 came the images of Boston taken from a balloon (J.W.Black and A.King) and the first military use during the US Civil War (T.Lowe). For the purposes of practical photogrammetry, photography was first used in France for mapping in 1861. For mapping, the method of cross-sectional photogrammetry "*métrophotographie*" was used. The name "photogrammetry" dates back to 1858, when it was used by the German A. Meydenbauer, who is also considered the pioneer of photogrammetric documentation of historical objects. Commissioned by the Prussian state, he later created a unique archive of about 16,000 photogrammetric images of historic buildings between 1885 and 1909, some of which are still extant, and developed practical graphic methods for photogrammetric evaluation, especially of building facades. In 1873 the English photographer J.Burger used gelatine emulsion for photographic plates and in 1874 R.Kennett produced dry plates. The invention of film by G.Eastman in 1884 (paper film) and its introduction into practice in 1889 (celluloid film) brought great progress, especially for later aerial photogrammetry. In the same year G.Eastman constructed the first roll film camera. In 1886 the pictures of Kronstadt were taken by A.M.Kovaňek, in 1890 the first classical aerial photocamera was constructed by the French company Pathé. The first practically used phototheodolites were constructed independently by Porro (Italy, 1865), Koppe and Finsterwalder (Germany, 1896, 1895). In 1904 the Zeiss Jena company constructed the phototheodolite 19/1318, which was practically produced under the name PhoTheo until the 1960s and for terrestrial photogrammetry. Later these cameras were replaced by more modern cameras like the UMK Jena or Wild.

Technologically, intersection photogrammetry was used, which represents forward intersection as we know it from geodesy. However, this way of working has a major drawback in the difficulty of identifying corresponding points on the images. A new technology was therefore sought to remove these shortcomings. It was found in the late 19th century in the relatively simple principle of stereoscopy. In 1892, F. Stolze took stereo images and proposed the principle of spatial measurement marks. In 1894 Hauck proposed a stereoscopic evaluation device. The practical pioneer of stereophotogrammetry was C.Pulfrich (Zeiss Jena), who in 1901 constructed the first device for stereoscopic measurement of image coordinates - the stereocomparator. This is still the most accurate instrument for measuring on images. Stereophotogrammetry made it easier to identify points on images and increased the accuracy of photogrammetry enormously. The design of the stereocomparator also laid the foundation for later analogue evaluation machines, and for the introduction of elements of mechanization and automation in evaluation work. The stereocomparator, however, allowed only spot evaluation of images and required laborious computational and imaging work. The improvement of the stereocomparator and the mechanization of the computational work by introducing elements of mechanical analogy was greatly contributed to by E. Orel, a member of the former Geographical Institute in Vienna, who in 1909-1911 constructed the first "*Autostereograph*", which was produced from 1909 in the Carl Zeiss Jena works as the "*Stereoautograph*". This simplified and facilitated the construction of the position and elevation components of the map from stereophotogrammetric images. Terrestrial photogrammetry could only be used advantageously for mapping smaller areas of clear view or in the mountains. Larger portions of the land surface can be more economically mapped using aerial photogrammetry, where the photographic instrument is placed in an aircraft (or formerly a balloon). The theory of aerial photogrammetry was developed in the late 19th century by Austrian Th.Scheimpflug. In 1911 he also constructed the first redrawer for redrawing a tilted image of flat terrain to the scale of a map.

Aerial photogrammetry came into being only at the beginning of the 20th century with the development of aviation (Wright brothers, 1903). In 1909 L.Blériot flew across the English Channel and in March 1912 Captain C.Pizza took the first ever photographs of the Turkish lines from an aeroplane for military use during the Italian-Turkish conflict. Very quickly this technology with the interpretation of images spread already in the First World War.
At the beginning of World War I, more than half of all aircraft were used for aerial photography. A "projection multiplex" was used to evaluate stereoscopic images.

In 1915, Gasser produced the first double projector for processing pairs of aerial images. In the interwar period, there was a rapid development of analogue plotters that performed reconstruction (analogy) of the state during the imaging and subsequent evaluation of the image content from the model form to the selected map output. Different designs were developed on the basis of optical (double projector, multiplex), optical-mechanical (Stereoautograph) and mechanical (Wild A5). In 1920 R.Huggershoff constructed the first *Autocartograph*. Already in 1923, the first *Stereoplanigraph* was constructed at the Carl Zeiss factory in Jena according to Bauersfeld's design. Independently of these German constructions of photogrammetric analogue plotters, constructions of photogrammetric instruments and aids appeared elsewhere in the world. In France the first *Stereotopograph* of Poivilliers was constructed, in Switzerland the first *Autographs* of Wild, in Italy the first machine constructions of Nistri and Santoni.

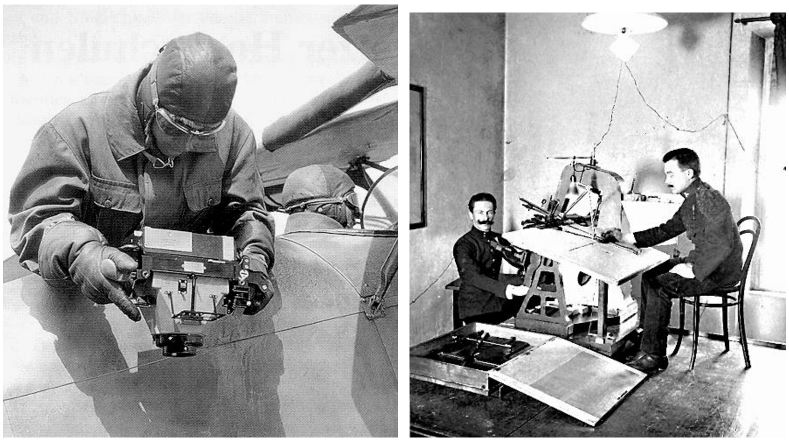


Fig.1.2: Historical photogrammetry - military photography and image processing

A great boom in aerial photogrammetry was recorded in Russia after Lenin's decree on the establishment of the State Geodetic Service (1919), as large parts of the territory of the former Russia were not sufficiently mapped and some not even at all. In terms of speed, aerial photogrammetry was adopted as the basic mapping method. Due to the isolation of Russia after the Revolution, it was necessary to organise research, to look for new technological methods and to create the necessary base to produce photogrammetric instruments and tools. Their designer was mainly F.V.Drobychev. Russian photogrammetric procedures and instruments at that time were different from the rest of the world. They were in many cases less accurate, but simpler and much faster to work with than classical procedures. In a relatively short period of time, it was possible to map huge areas.

Technique and technology began to develop very rapidly. In 1935, the first Kodakchrome colour film was launched. However, the development of photogrammetry was temporarily interrupted by the Second World War. During the Second World War, some new chambers and machines were constructed and methods of using photogrammetry were developed, but mainly for military purposes. Systematic research and further development of photogrammetry for non-military and economic purposes did not take place again until after 1945.

Other types of photogrammetric plotters were developed like "*differential redrawers*", which converted the central projection to orthogonal projection also in the area with vertical division ("*orthophoto*") and enabled the development of a new "*integrated method „of* aerial photogrammetry.
Around 1960, the first electronic correlation systems were developed which allowed for semi-automatic processing ('*stereomats*'). With the development of computer technology, an evolutionary shift towards analytical methods began. Analytical methods of photogrammetry had been known for a long time but were not used because of their numerical complexity. It was necessary to wait for sufficiently fast computers with significant memory capacity. The reason for the effort to construct an analytical plotter was that the measurement of image coordinates on stereo or monocomparators was absolutely the most accurate. Further, if we have the image coordinates of individual points, we can correct them numerically for various effects and the outputs can be stored in digital form. Newly developed analytical methods based on the work of Gast (Germany, 1930) were presented by Schmid and Schut (USA,1953,1958). These led to the construction of precision comparators (PSK, Kern MK1), to the development of aerotriangulation

programs (e.g. Ackerman 1964, Brown 1967) in versions for block or bundle alignment (e.g. PAT-M, BLUH, etc.). The final product was the construction of an analytical plotters that combined the basic strengths of the analytical methods. The principle of an analytical plotter based on the solution of a direct relationship between image and geodetic coordinates was put forward by the Finnish photogrammetrist U. Helava as early as 1957, but a commercially successful design had to wait a full twenty years. The basis of the analytical plotter was practically a stereocomparator with motorized movement of the images in the carriers, digital readout of the position of the controls and a powerful computer with an operating program (originally AP/C - Bendix/OMI 1964, Planicomp C100, Zeiss 1976, DSR-1, Kern 1980, Aviolyt AC-1, Wild 1980). Analytical plotters saw a rapid development after 1980, when computing technology took the form of a sufficiently fast personal computer. Due to the evolution of technology and advances in technology, analytical photogrammetry had a short life. Already in the 1990s it was replaced by digital technology.

The truly revolutionary change came in the mid-1980s. The breakthrough in computer technology enabled the first digital systems to be developed and digital photogrammetry ("*softcopy photogrammetry*") was born. After 1988, stereo-photogrammetric workstations capable of processing virtually any measurement image in a very loose configuration were developed (e.g. Imagestation/ Intergraph, Leica/Helava, Phodist/Zeiss, etc.). With the new possibilities of computer technology, intersectional photogrammetry via *réseau* cameras was reintroduced (e.g. RolleiMetric, 1986). New applications, simplification of image capture and cheapening of the whole process were brought by new digital cameras (after 1995). The ease of use of digital cameras, their constantly improving parameters and the existence of programs for direct 3D evaluation based on conventional images (e.g. Photomodeler) lead to the availability of digital low-cost photogrammetry to a wider professional public. A breakthrough in technology can be found after 2010, when automated photogrammetry technologies based on SfM (Structure from Motion) and MVS (Multi View Stereo) principles became widespread. This allows even complete laymen to use digital automated photogrammetry to create high quality textured 3D models. Nowadays, Agisoft Metashape, RealityCapture, etc., are prominent representatives. Another distinctive feature has become the merging of photogrammetry and remote sensing, which brings satellite technology into the field of photogrammetry as well. The use of artificial Earth satellites, until recently only in the field of small and medium scale mapping, has been extended to large scale mapping thanks to new operating systems with resolution below 1 m (after 1999) and in many countries they are used for cadastral surveys. Satellite multispectral and panchromatic imagery is used for mapping and thematic mapping. In addition to the now obsolete conventional film cameras, scanning radiometers (scanners), which operate over a wider spectral range and allow stereoscopic imaging, and SAR ('*Synthetic Aperture Radar*') systems, which can be used to produce elevation maps in the form of DRM (Digital Relief Model), are mainly used. Laser scanning, used in both terrestrial and mobile terrestrial and airborne applications, became a novelty after the turn of the millennium. The result, as with automatic digital photogrammetry, is a point cloud. If laser scanners are also equipped with a camera, the point cloud is textured, as in photogrammetry, and used to create 3D mesh models. In the case of larger objects, it is possible to talk about the creation of a digital twin.

2 Basics of photogrammetry

2.1 Overview and classification of photogrammetric methods

Photogrammetry as a method of non-contact determination of spatial coordinates of objects has been performed for about 150 years. During this period, it has undergone a very heterogeneous development. The first half-century was marked by the search for the application of a new method, exploration of the foundations and taking images, technologically only intersection methods of terrestrial photogrammetry were used. The beginning of the 20th century brought stereoscopy and aerial photogrammetry, which led to the construction of a few very sophisticated analogue plotters for image evaluation. This is how photogrammetry developed for the next half century. In the last fifty years, with the development of computers, the former concept of photogrammetry has changed completely, gradually moving to analytical methods and, since the 1990s, to digital methods, which marked a major technological change and became completely dominant at the turn of the century. Satellite photogrammetry and laser scanning technology were developed, which, in conjunction with digital photogrammetry, brought a completely new view of 3D object documentation.

During its development, photogrammetry has been divided into different types according to the methods of image acquisition and evaluation. In different parts of the world, photogrammetry is classified differently. Classical European photogrammetry was divided into terrestrial and aerial photogrammetry, because terrestrial photogrammetry started in Europe and because technologically, they were different processes. Equipment for e.g. terrestrial photogrammetry could not be used for aerial photogrammetry. This division has traditionally been presented in Central and Eastern Europe, as analogue equipment and procedures that corresponded to this division were used practically until the 1990s. This entailed different designations, e.g. coordinate axes, angles, etc. Nowadays, this division has little relevance as all types are now handled with the same technology and even on the same equipment. In Western countries, no great distinction was usually made between terrestrial and aerial photogrammetry, the theory being derived for aerial photogrammetry, and terrestrial photogrammetry was regarded as a special application whose share in the total photogrammetric work was only a few percent. However, in the last few years the situation has changed, with the development of high-quality digital cameras terrestrial applications are increasing and separate programs with their own markings or specialized modules for terrestrial applications are emerging. Currently, photogrammetry can be divided according to:

- position of station

- the number and configuration of the images to be evaluated

- the technological method of processing

- type of output

The individual types will be briefly described in the following text, more detailed information is the focus of the following text.

1. According to the position of the station from which the image was taken, photogrammetry can be divided as follows:

Terrestrial ('ground-based', close-range) photogrammetry

In the terrestrial photogrammetry method, the station is usually stationary, located on the Earth. Historically, it has been possible to geodetically determine the exact coordinates of the stand and the spatial orientation of the image (they were not in motion) when taking photographs. The images were taken based on precise alignment and positioning of instruments; therefore, processing such images is easier. However, the disadvantage of terrestrial photogrammetry is that the individual objects of measurement are mutually occluded, and the image often contains a significant part of non-evaluable areas (hidden spaces), and it has another significant disadvantage - the accuracy of measurement in the spatial component (distance to the object) **decreases with the square of the distance**. For this reason, in particular, terrestrial photogrammetry is suitable for objects that are approximately equidistant (house facades, steep riverbanks, quarry walls, rocks, etc.). The range of terrestrial photogrammetry depends on the camera and is up to 500 m in extreme cases, but tens of metres in classical cases. Recent trends are special applications of terrestrial photogrammetry, which can be found in a few completely different fields (medicine, design, engineering, etc.), the great development in the documentation of e.g. monuments is mainly due to affordable digital cameras and processing programs. This whole field is called "*Close Range Photogrammetry*" in the English literature and its importance is growing.

Aerial photogrammetry

In the aerial photogrammetry method, the station for taking the image is located in an aircraft or other moving vehicle. The image shows a much larger area than in terrestrial photogrammetry. The disadvantage is that the spatial position of the image at the time of acquisition cannot usually be determined with sufficient precision and therefore the processing methods will be more complex than with terrestrial photogrammetry. As the images taken are mainly approximately perpendicular, the distance from the point of photography to the objects (relative to the flight height) is approximately the same and therefore the accuracy of the evaluation is approximately the same. It is in this area that significant progress has been made recently, due to the introduction of GNSS/IMU devices that allow the determination of the exterior orientation features of individual images in flight.

Satellite photogrammetry

Satellite photogrammetry originated in the 1960s, based on spy and interpretive imagery from specialised satellites. Satellite imagery was also used in our country to create photomaps. Practical civil application came after the launch of the Spot-1 satellite in 1984, as the satellite was equipped with an electronic scanner with a resolution of 10 m in panchromatic mode with the possibility of creating stereo images. However, the images obtained in this way could not be evaluated on conventional equipment and special software in the field of digital photogrammetry had to be developed. Today satellite photogrammetry is a special but otherwise common technology, and the resolution of today's commercial satellites is about 30 cm!

Drone photogrammetry

After the turn of the millennium, the first designs of unmanned remotely piloted flying devices, popularly called drones, began to appear. The correct name RPAS (remotely piloted aircraft system) has not become very popular, and UAV or UAS (unmanned aerial vehicle, or unmanned aerial system) is often used. By system, we mean the whole, including the ground segment to be piloted. Drones have expanded considerably since 2010 and there are now several commercial high-quality systems not only for photogrammetry, laser scanning and remote sensing. Multicopters, which are cheaper and can easily fly around smaller objects, dominate, but winged systems with greater endurance in the air and longer range are also in use. From the point of view of photogrammetry, especially multicopters for heritage purposes are excellent helpers.

2. According to the number of images evaluated, photogrammetry is divided into:

Single-image photogrammetry (obsolete)

Single-image photogrammetry uses only single measurement images. Since only plane coordinates can be measured on a single image, single-image photogrammetry can again only determine the plane coordinates of the object of measurement and can be used when the object of measurement is plane or close to plane. The relation describing the solution of single-image photogrammetry is called collineation and is expressed by a projective transformation. Terrestrial photogrammetry uses single image methods for the creation of photomaps of planar objects, e.g. not very rugged facades of houses, in aerial photogrammetry the axis of view is mostly vertical, therefore single image methods can be used to obtain the positional component of a map of a flat area; however, oblique images can also be processed in the same way, again assuming the flatness of the area being evaluated. In the case of spatial division of such an area, radial displacements of individual objects occur, which prevent an accurate evaluation. However, for certain work this is a very simple and inexpensive method.

Multi-image photogrammetry

Multi-image photogrammetry is used for 3D processing and always requires at least two overlapping images. Only 2D coordinates can be determined from a single image, and to get to 3D coordinates we need another measurement - that is another image. The object to be measured must be simultaneously displayed in both images and from the image coordinates of the same object in both images, its 3D spatial position can be calculated.

If the images have at least approximately parallel shot axes, stereoscopy can be used. To process the content of the images, an artificial stereoscopic perception is used, which makes it possible to create a spatial model of the object of measurement, we speak of stereophotogrammetry. Stereophotogrammetry, due to its versatility, is the most used today.

If the axes of the images are convergent to each other, we speak of a multi-image spatial intersection (resection, *Multiphoto Orientation*). Technologically, this is intersection photogrammetry. A convergent set of oriented images can only be evaluated pointwise (or by individual elements - partial primitives in the image) provided that the same point or object can be identified in at least two images.

1. According to the method of image processing, i.e. according to the method of converting image coordinates to spatial coordinates in the chosen coordinate system, photogrammetry can be divided into the following technologies:

Analog (historical) methods

In this technology, which is no longer used today, an analogous state to that of the actual imaging was created mechanically, optically or by a combination of the two. Analog image processing requires the use of precise, complex, single-purpose analogue plotters, which are no longer used today. For completeness, the production of analogue plotters at the Zeiss Jena factory was discontinued only in 1990.

Analytical methods (out of date)

Here, a difference must be made between analytical evaluation of the image content and analytical plotters.

Analytical image content evaluation uses a spatial transformation, which is solved on a computer, to convert the image coordinates into a geodetic system. The image coordinates are measured on relatively simple but accurate comparator-type machines, and the transformation is nowadays performed on any powerful computer. Virtually any image (taken by different cameras and arbitrarily rotated) can be processed in this way. For stereophotogrammetric analytical processing, it is advisable to use images with at least approximately parallel axes of view and sufficient overlap for the best possible stereoview. In this case it is not necessary to have signal points for detailed evaluation. For methods using the principle of intersection photogrammetry, it is again appropriate to use images with a suitable angle of intersection between the axes of the images (analogous to intersection from angles); detailed points must be naturally or artificially signalled.

Analytical plotters use the design of a stereocomparator in conjunction with a computer. Work is carried out on the original images and transformation keys are calculated after the necessary image orientations. The evaluator controls the model coordinates from which the image coordinates are computed, to which the machine automatically adjusts under stereo vision conditions. At the same time, the geodetic coordinates of the scan points are calculated.

Digital (present) methods

Digital technology uses digital images. Spatial transformation is also used to convert the image coordinates into a geodetic system, which is solved on a computer. The image coordinates are measured directly on the screen. The simpler systems can make do with a conventional computer and a program; for stereo methods, the computer must be supplemented with hardware accessories to enable stereo vision.

3. According to the recording of the output values of photogrammetric image processing, photogrammetric methods can be divided into:

Graphical

In graphical methods, the result of the image processing is directly marked graphically on a drawing table connected to the processing instrument (analogue or analytical plotters). Graphic methods of processing are relatively fast for an experienced evaluator, during the mapping process a cartographic original of the positional or even elevation component of the map is created directly. However, such output is nowadays out-of-date, because the result cannot be further processed directly by computer technology and cannot be reproduced or edited in good quality. Moreover, the result has only graphical accuracy (about ± 0.2 mm in the scale of the original). It is no longer used today.

Numerical

This basic method of evaluation today consists in automatically registering the coordinates of the individual points to be evaluated in the computer memory or on another data medium and processing them either directly or in another processing system into the final form. The results are in vector form (lines, points, polygons, surfaces, attributes) or in raster form. The advantage is their portability, storage, editing, etc.

3 Theory of optical imaging

3.1 Ideal projection

The best approximation of central projection is the pinhole camera (camera obscura), the principle of which was known and used already during the Renaissance. Its use has not been widespread, mainly because of its very low luminosity. The pinhole camera does not contain a lens and, provided the aperture is small enough, it is a precise central projection. This means that the object and image angles are equal and the basic relations (3.1) hold (neglecting light bending). An idealised physical abstraction, the so-called "thin" lens, satisfies the same assumptions.

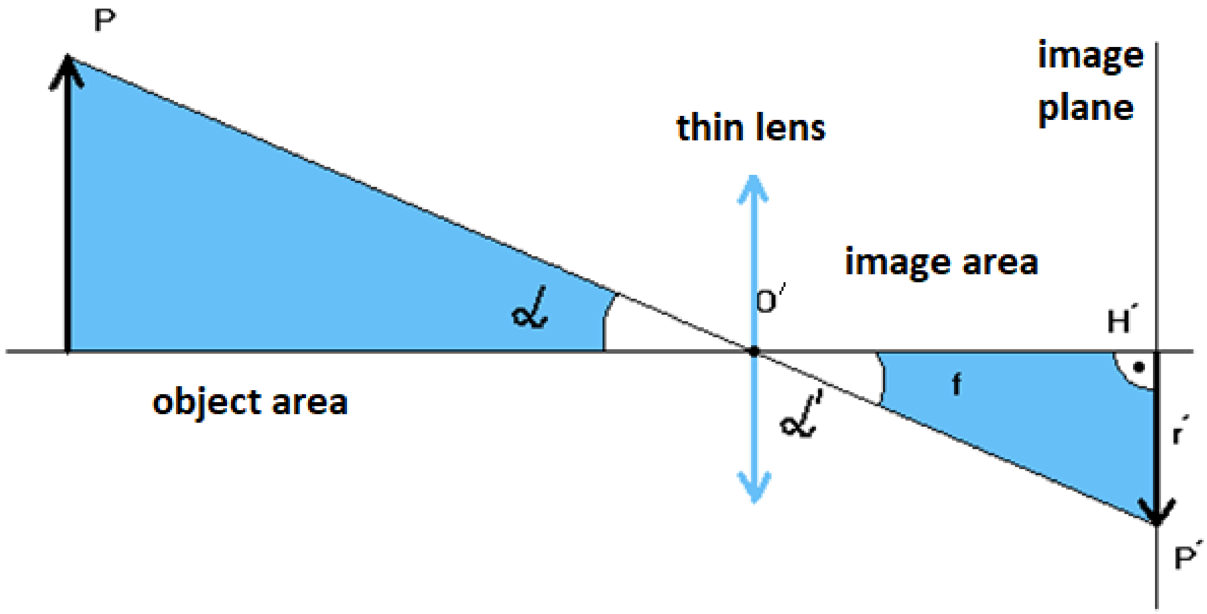


Fig.3.1: Ideal central projection.

$$\alpha = \alpha'$$
$$r' = f \cdot tg\alpha \qquad (3.1)$$

For further numerical solution we need to know the value of *f (*the so-called camera constant*),* which in the idealised case is equal to the perpendicular distance from the centre of the projection to the image plane.
To a first approximation, the camera constant *f* can be set equal to the focal length of the ideal lens, and the position of the point *H'* from which the radial distance *r'* is measured can be taken as the intersection of the optical axis with the image plane. The scale of the image through the lens can be calculated, e.g., as:

$$1 : m_s = \frac{r'}{r} = \frac{b}{a} \qquad (3.2)$$

3.2 Lens imaging

In photographic imaging, the central projection is done optically. Optical projection would match geometric centre projection only when using a "pinhole lens".

A real lens consists of several optical elements and its thickness is certainly not negligible. Each lens has a defined optical axis passing through the centre of the lens on which the centres of curvature of the individual lenses are supposed to lie (Figure 3.2).

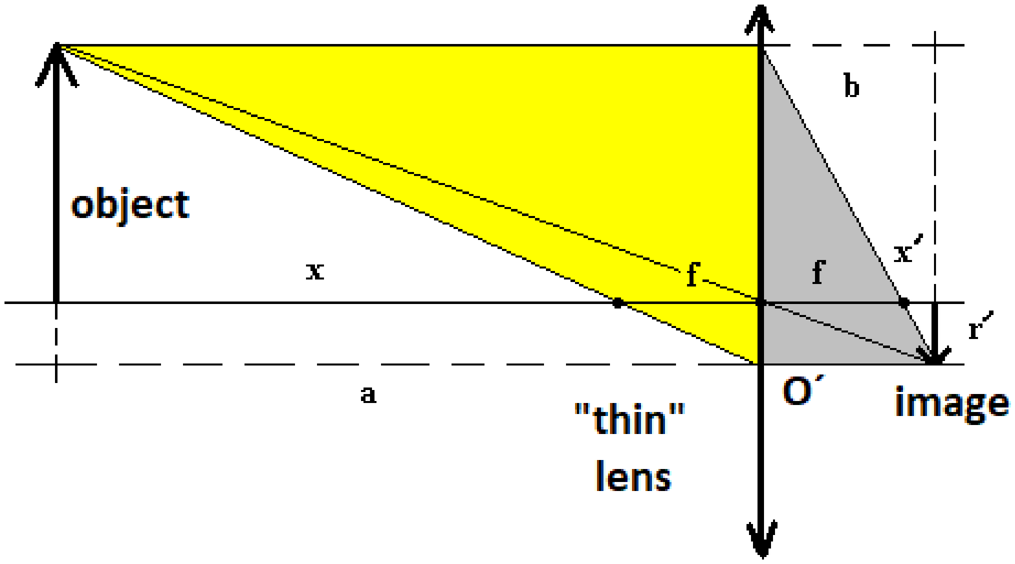


Fig.3.2: Thin lens imaging

For an ideal lens, the lens equations that can be derived from the figure apply:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} \tag{3.3}$$

or its Newtonian form:

$$x \cdot x' = f^2, a = (x + f), b = \left(x' + f\right) \tag{3.4}$$

where *a* is the object distance of the point, *b* is the image distance of the point, *f* is the focal length of the thin lens.

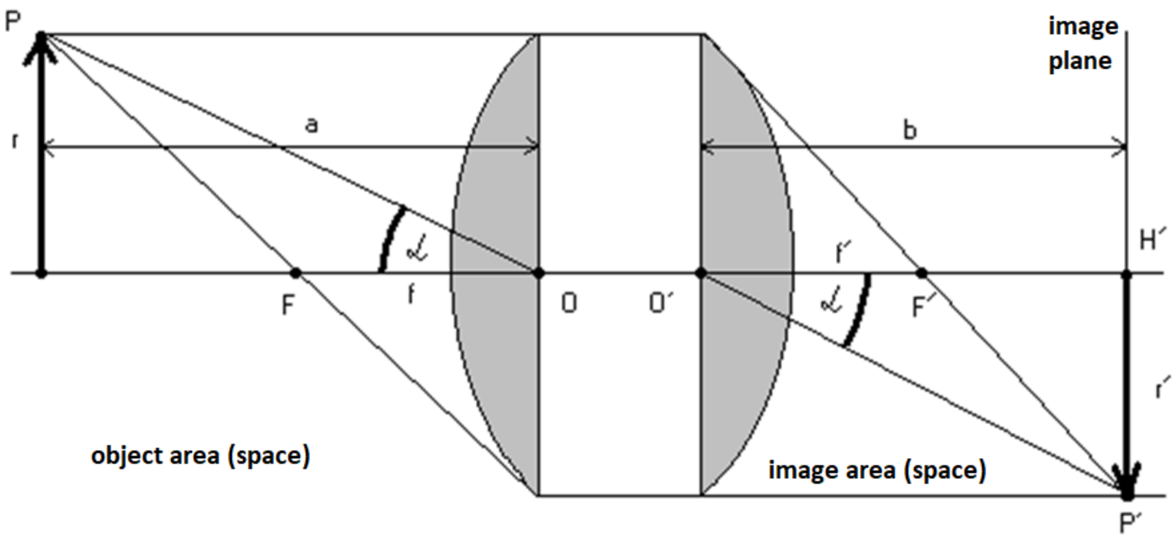


Fig.3.3: Imaging with an ideal lens

In the figure, *f´* is the image focal length of the lens and *f is* the subject focal length. For compound lenses with a larger diameter, accurate centre projection is not performed. In geometric centre projection, the image of a point is again a point which is the intersection of the central projection ray with the projection plane. In optical projection, the image is formed by the rays emanating from point *P* in the object space striking the lens, and the projection is made by those which pass through the aperture, the image of which in the object space is the entrance **pupil**. The entire cone of ray bundles, the base of which is the entrance pupil and the apex of which is the projected point *P,* participates in the display. This cone corresponds to a cone in image space with its base in the exit pupil and its apex at point *P´* (the image of point *P*). The position and size of the two pupils depend on the aperture used, hence the variable position of the projection centres.

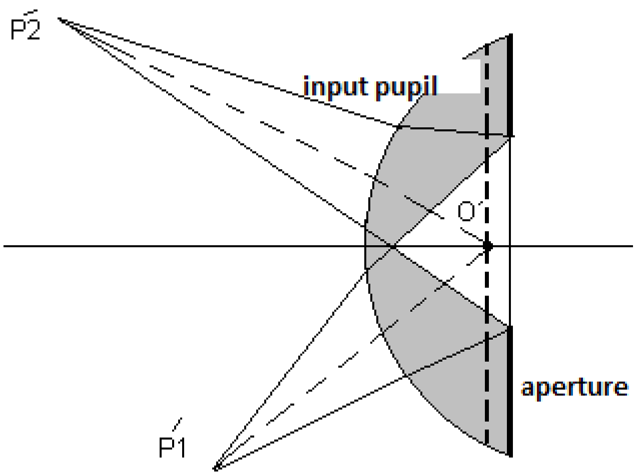


Fig.3.4: To define the centre of the exit pupil as the projection centre

In photogrammetric practice, objects are usually so far away that the rays entering the lens can be considered parallel. In this case, the image plane is identical to the focal plane of the lens (*b=f*). However, even in this case, it is not an exact point representation due to so-called *aberrations* (i.e. deviations from the point representation - see below). Aberrations cause the beam entering the lens at an angle α exits the lens at an angle α´, which is slightly different from the angle of entry. In terms of photogrammetric calculations, a photogrammetric-mathematical projection centre[1] O´M, to which is related the camera constant, which is usually calculated together with the effect of radial distortion during lens calibration, is introduced for conventional lenses to minimize the α- α´ difference.

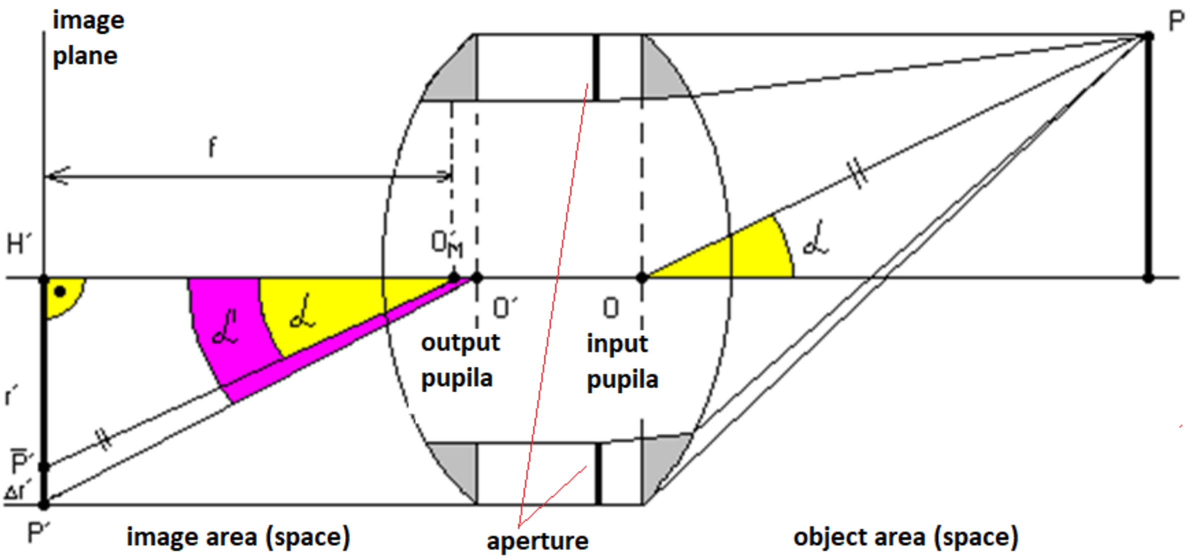


Fig.3.5: Beam bundles- lens imaging

Thus, the camera constant in photogrammetry is a fictitious value that is related to each lens and its distortion, given by the sum of its defects. This can be interpreted as a variation of the camera constant or the projected image angle that varies with r´.

Further, in the above figures, O´ is the physical projection centre, O´M is the mathematical projection centre, H´ is the main imaging (autocollimation) point, the H-object is the main autocollimation ray, and r´ is the radial distance of the imaged point.

[1] K.Kraus, 1994, Photogrammetrie, Band I.,Dümmler/Bonn, ISBN 3-427-78645-5

$$r' = f \cdot \tan\alpha + \Delta r'$$
$$r' = \sqrt{(x' - x_0' + \Delta x')^2 + (y' - y_0' + \Delta y')^2} \tag{3.5}$$

where Δx´ , Δy´ are coordinate corrections for lens distortion (distortion).

3.2.1 Depth of focusing

Most surveying cameras in photogrammetry are focused at infinity and the objects in the images are sharp from a certain distance. The depth of field is determined by the size of the aperture used in the photography. The larger the aperture number, the greater the depth of field. The minimum distance of objects that will still be sharp in the image is given by the relationship:

$$y_{\min} = \frac{f^2}{\left(\frac{f}{A}\right) \cdot \Delta u} = \frac{f^2}{n \cdot \Delta u} \tag{3.6}$$

where n is the aperture number and Δu is the dispersion ring ("blur or unsharpness"). It should not be larger than the diameter of the measuring mark used on the old photogrammetric plotters (0.02-0.05mm); A is the diameter of the input pupil.

typ komory	f [mm]	n	y_{min} (Δu=0.02mm) [m]
Digital Canon Mark II D5	40	5.6, 8, 11, 16, 32	14.2,10, 7.2, 5, 2.5
Analogue UMK 10/1318	100	8, 16, 32	63, 32, 16

Tab. 3.1: Depth of focusing

1.1 Summary of the effects on the geometry of the lens imaging

The quality of the metric lens greatly influences the accuracy of the image coordinates. The use of any lens results in a violation of the ideal centre projection. Deviations of the actual display from the ideal are called optical defects or aberrations. Optical aberration can be divided into: (a) monochromatic (monochrome),

(b) colour.

Furthermore, aberration arising from:

(c) point imaging (spherical aberration, astigmatism and coma),

(d) object imaging (field blur and image distortion).

3.2.2 Spherical aberration

This defect is caused by the fact that rays passing through the lens at different distances from the optical axis refract differently (Figure 3.6). They do not intersect at a single point but form a so-called caustic surface around the optical axis - the point does not appear as a point but as a small scattering ring. The size of the defect is determined by the length of the segment $_{FK}$, *F*. The spherical defect can be compensated for by a combination of a *converging lens* and *concave* lens. However, it cannot be completely eliminated.

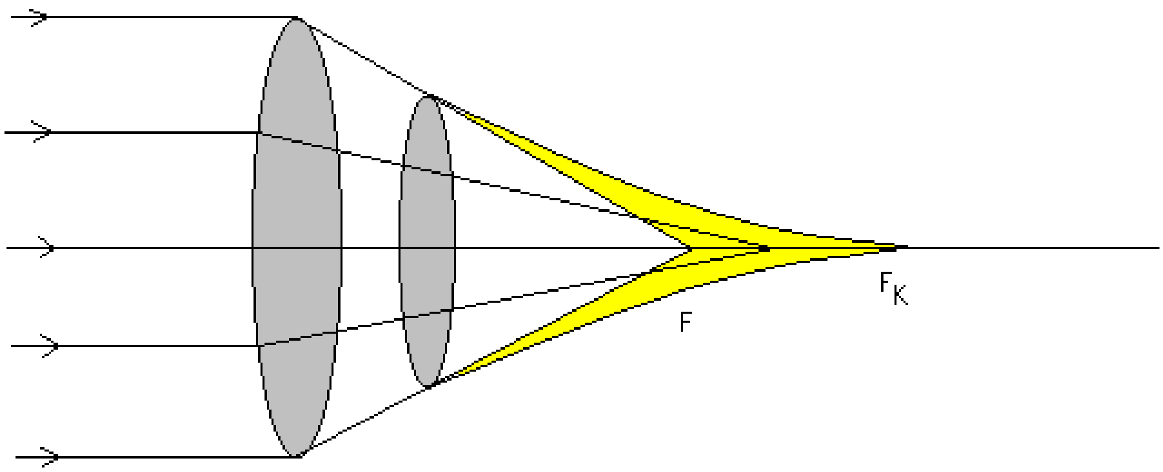


Fig.3.6: Lens imaging - spherical aberration

3.2.3 Aspheric aberration (coma)

If an oblique and wide beam of rays strikes the optical system, then a caustic with one plane of symmetry is formed instead of a point (Figure 3.7). This asymmetric aberration has a great effect on the image quality, especially in large systems. The coma is already evident at small distances of the points from the axis. For larger distances, coma is combined with astigmatism.

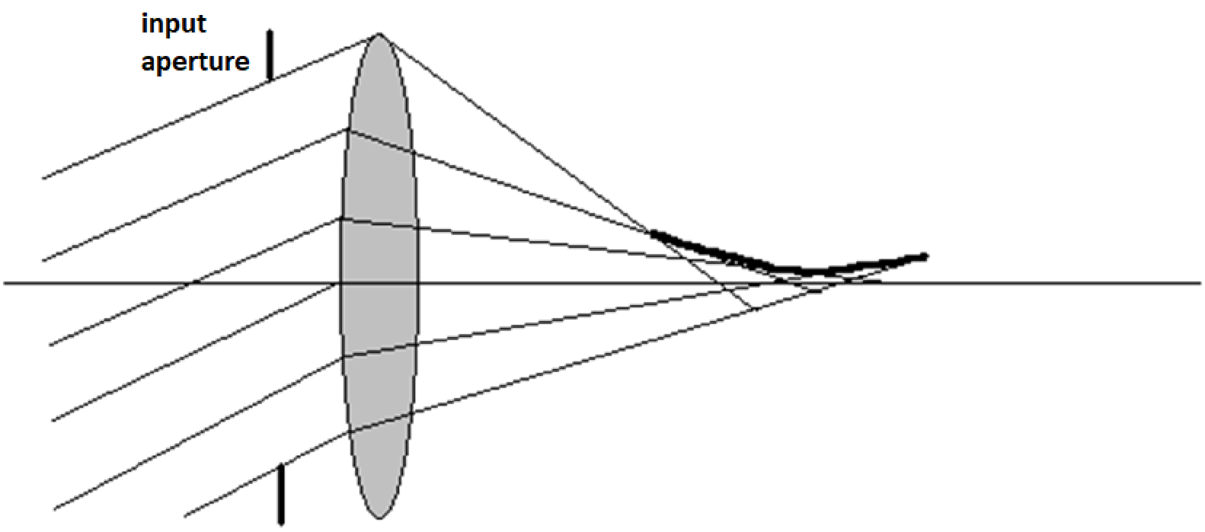
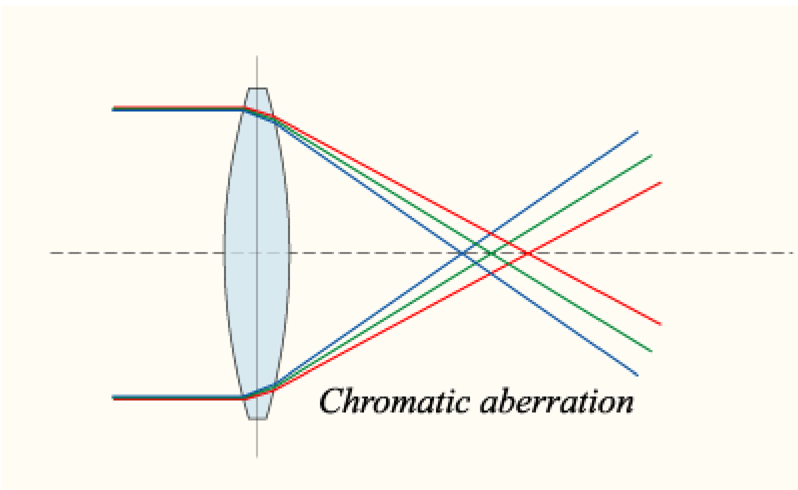


Fig.3.7: Lens imaging - aspherical aberration

3.2.4 Lens colour aberration

This defect is caused by the different refractivity of rays of different wavelengths. Visible light is broken down into its coloured components, with red being the most refractive and blue the least. As a result of this defect, the image is not point-like, i.e. instead of a point we get scattered rings that are differently coloured at the edge. This defect in the lens can be compensated by composing the lens from several components (*convergent* and *concave lens)* using different types of glass that differ in refractive index *(crown* and *flint glass*), (Fig.3.8).

In the visible light range, these defects can be successfully compensated for, but for the invisible optical range (UV and IR light) the *lens-frame plane* design must be modified for a sharp image.



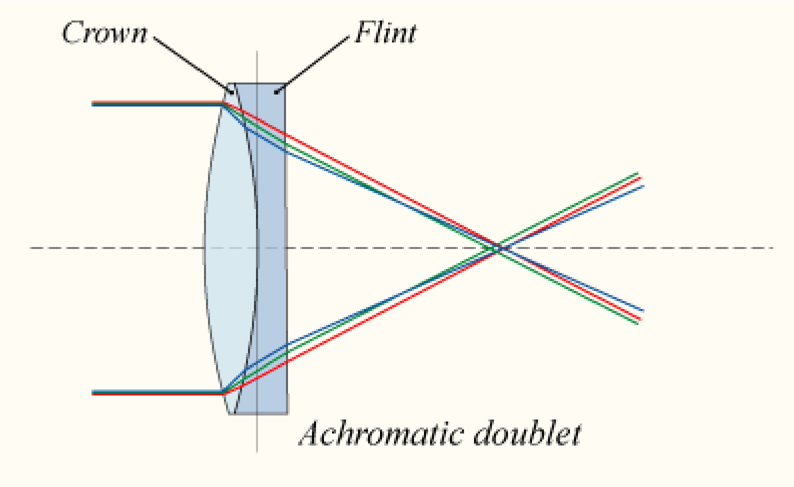


Fig.3.8: Lens imaging - chromatic (colour) aberration

Chromatic aberration does not occur with mirror lenses.

3.2.5 Astigmatism

Astigmatism is an aberration of the optical lens whereby when a plane perpendicular to the optical axis is viewed, points in mutually perpendicular axes do not appear at the same distance. Astigmatism also causes a different image when the beam strikes the optical system perpendicularly or at an angle.

3.2.6 Lens distortion

The defects mentioned in the previous paragraphs only affect the quality of the display and only secondarily affect the accuracy of the measurement. Defects which affect the geometry of the image, and which therefore have a decisive influence on the accuracy of the measurement are called **lens distortion**. There are two distortions:

- radial

-tangential

Lens distortion is caused by a combination of geometric inaccuracies in the manufacture of the lens. Today's lenses are asymmetrical, consisting of 8-30 elements that cannot be aligned absolutely precisely to the ideal optical axis. For this reason, the angle of the incoming beam is not exactly the same as that of the outgoing beam and the position of the imaged point differs slightly from the correct position. For precision work and for lenses with large distortion values, these defects must be corrected. Determination of the distortion is either carried out directly by the manufacturer (usually by measuring in eight radial directions) or can be determined by analytical methods (using a precisely defined point field). Modern photogrammetric software usually allows corrections to be introduced to minimise the effect of lens distortion, or it can calculate the distortion from an excessive number of measurements on the images. This gives the possibility to use, especially for close photogrammetry, also non-metric cameras where the lens distortion affects the measurement results very significantly.

Radial distortion

The displacement of a point of radial distance *r'* on an image by the value *Dr´* is called radial distortion. Its course is usually not exactly rotationally symmetric, but we assume this symmetry when compensating for it. In modern photogrammetric metric lenses, it reaches values of 5-10mm. Due to rotational symmetry, it is sufficient to determine the distortion in one radial direction. The distortion is usually expressed by a characteristic curve for each octant or curves of the same distortion - isolines - are constructed.

Tangential distortion

The second type of lens distortion is tangential distortion, which is caused by inaccurate centration of individual lenses. It acts perpendicular to the radial direction and causes irregular ill-defined local shifts. This distortion is virtually impossible to compensate for easily and is not commonly considered (the problem is that we would need to know the distortion values over the entire area and when removing the distortion, we would have to interpolate a correction in the area table for each measured point). With good modern lenses, it is reasonable to assume that the effect of tangential distortion is negligible. For metric lenses, the accuracy of the offset of the individual elements is usually so high (better than 5^{cc}) that this assumption is justified.

Historically, the effect of distortion has been eliminated by various methods, ranging from graphic corrections to compensating elements. Today, the effect of distortion is expressed analytically (Fig.3.9 and 3.10).

Analytical methods for lens distortion suppression

Due to the transition of almost all processing on the computer, the principle of corrections from distortion to measured values is used. It must be remembered that lenses, in order to be used at all for conventional close-range photogrammetry, must meet the requirement of at least approximately symmetrical radial distortion (i.e. tangential distortion will be minimal or zero). Only good quality lenses will meet this requirement (amateur cheap cameras are recommended for marginal use).

The analytical method of suppressing the effect of radial distortion by expressing it as a polynomial is currently the most common. The course of the radial distortion is known from the manufacturer or is determined by laboratory measurement or by using calibration procedures directly during measurement and evaluation ("*on field calibration*"). The measurement involves measuring the image coordinates of individual discrete points whose position is known (the so-called control points). The observed differences between the measurements and the control points are attributed to the effect of radial distortion. The measurement obtained is thus a set of points, which is interleaved with a suitable function that best represents the set of points. For example, a simple polynomial function can be used:

$$x' = x'_{měřená} + \Delta x'$$
$$y' = y'_{měřená} + \Delta y' \tag{3.7}$$

In general, the effect of radial distortion can be expressed by a polynomial, e.g.:

$$\Delta x' = a_0 + a_1 x' + a_2 y' + \dots$$
$$\Delta y' = b_0 + \dots \tag{3.8}$$

The forms used are expressed as:

$$r'^2 = x'^2 + y'^2 \tag{3.9}$$

$$\Delta x' = x' \cdot \left(a_1 r'^2 + a_2 r'^4 + a_3 r'^6\right) + b_1\left(r'^2 + 2x'^2\right) + 2b_1 x'y'$$
$$\Delta y' = y' \cdot \left(a_1 r'^2 + a_2 r'^4 + a_3 r'^6\right) + b_2\left(r'^2 + 2y'^2\right) + 2b_2 x'y' \tag{3.10}$$

This form is too detailed, for most lenses only the coefficients a_1, a_2 can be used. The RolleiMetric system uses, for example, this correction:

$$r'^2 = x'^2 + y'^2 \tag{3.11}$$

$$\Delta r' = a_1 \cdot r' \cdot \left(r'^2 - r_0'^2\right) + a_2 \cdot r' \cdot \left(r'^4 - r_0'^4\right) \tag{3.12}$$

r'_0 is the radial distance, where the radial distortion $\Delta r'$ *is* still (off-centre) equal to zero (its sinusoidal shape is common).

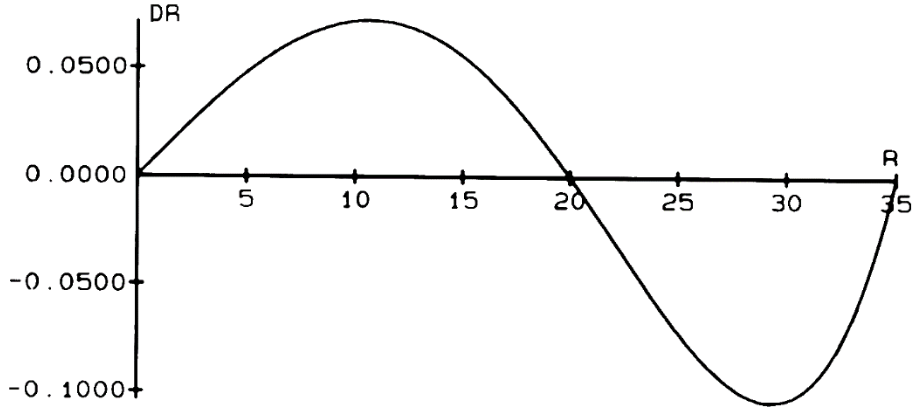


Fig.3.9: Lens imaging - radial distortion (RolleiMetric 6006, 40mm lens)

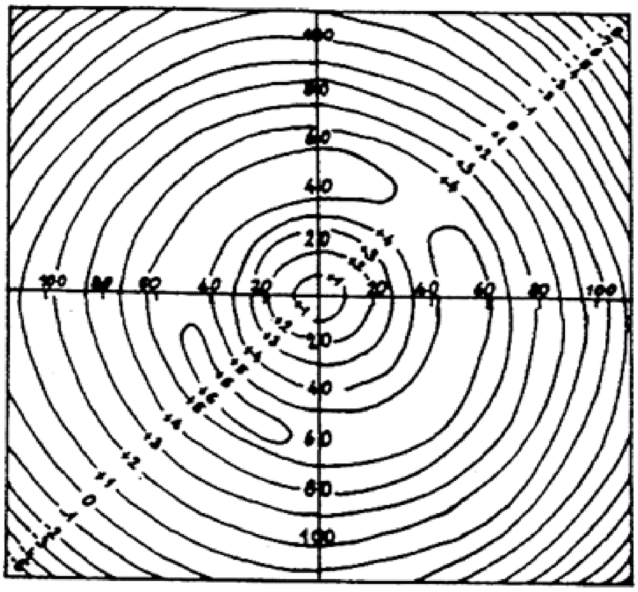


Fig.3.10: Lens imaging - radial distortion expressed by isolines

3.2.7 Dimensional shrinkage of photographic material

The dimensional stability of the material on which the image is recorded is a decisive factor in photogrammetric measurements. It is particularly influenced by the substrate on which the light-sensitive medium is deposited. Glass plates are the most dimensionally stable, followed by film and finally paper. In photogrammetry, only digital recording on a silicon semiconductor chip, which has the properties of high-quality glass, is currently used. Previously, for almost 100 years, conventional film was used as a carrier for the sensitive layer. In terrestrial photogrammetry, film materials were no longer used after the turn of the millennium, in aerial photogrammetry about 10 years later for technical reasons. Film had several advantages: it was lightweight, flexible, easier to develop than glass plates, and less costly. However, it showed dimensional shrinkage. For reference glass has a dimensional shrinkage of 3-5 μm, modern PET based films up to 5 μm. However, historical analogue film materials showed significantly more deformation.

The shrinkage of photographic material is divided into:

(a) regular over the entire image area - this shrinkage is very easy to detect by comparing known lengths

b) differential - this is a regular shrinkage which is different in the x' and y' axis directions

c) irregular - this shrinkage is practically impossible to exclude without a special device (réseau - grid).

The shrinkage of photographic film material is currently negligible, but it must be considered, for example, when evaluating older (archival) images or images from non-measurement cameras. If the shrinkage is regular, it is very easy to exclude it by comparing it with the actual coordinates of the fiducial marks. The correction is made by an affine transformation or, if the scale change for both axes is the same, a similarity transformation is used, or a regular shrinkage is corrected by a small change in the camera constant. If it happens that the shrinkage is different in the two axes (e.g. by stretching the material in one direction when rewinding the film), it can be further supported e.g. as additional parameters in the alignment (different scale for each axis) or in other ways, e.g. by DLT (direct linear transformation). Irregular shrinkage is difficult to remove; if the camera is equipped with a set of additional markers (réseau camera), it can again be captured by a transformation, e.g. Helmert.

A change in the position of a point on the image can also be caused by the unevenness of the image, which is caused by the deflection of the glass, or in the case of film, by imperfect alignment during taking the picture. The sagging of the sensitive layer causes a local change in the scale of the image or a change in the projection distance. The flatness of glass plates or PET film is 5-20µm. This error cannot be routinely eliminated.

3.2.8 Cumulative effect of refraction and curvature of the Earth

The radial displacements of objects in the image caused by the curvature of the Earth are determined by the choice of the coordinate (Cartesian) system in the selected projection plane. However, the image is taken over a real body that can be approximated by a rotating ellipsoid or a reference sphere. Due to the distances involved in terrestrial photogrammetry and its accuracy, the **influence of the curvature of the Earth does not need to be considered**. The effect of the curvature of the Earth is partly compensated by the error from refraction, since the two effects work against each other. However, for aerial photogrammetry, these effects must be considered and the well-known equations from classical geodesy can be used after modification.

3.2.9 Photogrammetric projection

Assuming an ideal lens, the photographic image is an accurate central projection of the object being photographed. Each point of the object to be photographed corresponds to a point in the image plane. The rays passing between the corresponding points (the point and its image) intersect at a single point - the centre of projection (projection centre O).

However, the practical design of lenses differs substantially from the physical idealization, which creates problems in solving practical photogrammetry tasks. One of the goals of photogrammetry is to transform the central projection into an orthogonal (parallel) projection as required by the map projection. To solve this task, it is necessary to define precisely the basic elements of the central projection - i.e. the projection centre. Its definition, together with other parameters, forms the so-called elements of the internal orientation of the metric camera. It is the precise knowledge of these elements that is the outstanding feature of the metric camera. The prospective central projection of an object can be derived using the principles of descriptive geometry and is shown in Figures 3.11 and 3.12.

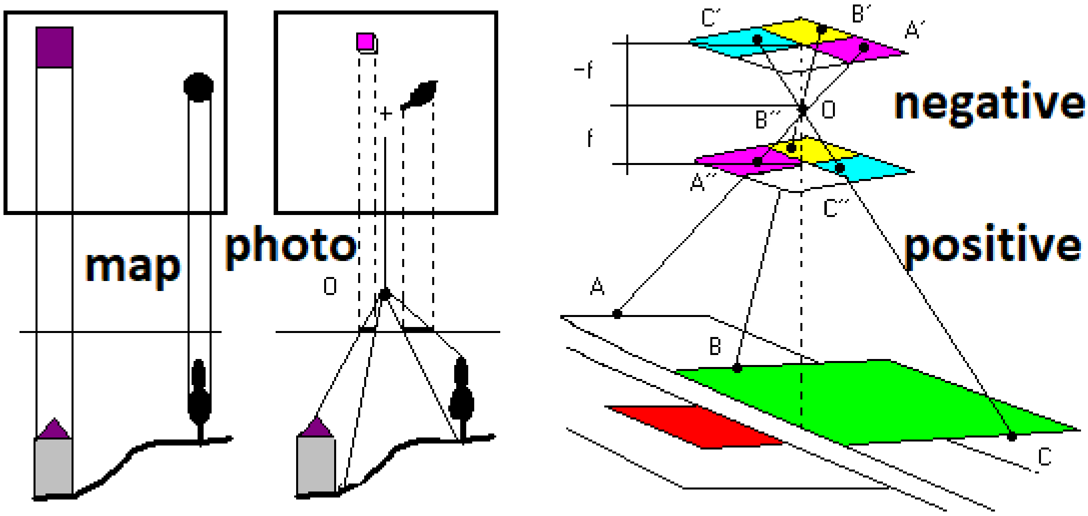


Fig.3.11: Central projection

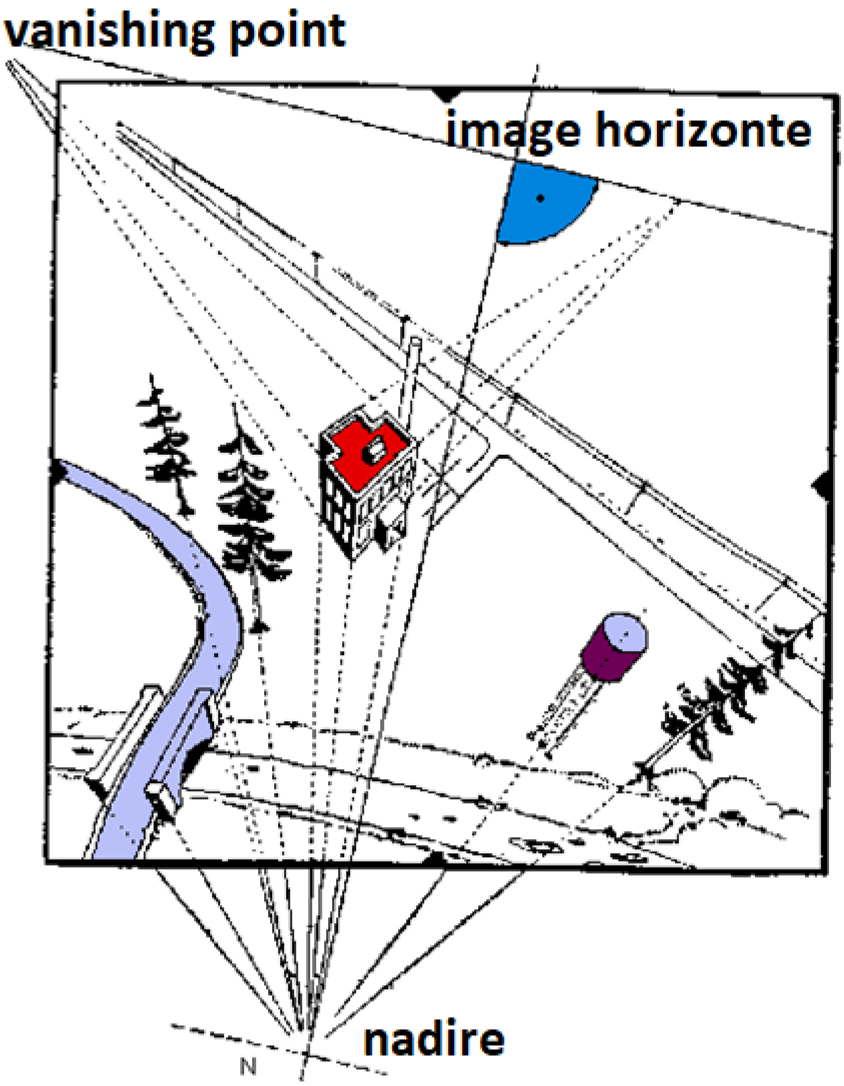


Fig.3.12: Photographic image. The vanishing point of the vertical object lines is the image nadir.

3.2.9 Elements of internal orientation

The distance of the objects to be imaged is usually so great that their images are formed on the back focal plane of the lens, and the image distance is thus practically equal to the focal length of the lens. For this reason, classical photogrammetric cameras were permanently focused on infinity, resulting in a simpler camera design and preserving the internal orientation of the camera. When cameras are equipped with the ability to refocus, this is done after certain precise steps to which the increment of the camera constant is known. Not suitable are zoom lenses, where the camera constant cannot be defined precisely and in advance (except for the extreme positions). Special calibration procedures must be used for such lenses.

The entrance or exit pupil of a lens is the aperture image of the lens, formed by the object or image part of the lens; practically it always lies inside the lens. According to the preceding text, the projection centre (object centre of projection) is not a single point. The practical definition of a projection centre is different - projection centres are a mathematical abstraction. Since in photogrammetry the relationship between the image coordinates i and the object rays is involved, it is necessary to define projection centres based on this relationship. Projection centres defined as the centres of the input and output pupils, with respect to the distortion of the lens or mathematical projection centre, satisfy this requirement.

The coordinates of the main image point and the camera constant are called the **elements of the internal orientation of** the metric camera and define the geometry of the rays inside the camera. Knowledge of the radial distortion is often added to the internal orientation elements.

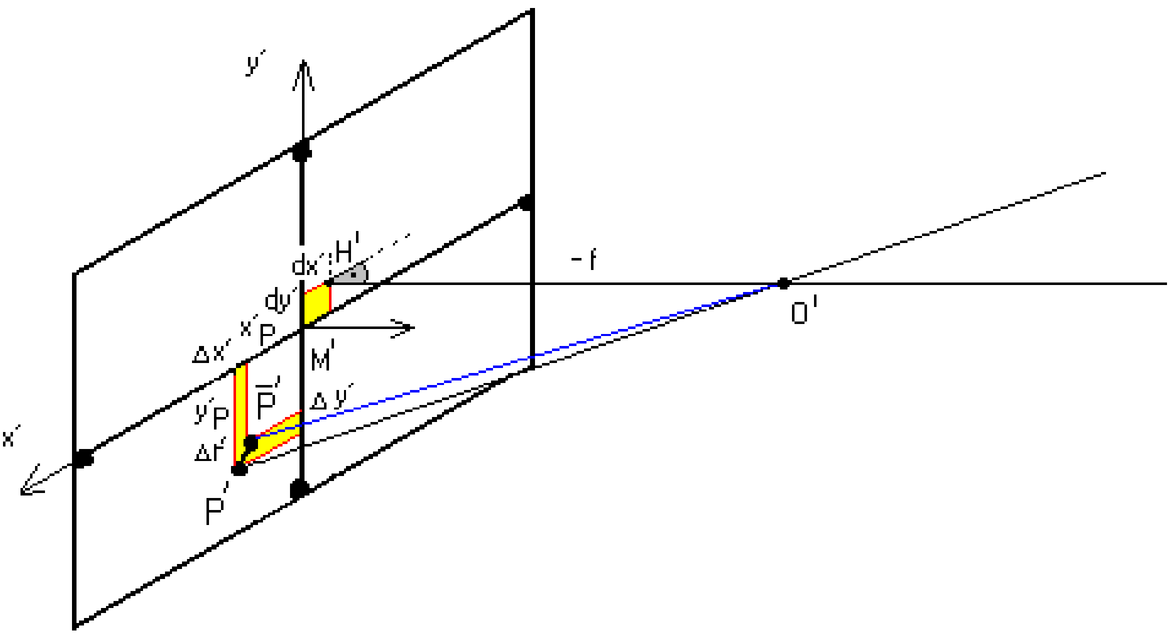


Fig. 3.13. *Definition of the elements of the internal orientation in the general configuration (image coordinates x ´, y ´); The axis of view is the perpendicular to the image plane passing through the object projection center.*

Principal (image) point

The principal image point *H´* is defined as the intersection of the image plane with the ray passing through the centre of projection in the object space and which is perpendicular to the image plane.

Camera constant

The camera constant *f* is the distance from the main image point *H´* to the centre of projection in the object space (centre of the exit pupil). The camera constant is usually denoted by *f or c* (for the positive under consideration) or *-f* or *-c* for the negative (see Figure 3.14). The marking *f or -f* is a matter of defining the origin and direction of the axes and need not be completely uniform. Usually, the origin is the centre of projection; since we often give the forward direction as positive, it is logical that from the centre of projection back (to the image plane of the negative) it is negative (i.e., *-f*). Furthermore, the value of f only corresponds approximately to the focal length of the lens, as already mentioned. In optics, and in many mathematical calculations, f is usually positive, since only its magnitude is considered.

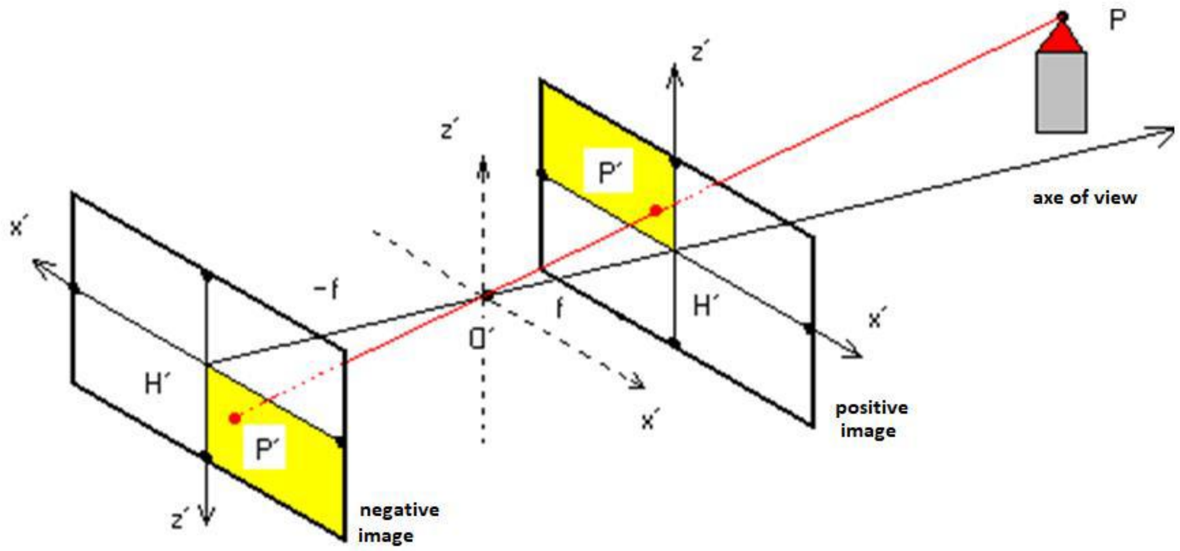


Fig.3.14. Camera constant, negative and positive, terrestrial configuration Elements of interior orientation : x0 ´,y0 ´, f (and known parameters of distortion)

Centre of photo

Classical photogrammetry used analogue images taken with special analogue photogrammetric cameras. As the main image point was difficult to define in practice, frame marks were introduced in the measuring chambers, the intersection of which indicated the centre of the image M´, from which the image coordinates were determined. Ideally, the image point should lie at the intersection of the fiducial marks. However, due to the technical design of the camera and the lens, the position of the principal point is deviated by a small amount from the centre of the frame given by the fiducial marks (fig. 3.15). This value was then determined by measurement or laboratory procedures and is usually known to a high accuracy (about 0.01 mm) for measurement cameras. The position of the centre of photo H ´, measured from the image centre M ´, is given in coordinates [x'_0, y'_0] (or [dx', dy']) for aerial photogrammetry or as [x'_0, z'_0] (or [dx', dz']) traditionally for terrestrial photogrammetry.

Fiducial marks are stable optical or mechanical structures and their image is transferred to the negative during exposure. For terrestrial photogrammetry, a system of four frame marks at the centres of the sides of the format in use was commonly used, while for aerial photogrammetry, eight frame marks were used at the corners and centres of the sides of the format. In special réseau cameras there is a planparallel plate in the image plane with a system of crosses (up to 121), which are used to define the frame coordinate system and to remove the effect of deformation of the film material.

The position of the frame marks is always clearly defined and their coordinates, together with the elements of the internal orientation, are usually included in the certificate issued for the measuring chamber.

Currently, only high-quality digital SLR cameras are used for terrestrial photogrammetry. Special and expensive digital cameras were developed for aerial photography after new Millenium, but their mass implementation was only after 2010. In general, digital cameras do not have fiducial marks, corner pixels are used, otherwise the same principles as for conventional film cameras apply.

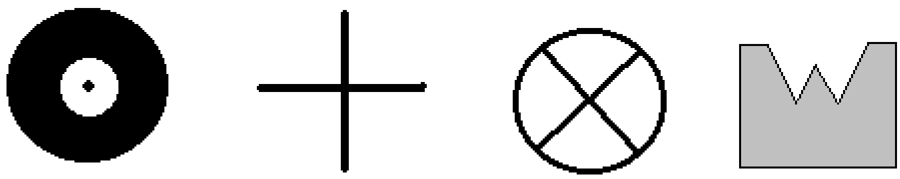


Fig. 3.15. Different types of fiducial marks (on historical images or in historical cameras only)

4 Mathematical fundamentals

4.1 Image orientations

The relations describing the geometry of the ray tracing inside the camera describe the elements of the interior orientation, the position of the camera in space and the direction of the view are described by the elements of the exterior orientation. Knowing and restoring or adjusting them allows a correct evaluation of the image content. The procedures that lead to the creation of the model are called image orientations.

Interior orientation

For a good evaluation of photographic measurement images, it is necessary to know and recover the elements of the ***interior orientation of*** the metric camera (the camera constant *f*, the position of the principal point *H´(dx´,dy´)* and possibly the form of the radial distortion). These quantities are usually known in advance (they are specified by the manufacturer for each measuring camera or can be determined by laboratory measurement). Images for which the elements of the internal orientation are known are referred to as metric images.

Exterior orientation

The elements of ***the exterior orientation*** are defined for each image (coordinates of the centre of the input pupil X_0, Y_0, Z_0, then inclinations ω,φ,κ).

For terrestrial photogrammetry in its classical version, the determination of the elements of the exterior orientation was relatively simple. The coordinates of the centre of the input pupil $_{X0}$, $_{Y0}$, $_{Z0}$ can be determined by any geodetic method. In classical analogue stereo-methods, the left position was usually geodetically located. For analogue special photogrammetric cameras, the inclinations ω,φ,κ could be set with sufficient accuracy by means of the orientation equipment and levelling devices, i.e., the elements of the exterior orientation were standardised, and the calculations were greatly simplified, since the elements of the exterior and interior orientation were known in advance.

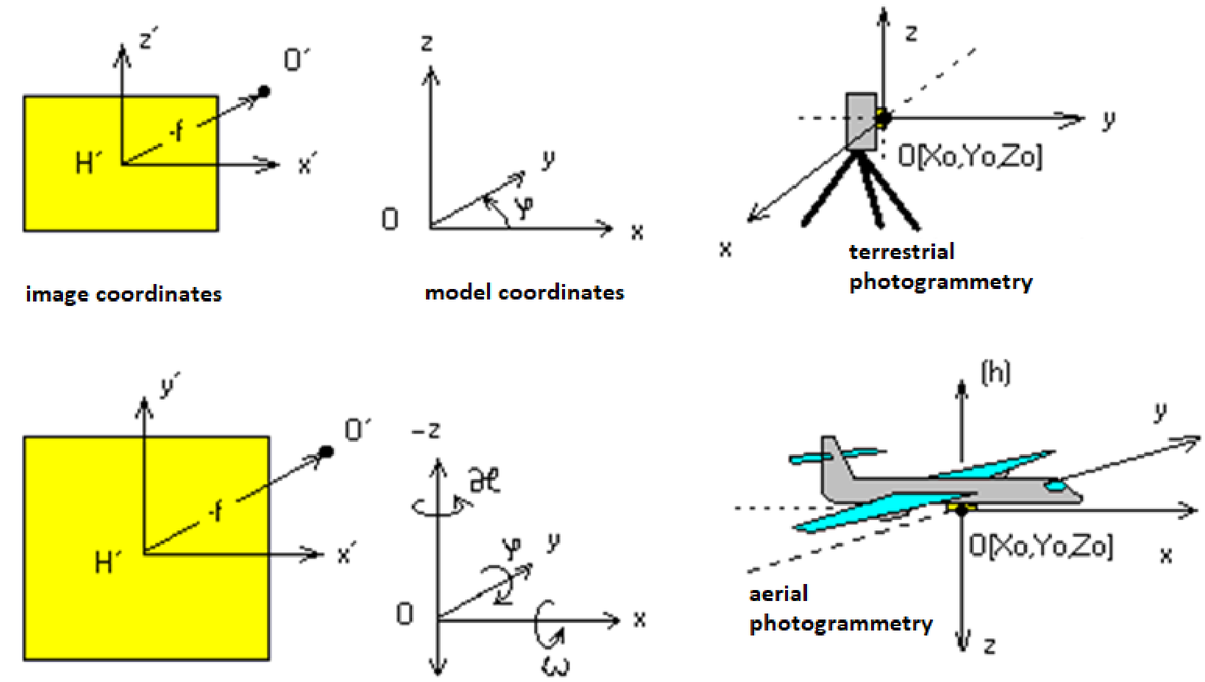


Fig. 4.1 Internal and external orientation elements for terrestrial and aerial photogrammetry

If we do not know the elements of the exterior orientation, they can be calculated classically in two steps as:

- **relative orientation** (relative orientation between the two stereo images, creating an arbitrary spatially oriented stereo model)

- **absolute orientation** (rotation and shifting of the model into the geodetic reference system)

Newer systems allow external orientation to be performed in a single step, using a direct relationship between image coordinates and geodetic coordinates **using the bundle adjustment method** (*Bundle Adjustment, German: Bündelausgleichung*)

4.2 Coordinate systems

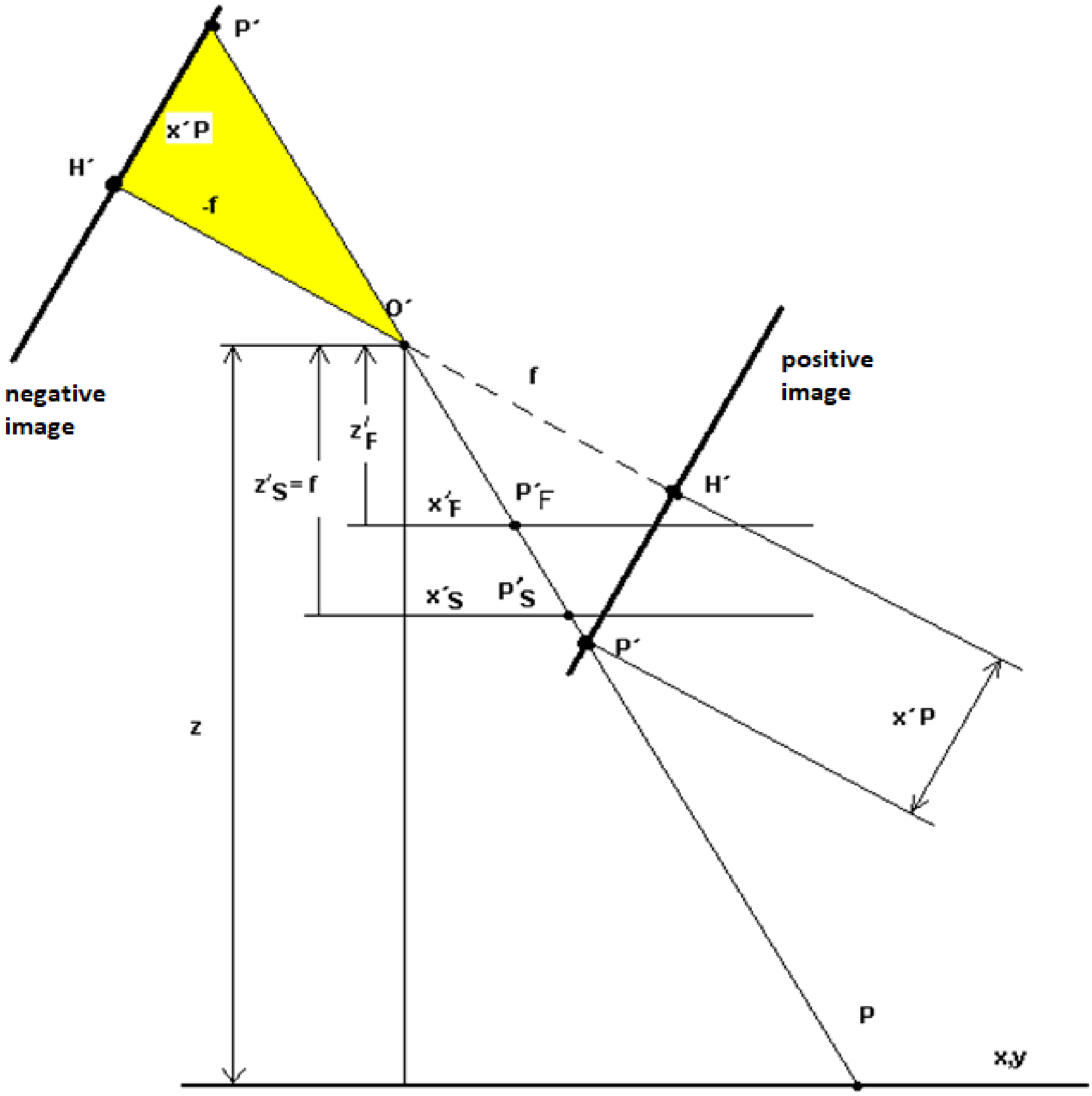


Fig. 4.2: Coordinate systems in photogrammetry

To understand the following text, it is necessary to define the coordinate systems used and the transformation procedures used in photogrammetry, which are referred to in the following text. Without perfect knowledge of these relationships and symbology, understanding the following text is difficult. Historically, slightly different symbology has been used for coordinate systems for terrestrial and aerial photogrammetry, although mathematically they are only one system. Much of the foreign literature does not make differences between terrestrial and aerial coordinate systems, uses the same symbolism, and does not use other coordinate systems. However, in general, two types of coordinate systems were (or still are) used in photogrammetry:

1) main coordinate systems (used)

- Image coordinate system: x' , y' , ($z' = -f$)
- Model coordinate system: x, y, z
- Geodetical system: X, Y, Z

- auxiliary coordinate systems (these are defined for easier coordinate transformations in the past)

- Fictional image coordinate system: x'_F, y'_F, z'_F

- Exact vertical image coordinate system: x'_S, y'_S, z'_S

4.2.1 Choice of coordinate systems in aerial photogrammetry

In defining model systems and deriving other relationships, we will follow the recommendations of the International Society for Photogrammetry and Remote Sensing (ISPRS), which is based on the principles of aerial photogrammetry:

-the x-axis is placed approximately in the direction of flight
-the direction of rotation is selected in a clockwise direction
-the x-axis is primary, the y-axis secondary and the z-axis tertiary
-the origin of the coordinate system is located at the projection of the left position of the stereo pair

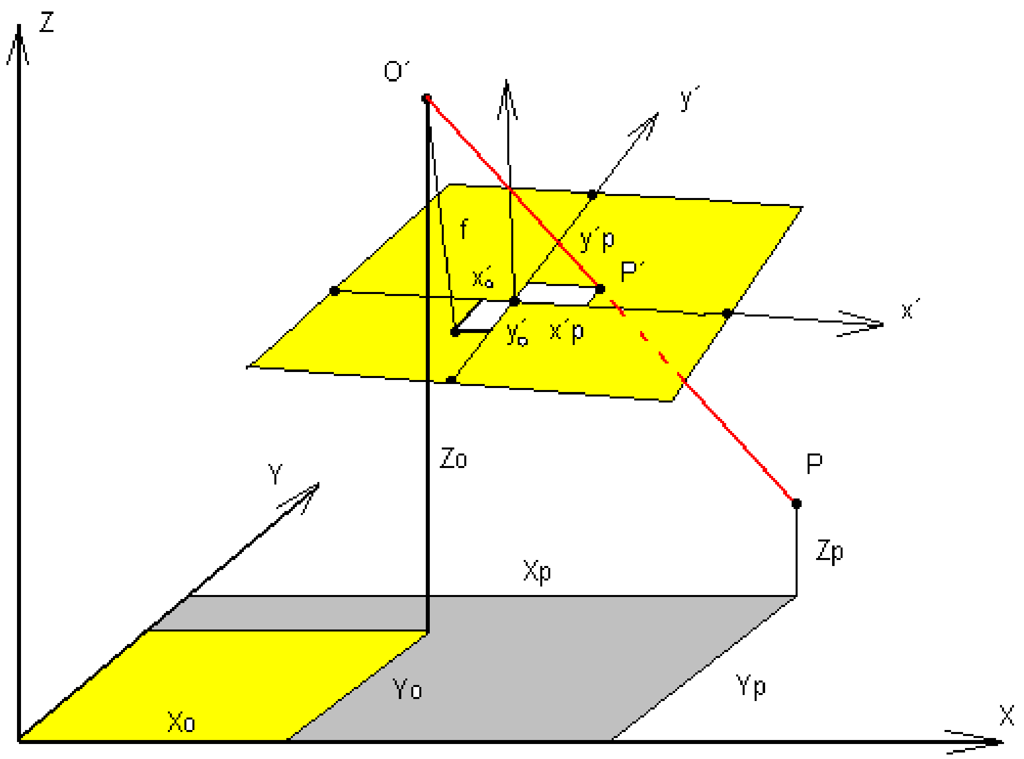


Fig. 4.3: Coordinate systems in aerial photogrammetry, direct transformation

4.2.1 Coordinate system definition in terrestrial photogrammetry

For the spatial processing of image content, terrestrial photogrammetry uses the same system as aerial photogrammetry, the (x,*y*) axes are the semi-major axes and the (z) axis is the vertical axis. While in aerial photogrammetry the spatial component is (z), in terrestrial photogrammetry it is used as the spatial component (y) (distance to the subject).

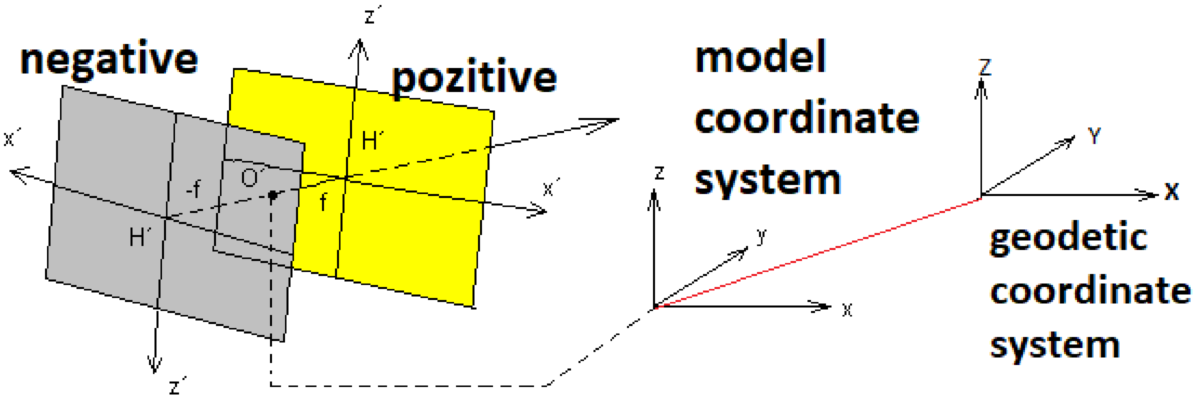


Fig. 4.4: Coordinate systems in terrestrial photogrammetry

It's up to agreement on what we call the coordinates. Traditionally, in Central Europe and in the historical literature, a distinction is made between terrestrial and aerial photogrammetry, although mathematically terrestrial photogrammetry is only a special case of the more general one, aerial photogrammetry. The modern world literature, mainly because of the same methods of evaluation of digital technology, has erased these differences and the spatial coordinate (z) is always in the direction of photography; in terrestrial photogrammetry, a transformation (y¬®*z*) must be made in the results. Another reason is the use of réseau cameras, which take both terrestrial and aerial images, and the introduction of both analytical and digital methods, where oblique images from an aircraft, helicopter, drone, platform or even from a tall building can also be processed (in this case neither vertical nor "classically" ground-based images are involved).

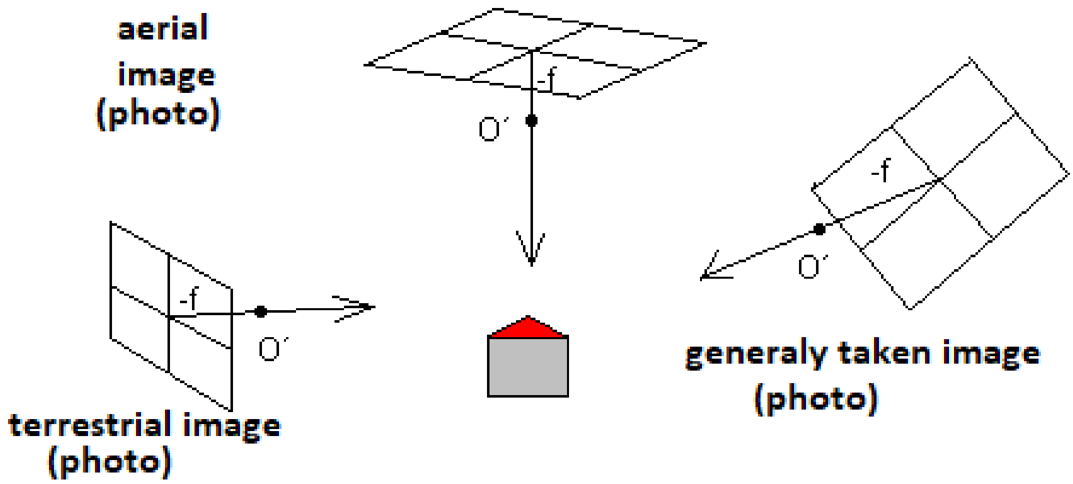


Fig.4.5: Classic terrestrial, aerial and oblique image

4.2.2 Image coordinate system

marking x′ , y′ , (z′ = -f) (left image) or x′′, y′′ , (z′′ = -f) (right image) or in the traditional concept x′ , z′ , (y′ = -f) (left image) or x′′, z′′ , (y′′ = -f) (right image)

The image coordinates are defined as follows: the origin of the coordinate system (i.e., where the measurement is taken from on the image) is located, in terms of the measurement, at the centre of the image M´, which is given by the line of frame marks on the image (hence we speak of the image coordinates). However, the image coordinates must be measured from the principal point H¢. Thus, if H´= M´, corrections for the offset of the origin of the coordinate system must be introduced (H´ has coordinates x'_0, y'_0 or dx′, dy′, or x'_0, z'_0 or dx′, dz′). The x' axis is placed between the horizontal fiducial marks, the y' (or z') axis lies perpendicular to the x' axis in the frame plane in the mathematical sense; the system is usually supplemented for mathematical reasons by a z' (or y') axis with a positive forward orientation in the object space (valid : z′ = -f, or y′ = -f).

Note: The object centre of projection is the correct mathematical origin of the 3D system with frame coordinates [0,0,0]. Its perpendicular projection to the image plane is H´. For practical purposes, the intersection of the fiducial marks M´ (these are 2D coordinates on the image) is considered the origin of the planar image coordinates. Since often H´≠ M´, measured frame coordinates with origin at M´ need to be corrected from this offset.

When digital images are used, M´ is defined as the intersection of the corner pixels.

Remarks:

thus, there can sometimes be double naming, especially of image coordinates - for example, in historical procedures and literature, using old comparators where image coordinates were measured from terrestrial images like x ',z' . For newer universal comparators, the naming of the same coordinates is x',y'. Since virtually all mathematical derivations are performed for an aerial photogrammetry system, we will mainly follow this newer approach when appropriate.

Thus, the resulting form of the image coordinates for a general point *P* will be:

$$\boldsymbol{x} = \begin{bmatrix} x' \\ y' \\ -f \end{bmatrix} = \begin{bmatrix} x'_P - x'_O + \Delta x' \\ y'_P - y'_O + \Delta y' \\ -f \end{bmatrix} \qquad (4.3)$$

where x'_p, y'_p *are measered image coordinates,* x'_o, y'_o are the coordinates of the principal image point and $\Delta x'$,$\Delta y'$ are the lens distortion corrections.

4.2.3 Model coordinate system

markings: x , y , z

The system of model coordinates. The axes and their direction and orientation are chosen according to the ISPRS recommendations or approximately in accordance with the resulting geodetic coordinate system. The coordinates x_0, y_0, z_0 are the model coordinates of the centre of the projection of the left image (input pupil).

4.2.4 Geodetic system

marking: X , Y , Z

The resulting coordinate system of the object, the geodetic system. The coordinates X_0,Y_0,Z_0 are the geodetic coordinates of the centre of projection of the left image (input pupil).

4.2.5 Fictitious image system

marking: x'_F , y'_F , z'_F (left frame) or x''_F , y''_F , z''_F (right frame)

A fictitious vertical image system where each point also corresponds to a different image distance z'_F generated from the general image. It serves as an auxiliary system when converting from an image coordinate system.

4.2.6 Exactly vertical image system

Marking: x'_S , y'_S , z'_S (left image), or x''_S , y''_S , z''_S (right image)

A system of **exactly** vertical images where all points have the same image distance equal to the camera constant *f* (valid : $z'_S = -f$). This system is used in the older coordinate system conversion procedure for aerial photogrammetry.

4.3 Determination of the effect of changing exterior orientation elements on image coordinates

This part is the basis of the mathematical approach to generally acquired images. Already a century ago the following relations were derived by Otto von Gruber. The result is series that describe with some degree of accuracy the influence of the elements of exterior orientation on the image coordinates, the so-called Gruber series.

These are the effect of rotations on the image coordinates and the effect of translations. Gruber series have become an important mathematical relation in photogrammetry for a long time, so they will be derived here.

4.3.1 Rotation matrix

In photogrammetry, transformation relationships between systems are used. The basis of modern spatial photogrammetry are direct relationships between image and geodetic coordinates. When they are converted to each other, they are generally *rotated, shifted* and *scaled*. While shift and scale change are relatively simple operations, spatial rotation is more complex and is the basis for further considerations.

4.3.2 Rotation in the plane

Plane transformations have already been introduced in geodesy; in the following, relations concerning the rotation of one coordinate system with respect to another will be derived. To understand the relations, we start from a two-dimensional idea that is commonly well known. Classical geodesy commonly uses transformation relations between Cartesian systems in the plane. The simplest is the ordinary (identity) transformation:

$$X = x \cdot \cos\alpha - y \cdot \sin\alpha \qquad (4.4)$$

$$Y = x \cdot \sin\alpha + y \cdot \cos\alpha$$

$$\boldsymbol{X} = \boldsymbol{R} \cdot \boldsymbol{x}., \boldsymbol{R}^T = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \qquad (4.5)$$

Where ***R*** is called ***the rotation matrix of*** the system. This matrix has a few distinguishing properties; it is orthogonal and orthonormal.

4.3.3 Rotation in space

Effect of rotation on all image coordinates

The total rotation matrix consists of the cumulative effect of three independent rotations, which we call $\boldsymbol{\omega}, \boldsymbol{\varphi}, \boldsymbol{\kappa}$.. By defining the axes and the order of rotations in deriving the spatial rotation matrix, we obtain the resulting expression of the matrix elements that would differ if the order were changed.

Three orthogonalization and three orthonormalization conditions are defined. Thus, the resulting matrix ***R*** contains a total of three independent optional parameters, which are just the rotations $\boldsymbol{\omega, \varphi, \kappa}$.

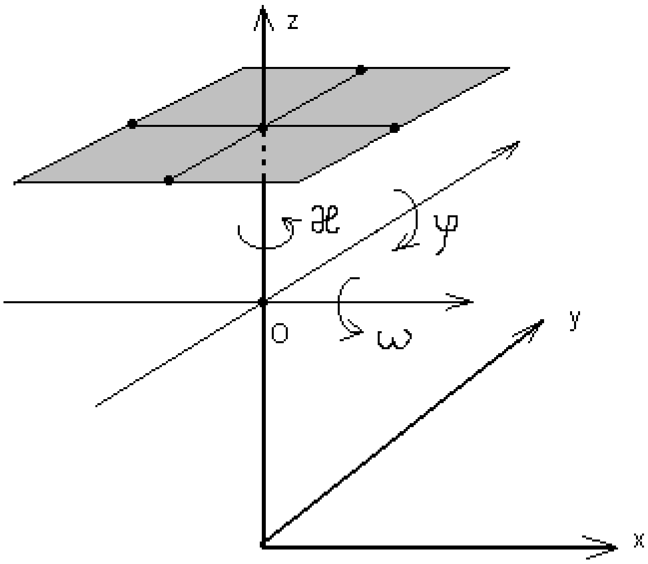


Fig.4.6: ω,φ,κ *(primary, secondary and tertiary rotation)*

4.3.4 Rotation about the primary *x´* axis

Consider a frame coordinate system that is tilted by an angle ω compared to the fictitious system. According to Figure 4.7, we can write:

$$\begin{aligned} x'_F &= x' \\ y'_F &= y' \cdot \cos\omega - z' \cdot \sin\omega \\ z'_F &= y' \cdot \sin\omega + z' \cdot \cos\omega \end{aligned} \tag{4.8}$$

or use a more elegant matrix form:

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \boldsymbol{x'_F} = \boldsymbol{R}_\omega \cdot \boldsymbol{x}' \tag{4.9}$$

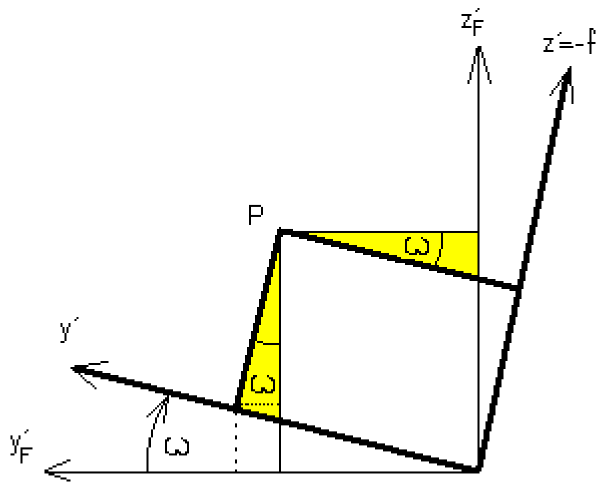


Fig. 4.7: Rotation of the system about the x´ axis by the angle ω

4.3.5 Rotation about the secondary *y´* axis

According to Fig. 4.8, for rotation about the *y´axis by an* angle φ, we can write:

$$x'_F = x' \cdot \cos\phi + z' \cdot \sin\phi$$
$$y'_F = y' \quad (4.10)$$
$$z'_F = -x' \cdot \sin\phi + z' \cdot \cos\phi$$

or use a more elegant matrix form:

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \begin{pmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \boldsymbol{x'_F} = \boldsymbol{R_\phi} \cdot \boldsymbol{x'} \quad (4.11)$$

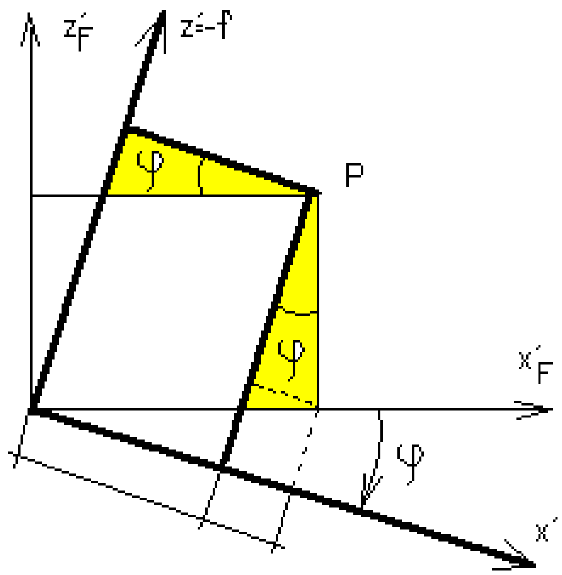


Fig.4.8: Rotation of the system about the axis y´ by the angle φ

4.3.6 Rotation about the tertiary z´ axis

The last rotation is the rotation about the z´ axis by the angle k; according to Fig. 5.4 we can write:

$$
\begin{aligned}
x_F' &= x' \cdot \cos\kappa - y' \cdot \sin\kappa \\
y_F' &= x' \cdot \sin\kappa + y' \cdot \cos\kappa \\
z_F' &= z'
\end{aligned}
\tag{4.12}
$$

or use a more elegant matrix form:

$$
\begin{pmatrix} x_F' \\ y_F' \\ z_F' \end{pmatrix} = \begin{pmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \boldsymbol{x}_F' = \boldsymbol{R}_\kappa \cdot \boldsymbol{x}' \tag{4.13}
$$

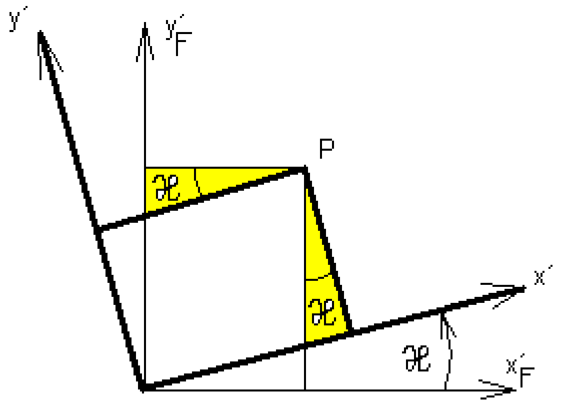


Fig.4.9: Rotation of the system around the z' axis by the angle κ

4.3.7 Resulting rotation matrix R

The resulting matrix $\boldsymbol{R}$ is given by the multiplication of the partial rotation matrices:

$$
\mathbf{R}_{\omega\varphi} = \mathbf{R}_\omega \cdot \mathbf{R}_\varphi \tag{4.14}
$$

$$
\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{pmatrix} \cdot \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{pmatrix} = \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ \sin\omega\sin\varphi & \cos\omega & -\sin\omega\cos\varphi \\ -\cos\omega\sin\varphi & \sin\omega & \cos\omega\cos\varphi \end{pmatrix} \tag{4.15}
$$

The total resulting rotation matrix will be:

$$\mathbf{R}_{\omega\varphi\kappa} = \mathbf{R}_{\omega\varphi} \cdot \mathbf{R}_{\kappa} \tag{4.16}$$

$$\begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ \sin\omega\sin\varphi & \cos\omega & -\sin\omega\cos\varphi \\ -\cos\omega\sin\varphi & \sin\omega & \cos\omega\cos\varphi \end{pmatrix} \cdot \begin{pmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} = \tag{4.17}$$

$$\boldsymbol{R}_{\omega\phi\kappa} = \begin{pmatrix} \cos\phi\cos\kappa & -\cos\phi\sin\kappa & \sin\phi \\ \sin\omega\sin\phi\cos\kappa + \cos\omega\sin\kappa & -\sin\omega\sin\phi\sin\kappa + \cos\omega\cos\kappa & -\sin\omega\cos\phi \\ -\cos\omega\sin\phi\cos\kappa + \sin\omega\sin\kappa & \cos\omega\sin\phi\sin\kappa + \sin\omega\cos\kappa & \cos\omega\cos\phi \end{pmatrix} \tag{4.18}$$

V symbolickém zápisu:

$$\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \tag{4.19}$$

Therefore, the following relations are applicable:

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \mathbf{R} \cdot \begin{pmatrix} x' \\ y' \\ z' = -f \end{pmatrix}, \qquad \begin{pmatrix} x' \\ y' \\ z' = -f \end{pmatrix} = \mathbf{R}^T \cdot \begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} \tag{4.20}$$

Mathematical Note:

$$\tan\omega = -\frac{r_{23}}{r_{33}}, \sin\phi = r_{13}, \tan\kappa = -\frac{r_{12}}{r_{11}}$$

Note here that $r_{13} = \sin\varphi > 0$ in the 1st and 2nd quadrants and further that $r_{13} = \sin\varphi < 0$ in the 3rd and 4th quadrants. Thus, the rotation of j is not uniquely determined. The quadrants of the other two rotations w,k are uniquely determined given the expressions from which we compute them. ***Thus, we get two sets of rotations ω,φ,κ to a single rotation matrix R.*** *Apart from these problems, a numerical problem may arise, namely in the case that $\cos\varphi = 0$ ($\varphi = 100^g$ or 300^g).*

However, this angle cannot practically occur in photogrammetry (except perhaps in a vertical fall of the aircraft).

4.4 Translation - shift in space

If we detect changes in the image coordinates, besides the rotation of the image system, there is also the shift of the system in space. This is easier to describe.

4.4.1 Effect of changing the x-coordinate

For a small shift of the image in the *x-axis* direction, Figure 4.10 applies:

$$x = \frac{z}{z'}x' = \frac{h}{f}x' \qquad (4.23)$$

where *h* is the flight altitude (constant in this case) and *f is the* camera constant, or we express this relation directly for the image coordinates and we differentiate this expression for all image coordinates:

$$x' = \frac{f}{h}x \qquad (4.24)$$

$$\Delta x' = \frac{f}{h}\cdot\Delta x, \quad \Delta y' = 0, \quad \Delta z' = \Delta f = 0 \qquad (4.25)$$

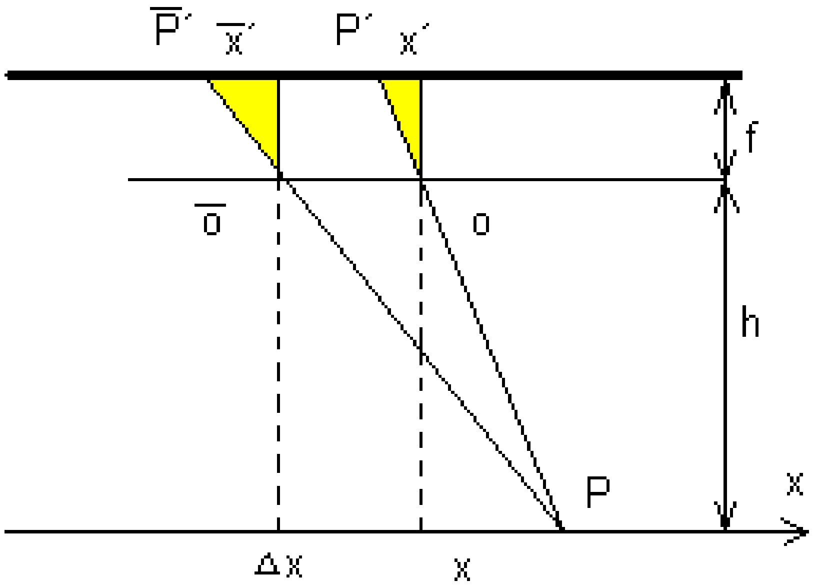


Fig.4.10 Effect of changing the x-coordinate

In this case, *f/h* is the scale of the image. Changes of *Δx, Δy, Δz* in the scale of the image are often referred to traditionally as ***db$_x$* , *db$_y$* , *db$_z$*** , because of the same meaning on old analogue machines. The expression goes to the shape:

$$\Delta x' = db_x, \quad \Delta y' = 0, \quad \Delta z' = \Delta f = 0 \tag{4.26}$$

Effect of y-coordinate change (same as for x-coordinate)

The translation in the y-axis direction is derived in the same way as for the x-coordinate:

$$\Delta x' = 0, \quad \Delta y' = \frac{f}{h} y, \quad \Delta z' = \Delta f = 0 \tag{4.27}$$

$$\Delta x' = 0, \quad \Delta y' = db_y, \quad \Delta z' = \Delta f = 0 \tag{4.28}$$

The effect of changing the z coordinate

However, when the flight altitude changes, there are changes in both image coordinates.

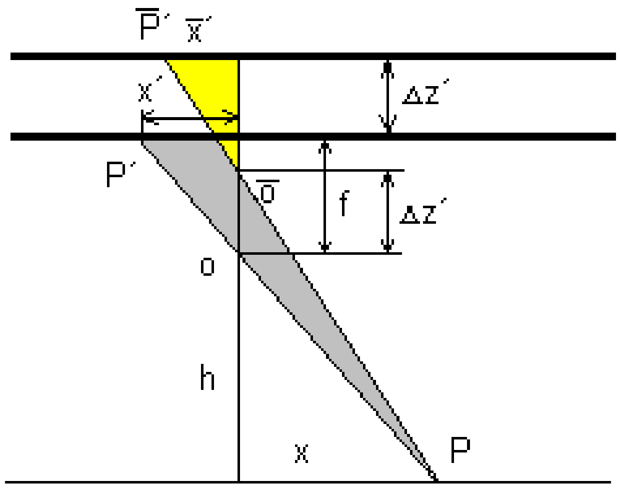


Fig.4.11: Effect of change of flight altitude on image coordinates

$$x = \frac{z}{z'} x' = \frac{h}{f} x', y = \frac{z}{z'} y' = \frac{h}{f} y' \tag{4.29}$$

$$x' = \frac{f}{h} x, \qquad y' = \frac{f}{h} y \tag{4.30}$$

$$\overline{x'} = \frac{f}{h+\Delta z} x, \qquad \overline{y'} = \frac{f}{h+\Delta z} y \tag{4.31}$$

If we subtract the equations (4.30) and (4.31), we get after modification based on relations:

$$\frac{1}{a \pm x} \cong \frac{1}{a}\left(1 \mp \frac{x}{a}\right)$$

expression for differences in frame coordinates:

$$\Delta x' = x' - \overline{x}' = \frac{x'}{h} \cdot \Delta z, \qquad \Delta y' = y' - \overline{y}' = \frac{y'}{h} \cdot \Delta z \tag{4.32}$$

We modify the expressions by inserting:

$$\Delta z = \Delta z' \cdot \frac{h}{f}$$

and we get the final form:

$$\Delta x' = \frac{x'}{f} db_z, \quad \Delta y' = \frac{y'}{f} db_z, \quad \Delta z' = \Delta f = 0 \tag{4.33}$$

4.5 Scale change

Scaling is a simple mathematical operation. If the change of scale *m is* the same for all axes, the relationship is trivial:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = m \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{4.34}$$

If the scale on the axes is not the same, the notation will be more complicated:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathbf{M} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad kde \quad \mathbf{M} = \begin{pmatrix} m_X & 0 & 0 \\ 0 & m_Y & 0 \\ 0 & 0 & m_Z \end{pmatrix} \tag{4.35}$$

Non-identical scaling numbers were used, for example, in roll film shrinkage, where the film shrunk more in the direction of the roll. However, non-equal axis scales (but the changes are small) can also be present in modern measurements, e.g. GNSS.

4.6 Relations between coordinate systems

The historical derivation is given here first.

The method of transforming image coordinates to geodetic coordinates is given in symbolic notation:

$$\mathbf{x}', \mathbf{y}', \mathbf{z}' \quad \rightarrow \quad \mathbf{x}'_F, \mathbf{y}'_F, \mathbf{z}'_F \quad \rightarrow \quad \mathbf{x}, \mathbf{y}, \mathbf{z} \quad \rightarrow \quad \mathbf{X}, \mathbf{Y}, \mathbf{Z}$$

It is a display from one system to another. It is necessary to define the **matrix of the display**, which is the rotation matrix (see below). The transformation procedure starts by converting the image coordinates into fictitious (three-dimensional) coordinates

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \mathbf{R} \cdot \begin{pmatrix} x' \\ y' \\ z' = -f \end{pmatrix}, \qquad \mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' = -f \end{pmatrix} = \mathbf{R}^T \cdot \begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} \tag{4.36}$$

The matrix ***R*** is orthogonal, $\boldsymbol{R}^{-1} = \boldsymbol{R}^T$. Furthermore, the similarity holds:

$$\frac{x'_F}{z'_F} = \frac{x'_S}{z'_S} = \frac{x'_S}{-f} = \frac{x - x_0}{z - z_0}, \qquad \frac{y'_F}{z'_F} = \frac{y'_S}{z'_S} = \frac{y'_S}{-f} = \frac{y - y_0}{z - z_0} \tag{4.37}$$

Substituting from (4.36) into (4.37) we obtain a **collinear relation**:

$$x'_s = -f\,\frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f}, \quad y'_s = -f\,\frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f} \tag{4.38}$$

Next, similar to (4.38), we can use the relations (4.37) to express the model coordinates:

$$x = x_0 + (z - z_0)\frac{r_{11}(x' - x'_0) + r_{12}(y' - y'_0) - r_{13}f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33}f}$$

$$y = y_0 + (z - z_0)\frac{r_{21}(x' - x'_0) + r_{22}(y' - y'_0) - r_{23}f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33}f} \tag{4.39}$$

or inverse frame coordinates (this form is more common because we can advantageously assign corrections to image coordinates and linearize the relation):

$$x' = x'_0 - f\,\frac{r_{11}(x - x_0) + r_{21}(y - y_0) + r_{31}(z - z_0)}{r_{13}(x - x_0) + r_{23}(y - y_0) + r_{33}(z - z_0)}$$

$$y' = y'_0 - f\,\frac{r_{12}(x - x_0) + r_{22}(y - y_0) + r_{32}(z - z_0)}{r_{13}(x - x_0) + r_{23}(y - y_0) + r_{33}(z - z_0)} \tag{4.40}$$

4.6.1 Direct relationship between image and geodetic coordinates

Another - modern - possibility is to derive a direct relationship between image coordinates and geodetic coordinates. The model coordinate system is in a collinear relationship with the image coordinate system as follows:

$$\frac{x' - x_0'}{-f} = \frac{x - x_0}{z - z_0}, \qquad \frac{y' - y_0'}{-f} = \frac{y - y_0}{z - z_0} \tag{4.41}$$

The model coordinate system can be converted to a geodetic system by rotation about three axes:

$$\begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} = \mathbf{R} \cdot \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}, \qquad \mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$
$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} = \mathbf{R}^T \cdot \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} \tag{4.42}$$

By substituting from (4.42) into (4.41) we obtain a **<u>direct relationship between image and geodetic coordinates, which is the basis of all modern photogrammetry</u>**:

$$x' = x_0' - f\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$
$$y' = y_0' - f\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} \tag{4.43}$$

Similarly, we arrive at the same result by considering a general transformation from classical geodesy. From the image coordinate system (omitting the correction for lens distortion, which is introduced directly in the measurement of image coordinates) we get to the geodetic system by rotating, scaling, and shifting the image coordinates (rotation, scaling, and translation):

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} + m \cdot R \cdot \begin{bmatrix} x' - x_o' \\ y' - y_o' \\ -f \end{bmatrix} \tag{4.44}$$

where R is the rotation matrix, m is the scale number, $\begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix}$ represents the shift from the origin of the geodetic system to the position of the input pupil.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} + m \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \cdot \begin{bmatrix} x' - x_o' \\ y' - y_o' \\ -f \end{bmatrix} \tag{4.45}$$

$$\begin{aligned} X &= X_o + m \cdot \left(r_{11} \cdot \left(x' - x_o'\right) + r_{12} \cdot \left(y' - y'\right) - r_{13} \cdot f\right) \\ Y &= Y_o + m \cdot \left(r_{21} \cdot \left(x' - x_o'\right) + r_{22} \cdot \left(y' - y'\right) - r_{23} \cdot f\right) \\ Z &= Z_o + m \cdot \left(r_{31} \cdot \left(x' - x_o'\right) + r_{32} \cdot \left(y' - y'\right) - r_{33} \cdot f\right) \end{aligned} \tag{4.46}$$

From the third equation in (4.46), we express the scale *m* and plug it into the first and second equations:

$$\begin{aligned} X &= X_o + (Z - Z_o) \cdot \frac{r_{11} \cdot \left(x' - x_o'\right) + r_{12} \cdot \left(y' - y'\right) - r_{13} \cdot f}{r_{31} \cdot \left(x' - x_o'\right) + r_{32} \cdot \left(y' - y'\right) - r_{33} \cdot f} \\ Y &= Y_o + (Z - Z_o) \cdot \frac{r_{21} \cdot \left(x' - x_o'\right) + r_{22} \cdot \left(y' - y'\right) - r_{23} \cdot f}{r_{31} \cdot \left(x' - x_o'\right) + r_{32} \cdot \left(y' - y'\right) - r_{33} \cdot f} \end{aligned} \tag{4.47}$$

$$m = \frac{Z - Z_0}{r_{31} \cdot \left(x' - x_0'\right) + r_{32} \cdot \left(y' - y'\right) - r_{33} \cdot f}$$

By inverting the relation, we obtain equation (4.43).

4.7 Photogrammetric series

Photogrammetric series belong to the theoretical foundations of photogrammetry and are the basis of many other derivations, so they should be given due attention.

Working with the complex rotation matrix has been difficult in the past. Therefore, ways were sought to simplify the calculations, but still be accurate enough. The way was to linearize the rotation matrix and simplify it under certain prerequisites.

Definition: photogrammetric series are expressions which, with a degree of precision given by the linearization of a complete relation, express the effect of elements of exterior orientation on image coordinates.

Photogrammetric series are intended to define the influence of the elements of exterior orientation on the image coordinates - i.e. the influence of rotation and translation. Translation is inherently a linear relationship, so it is necessary to linearize the apparently nonlinear rotation matrix. The rotation matrix (4.22) contains the sums and products of the goniometric functions. Assuming (which is commonly met in photogrammetric imaging) that the angles of rotation are sufficiently small (**within 2°-3°**), the following simplification can be used:

$cos\,(\alpha) \cong 1$ and $sin\,(\alpha) \cong d\alpha$

Once this is achieved, we find that differentially small angles allow us to construct a simplified spatial rotation matrix ***R***:

$$\mathbf{dR} = \begin{pmatrix} 1 & -d\kappa & d\varphi \\ d\kappa & 1 & -d\omega \\ -d\varphi & d\omega & 1 \end{pmatrix} \tag{4.48}$$

There are several ways to derive photogrammetric series. Here is the simplest one. The basis is the collinear relationship between the coordinates of the exact vertical image and the fictitious three-dimensional image coordinates. These fictitious image coordinates are replaced by the real measured image coordinates (there are only two of them, plus the camera constant) according to the following relations:

$$\frac{x'_S}{-f} = \frac{x'_F}{z'_F}$$

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = dR \begin{pmatrix} x' \\ y' \\ -f \end{pmatrix}$$

$$x'_s = -f\,\frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f}, \quad y'_s = -f\,\frac{r_{21}x' + r_{22}y' - r_{23}f}{r_{31}x' + r_{32}y' - r_{33}f} \tag{4.49}$$

if we further add the linearized simplified rotation matrix after the rotation matrix we get the notation:

$$x'_S = -f\,\frac{x' - y'd\kappa' - f d\phi'}{-x'd\phi' + y'd\omega' - f} \qquad y'_S = -f\,\frac{x'd\kappa' + y' + f d\omega'}{-x'd\phi' + y'd\omega' - f} \tag{4.50}$$

the next procedure is analogous for x'_S *and* y'_S

$$\left(-\,x'd\varphi' + y'd\omega' - f\right)x'_s = -f\left(x' - y'd\kappa' - f\,d\varphi'\right) \tag{4.51}$$

after division by *-f:*

$$\left(1 - \frac{y'd\omega'}{f} + \frac{x'd\phi'}{f}\right) x_s' = \left(x' - y'd\kappa' - fd\phi'\right) \tag{4.52}$$

adjust to a form suitable for development (multiplication):

$$x_s' = \left(x' - y'd\kappa' - fd\phi'\right)\left[1 - \left(\frac{y'd\omega'}{f} - \frac{x'd\phi'}{f}\right)\right]^{-1} \tag{4.53}$$

If we restrict the development to only first order members (i.e. only members where the differential value occurs at most once), we get:

$$x_s' = A \cdot [1 - B]^{-1} \approx A \cdot (1 + B) = A + AB \tag{4.54}$$

The resulting equations will then be:

$$\Delta x' = x_s' - x' = -y'd\kappa' - \left(f + \frac{x'^2}{f}\right) d\phi' + \frac{x'y'}{f} d\omega' \tag{4.55}$$

$$\Delta y' = y_s' - y' = x'd\kappa' - \frac{x'y'}{f} d\phi' + \left(f + \frac{y'^2}{f}\right) d\omega'$$

Finally, these expressions must be supplemented with the influence of translation according to ch.4.1:

$$\boxed{\begin{aligned} \Delta x' &= -y'd\kappa' - \left(f + \frac{x'^2}{f}\right) d\phi' + \frac{x'y'}{f} d\omega' + db_x' + \frac{x'}{f} db_z' \\ \Delta y' &= x'd\kappa' - \frac{x'y'}{f} d\phi' + \left(f + \frac{y'^2}{f}\right) d\omega' + db_y' + \frac{y'}{f} db_z' \qquad (4.56) \end{aligned}}$$

The expression (4.56) is called the "**complete photogrammetric series**", or historically the Gruber series, and is used in simplified theoretical derivations. The meaning and use of series was quite fundamental, especially in the era of analogue photogrammetry.

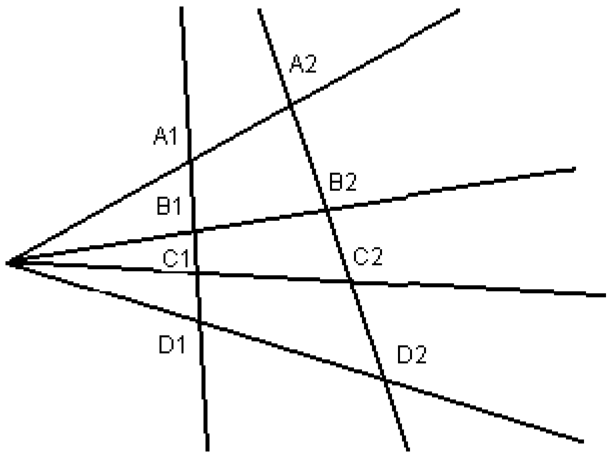
5 Single-image photogrammetry

Single-image photogrammetry has its use in both terrestrial and aerial and to some extent also satellite photogrammetry. It is a *relationship between two planes*, which must be considered in the choice of control points and in the processing.
Single-image photogrammetry is divided into terrestrial and aerial; it has the same fundamentals and mathematical rules for both types, only the method of use is different. It is the oldest technology of photogrammetry and has currently a limited using because it is possible to automatically create higher quality and more accurate digital orthophotos using modern digital photogrammetry methods.

Mathematical basis

If the image of a planar object is exactly vertical, the relationship between the image, the object and the map is simple, and the image differs only in scale. However, in most cases, the image is not exactly vertical (the axis of the image is perpendicular to the object), so the scale in the image is variable with the position in the image due to the variable distance to each point, resulting in a perspective view. It should be noted that practically no object is perfectly planar, and this causes radial displacements of detailed points depending on their spatial distribution. Thus, single-image photogrammetry produces photoplans that have limited accuracy.
For an ideal geometric relationship between two planes, there is a fundamental theorem for projective geometry. In its first variant, by Pappus of Alexandria (4th century AD) and second variant by B. Pascal's projective theorem (17th-century)

The basic is the cross ratio (Fig.xxx). Given four distinct collinear points A, B, C, and D, the cross ratio is defined as x(A, B, C, D) = AC/BC · BD/AD. It may also be written as the quotient of two ratios: x(A, B, C, D) = AC/BC : AD/BD. Pappus first proved the startling fact that the cross ratio was invariant—that is, x(A, B, C, D) = x(A', B', C', D').



$$\frac{\dfrac{A_1C_1}{B_1C_1}}{\dfrac{A_1D_1}{B_1D_1}} = \frac{\dfrac{A_2C_2}{B_1C_2}}{\dfrac{A_2D_2}{B_2D_2}} \qquad (5.1)$$

Fig. 5.1 Pappus theorem

The mathematical representation of reality is a **collinear transformation** between the image and the map. It is a transformation from 3D to 2D. It is logical to go from 3D to 2D using relation (5.3), but the inverse from 2D to 3D cannot be done without additional information (the inverse relation is not well defined).

Five control points (9 unknowns) are needed for the solution; dividing equations (5.2) by a constant $\hat{c}_3$ and applying the substitution $a_1 = \hat{a}_1 / \hat{c}_3$, $a_2 = \hat{a}_2 / \hat{c}_3, \ldots,$ $1 = \hat{c}_3 / \hat{c}_3$ yields the simpler expression (5.3), which is defined by only four control points (8 unknowns).

$$X = \frac{\hat{a}_1 x' + \hat{a}_2 y' + \hat{a}_3}{\hat{c}_1 x' + \hat{c}_2 y' + \hat{c}_3}, X\left(\hat{c}_1 x' + \hat{c}_2 y' + \hat{c}_3\right) = \hat{a}_1 x' + \hat{a}_2 y' + \hat{a}_3$$
$$Y = \frac{\hat{b}_1 x' + \hat{b}_2 y' + \hat{b}_3}{\hat{c}_1 x' + \hat{c}_2 y' + \hat{c}_3}, Y\left(\hat{c}_1 x' + \hat{c}_2 y' + \hat{c}_3\right) = \hat{b}_1 x' + \hat{b}_2 y' + \hat{b}_3 \tag{5.2}$$

$$X = \frac{a_1 x' + a_2 y' + a_3}{c_1 x' + c_2 y' + 1}$$
$$Y = \frac{b_1 x' + b_2 y' + b_3}{c_1 x' + c_2 y' + 1} \tag{5.3}$$

Expression (5.3) defines the central projection (also projective transformation), which is known, for example, from descriptive geometry. Note that **there are no** photogrammetric variables or constants in the above relation (f - camera constant, rotation angles, etc.); it follows that we can use virtually any camera even with unknown parameters - we just need to ensure that the images do not show too much distortion. If the distortion is noticeable, it should be removed beforehand, for which there are software tools.

Until the turn of the millennium, single-shot terrestrial photogrammetry was clearly of great use in the creation of photographic plans of not very spatially distributed objects such as facades of houses.

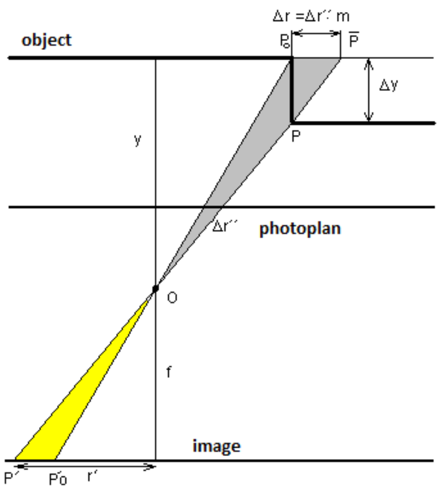
The resulting photoplan (its accuracy) will be influenced by the scale of the image and especially by the depth of the object being targeted. When applying single-image photogrammetry, it is not possible to target an object with too much depth (the method is not suitable for houses with many balconies, staircases, etc.). In the case of spatially distributed objects, the difference between the central projection of the image and the orthogonal projection would be too significant and would cause radial shifts of the detailed $\Delta r''$ according to the relation:

$$\Delta r = \Delta r'' \cdot m_F, \quad \frac{\Delta r}{\Delta y} = \frac{r'}{f}, \quad \Delta r'' = \frac{\Delta y \cdot r'}{f \cdot m_F} \text{ or } \Delta r'' = \frac{\Delta h \cdot r'}{f \cdot m_F} \tag{5.4}$$

. where mF is the scale number of the photoplane. When the maximum differences Δr''max in the photoplane are required (given a predefined value), the spatial (or depth) distribution of the object must not be greater than Δy:

$$. \ \Delta y_{\max} = \frac{f \cdot m_F \cdot \Delta r''_{\max}}{r'} \quad \text{or} \quad \Delta h_{\max} = \frac{f \cdot m_F \cdot \Delta r''_{\max}}{r'} \tag{5.5}$$

It is clear from the above formulas that the selection of the appropriate camera is important. The use of long-focus cameras, however, introduces the problem of a small angle of view, which is particularly pronounced when imaging monuments, where we are usually limited by the maximum possible distance from the object (e.g. the width of the street). Furthermore, it is obvious that the distortion caused by radial displacement of the points increases away from the centre of the image and it is therefore desirable to place the parts with spatially distributed details preferably in the centre of the image. In case the object is deeply segmented and the differences in the *y-coordinate* exceed a certain limit, it is possible to redraw the object in layers (in parts that have approximately the same height or distance from the standpoint; again, for each layer we need at least 4 control points). If it is not even possible to use the redrawing in parts, it is necessary to use the stereophotogrammetric method or to create an orthophoto, which is commonly used today.



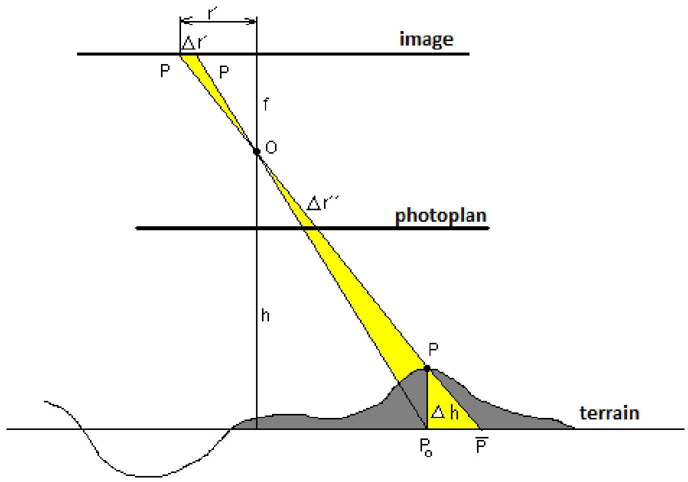


Fig.5.2: Influence of the depth (spatial) distribution of the object

The use of the single-image method for aerial photogrammetry is limited to documentation or interpretation work for flat terrain. The use is possible in the form of photomaps or photomosaics (several connected photomaps) only for flat areas without a requirement for high accuracy (used in the so-called combined mapping method).

For the long-time used redrawers, a good quality control map base with at least four contrast control points was necessary. The control map base was mounted on a projection plate, the magnification was adjusted approximately, and the redrawing process was iteratively manually performed, based on the necessary tilt of the table and the image (the so-called Scheimpflug condition); without this, general photos cannot be rectified correctly (only if they are perfectly accurate vertical images). Analog image redrawing was completely abandoned in the 1990s and replaced by digital redrawing, which is much faster and more accurate. The procedure is simple and can be applied on virtually any *software* that can perform a **collinear** transformation of the image based on the control points. Even so, it is now very little used and has been replaced by digital orthophotos.

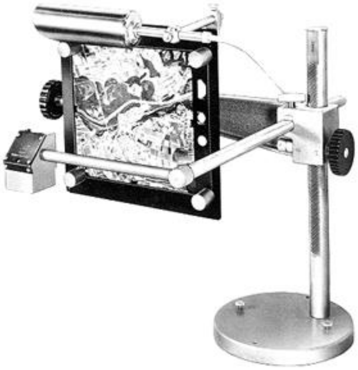
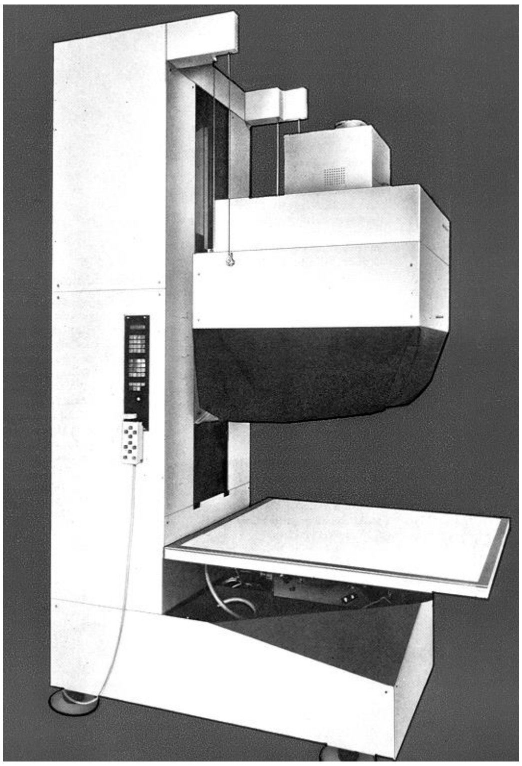



Fig. 5.3 Historically, analogue devices, croppers (left) and rectifiers (right) were designed for single-image photogrammetry. They were abandoned after the transition to digital technology.

The mathematical solution is based on the calculation of the coefficients of the collinear transformation using control points. A minimum of four suitably placed control points are needed in the image (preferably in the corners of the image, no three of these points must lie on a straight line, all control points must lie in the same plane). If there are more control points, the alignment will be performed. The basic equations are:

$$X = \frac{a_1 x' + a_2 y' + a_3}{c_1 x' + c_2 y' + 1} \qquad Y = \frac{b_1 x' + b_2 y' + b_3}{c_1 x' + c_2 y' + 1} \tag{5.11}$$

$$\begin{aligned} X &= a_1 x' + a_2 y' + a_3 \qquad\qquad\qquad\qquad - c_1 x'X - c_2 y'X \\ Y &= \qquad\qquad\qquad\quad b_1 x' + b_2 y' + b_3 - c_1 x'Y - c_2 y'Y \end{aligned} \tag{5.12}$$

The equations are written as a system of eight linear equations:

$$
\begin{pmatrix}
x_1' & y_1' & 1 & 0 & 0 & 0 & -x_1'X_1 & -y_1'X_1 \\
0 & 0 & 0 & x_1' & y_1' & 1 & -x_1'Y_1 & -y_1'Y_1 \\
x_2' & y_2' & 1 & 0 & 0 & 0 & -x_2'X_2 & -y_2'X_2 \\
0 & 0 & 0 & x_2' & y_2' & 1 & -x_2'Y_2 & -y_2'Y_2 \\
x_3' & y_3' & 1 & 0 & 0 & 0 & -x_3'X_3 & -y_3'X_3 \\
0 & 0 & 0 & x_3' & y_3' & 1 & -x_3'Y_3 & -y_3'Y_3 \\
x_4' & y_4' & 1 & 0 & 0 & 0 & -x_4'X_4 & -y_4'X_4 \\
0 & 0 & 0 & x_4' & y_4' & 1 & -x_4'Y_4 & -y_4'Y_4
\end{pmatrix}
\cdot
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ b_1 \\ b_2 \\ b_3 \\ c_1 \\ c_2 \end{pmatrix}
=
\begin{pmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \\ X_4 \\ Y_4 \end{pmatrix}
\tag{5.13}
$$

In matrix form, the solution will be simple:

$$
\begin{aligned}
\mathbf{A} \cdot \mathbf{a} &= \mathbf{X} \\
\mathbf{a} &= \mathbf{A}^{-1} \cdot \mathbf{X}
\end{aligned}
\tag{5.14}
$$

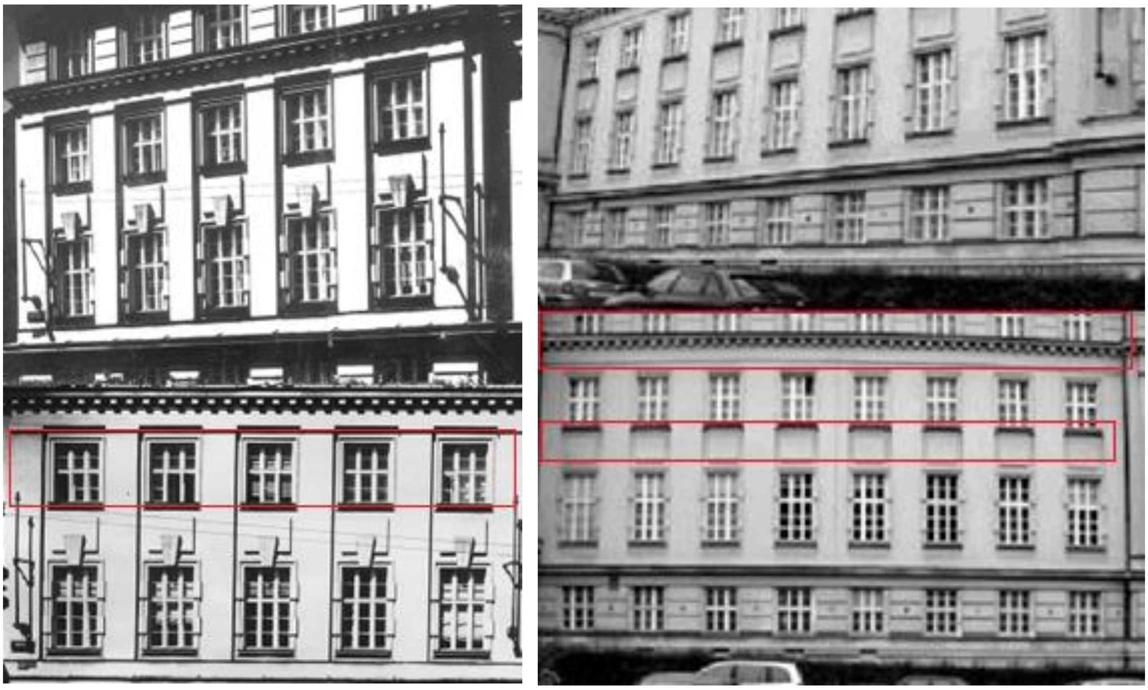


Fig.5.7: Original image and photo-plane, taken by the old measuring camera (left), image, taken by an ordinary photographic camera and its digitally redrawn form - the distortion of the image due to the unremoved radial distortion is clearly visible (right)

***

It is useful to note the following relationships (analogies) between the image and its product and the map product :

photograph - image *(it has no scale or orientation)*

photo plan, ortho-plan - plan *(has scale, orientation, can be measured inside it)*

photomap, orthophotomap - map *(has all the elements of a map such as scale, coordinate system, orientation, frame and out-of-frame data)*

6 Intersection photogrammetry

Intersection photogrammetry is one of the oldest photogrammetric methods. Basically, it is a forward intersection solved by means of metric images. The historical solution is as follows:

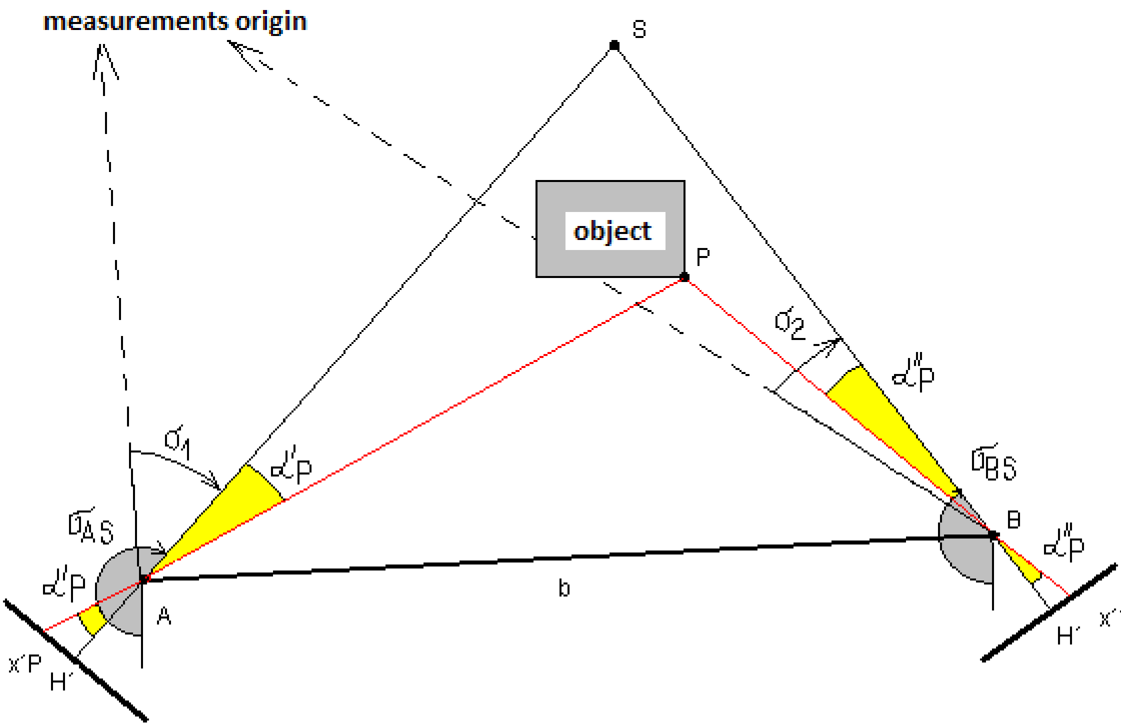


Fig. 6.1: The principle of old intersection photogrammetry

The horizontal angles of the projection rays with the line of sight are calculated from simple equations:

$$tg\alpha' = \frac{x'}{f},\quad tg\beta' = \frac{z'}{\sqrt{\left(f^2 + x'^2\right)}} = \frac{z'}{f}\cos\alpha' \tag{6.1}$$

If we know the geodetic coordinates of the survey stations and the intersections of the view axes (it is necessary to choose a naturally or artificially signalled point as the intersection of the view axes and measure to it with a high precision of about 10 ^{cc}), we can calculate the bearing of the AS and BS lines by adding or subtracting the angle at the left and right survey stations determine the measuring angles for the technology of forward intersection of measuring angles.

The altitudes are obtained twice as in the trigonometric measurement of altitudes. The images are measured monocularly and therefore difficulties will occur in identifying the same points on the left and right images when surveying terrain without distinctive points.

In current systems, this solution is modified with respect to today's possibilities of digital photogrammetry, and according to formula (6.1) it is not calculated, but an analytical solution is used (e.g. the well-known software Photomodeler). The solution is based on the basic photogrammetric equation (6.2). All photographic work consists of taking a suitable number of images with convergent axes of view that encircle the object to be imaged. The images must have sufficient overlap to identify the tie points.

R is the spatial rotation matrix, X, Y, Z are the geodetic coordinates of the points, X_0, Y_0, Z_0 are the coordinates of the projection center, *x', z', (-f)* (for terrestrial photogrammetry) are the measured image coordinates, and x'_0, z'_0 are the coordinates of the principal point. Calculation of the unknowns $\omega, \varphi, \kappa, X_0, Y_0, Z_0$, or *f*, x'_0, z'_0 (=dx', dz') for each image is done by iterating using the coordinates of known control points; this means that we need approximate values of the unknowns before calculation. A correction for radial distortion ($\Delta x´, \Delta z´$) should also be introduced into the calculation.

A necessary requirement is to measure the tie points in the images, which are used to create the model in a similar way to the orientation points in the stereo method for relative orientation. The overall calculation uses a bundle adjustment in the form:

$$\begin{pmatrix} x' - x'_0 + \Delta x' \\ z' - z'_0 + \Delta y' \\ -f \end{pmatrix} = m \cdot \boldsymbol{R}^T \cdot \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} \tag{6.2}$$

In addition, enough control points must be geodetically measured on the object. The minimum number for transformation into the geodetic system is seven measured values (practically 3 points). The spatial processing of the image content can be done from two images without control, three or more images with convergent view axes give us the possibility of control and adjustment; from the point of view of accuracy, the rules for intersection from measurement angles must be respected. More images mean more work, but also more precise positioning of the points to be determined. **Intersection photogrammetry can only be used for artificially or naturally signalled points.**

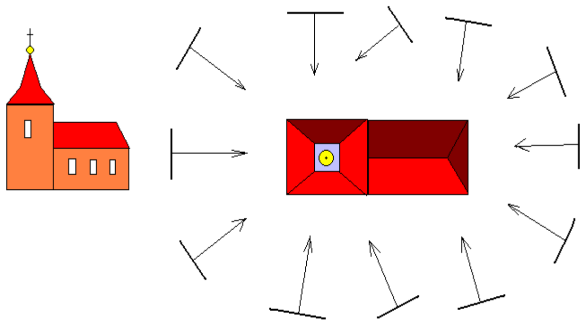


Fig.6.2: Intersection photogrammetry - scheme of photo capturing

7 Stereoscopy

Photogrammetry has applied the stereoscopic method since its introduction by Pulfrich in 1901. It allows to evaluate based on stereoscopic perception also non-signalized points, thus completely revolutionizing the possibilities of photogrammetry. Stereoscopic observation and processing of photogrammetric images is called artificial stereoscopic perception. A healthy person is endowed with natural stereoscopic vision, which is based on simultaneous observation of an object with both eyes. The principles of stereovision, its rules, the creation of artificial stereoscopic perception and its use in measurement are the subject of the following paragraphs.

Natural stereoscopic vision

Observation with one eye is called monocular observation, observation with both eyes is binocular observation. Binocular viewing of landscapes or stereoscopic images gives humans the rare ability to automatically determine the relative distances of objects. This can also be done approximately by monocular observation, but only on the basis of an idea of the size of objects and an innate experience of perspective.

The design of the eye and its function is a perfect and as yet unsurpassed system that can be modelled quite solidly by current technical tools (adaptive optics, automatic aperture, CCD elements, light pipes, computer).

Stereoscopic perception

The basis of successful stereoscopic perception is correct and simultaneous eye *accommodation* and *convergence*. When observing point *P*, it is necessary that the eye axes intersect at the observed point *P*. The eye axes then form the so-called convergence angle g. However, to see the points sharply, it is necessary to refocus the optical system to a given distance - to accommodate the eye, i.e. to change the radius of curvature of the eye lens using the eye muscles. According to previous extensive research, over 90% of people have good stereoscopic perception.

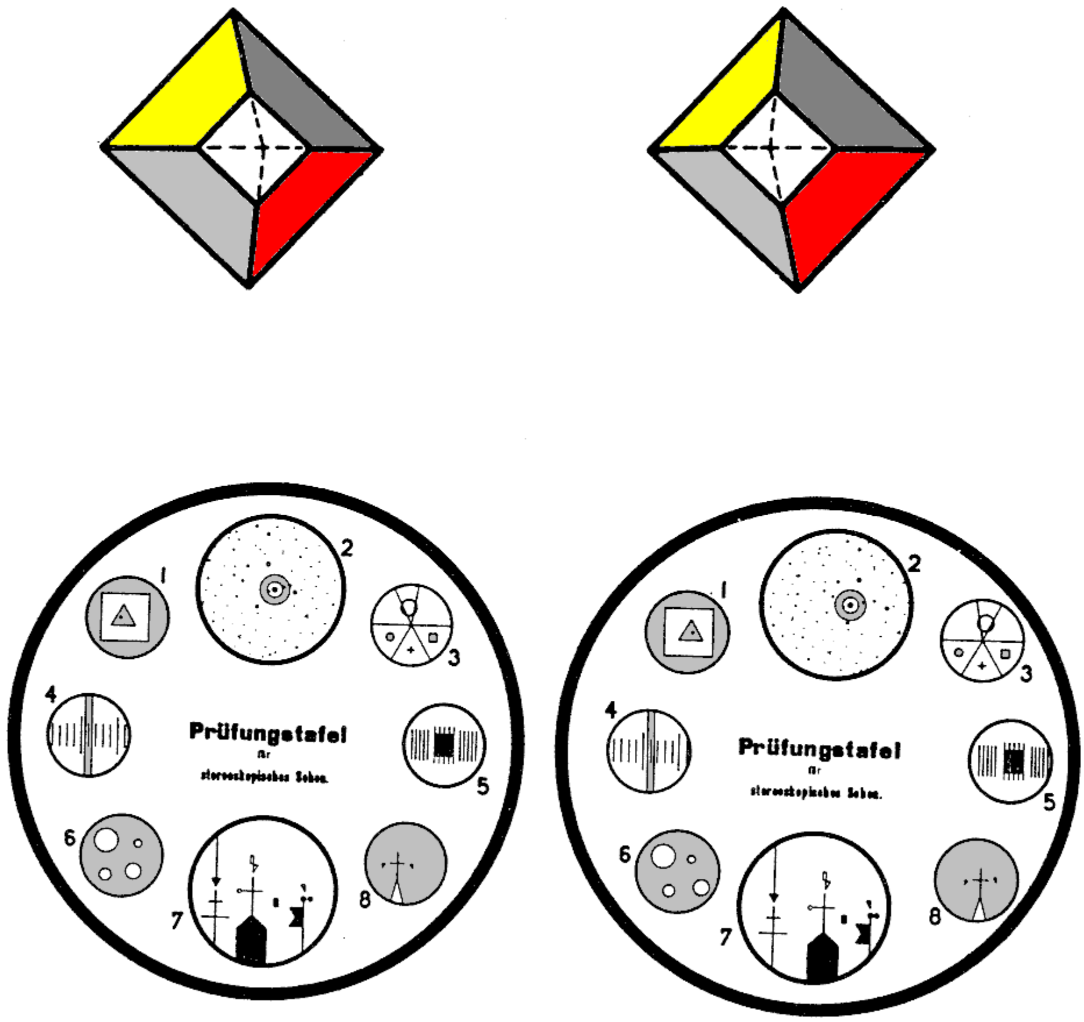


Fig.7.1: Test stereoscopic figure: an artificial stereoscopic perception should occur when viewed from a normal reading distance; vertical parallaxes can be removed by rotating the figure

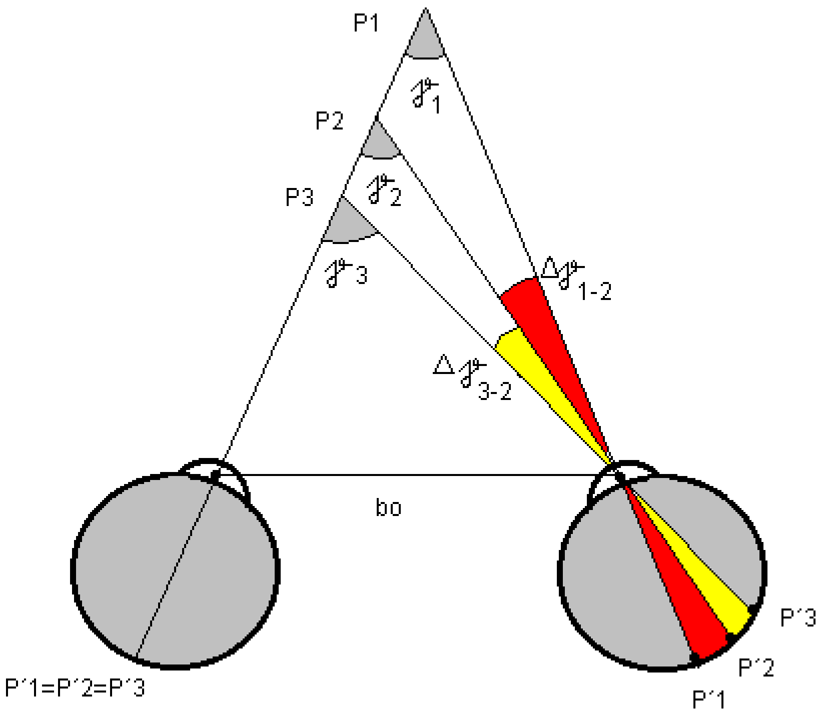


Fig.7.2: Stereoscopic capabilities of the eyes

According to Figure 7.2, point P1 corresponds to the convergence angle γ1, point P2 to the convergence angle γ2, and point P3 to the convergence angle γ3. The convergence angles γ1 and γ3 differ from the convergence angle γ2 by small values Δγ1-2 and Δγ3-2, which we call angular parallaxes according to the equation:

$$\begin{aligned} \Delta\gamma_{1-2} &= \gamma_1 - \gamma_2 \\ \Delta\gamma_{3-2} &= \gamma_3 - \gamma_2 \end{aligned} \qquad (7.1)$$

For the more distant point $_{P1}$ compared to point $_{P2}$ the angular parallax is negative, for the closer point $_{P3}$ the angular parallax is positive, for points as distant as point$_{P2}$ the angular parallaxes are zero. The differences of $_{Pi}$ point images in the direction of *x´ (x´´)* axes on the retina of the eye are physiological parallaxes, the differences of $_{Pi}$ point images in the direction of *z´ (z´´) axes* do not occur in healthy individuals (but they can be induced by gentle pressure of a better unsharpened object - e.g. a finger on the eye). These physiological parallaxes correspond to ***horizontal and vertical parallaxes*** in artificial stereoscopic phenomena. These are defined using an image coordinate system as the difference in the image coordinates of the corresponding point (*homologous point*) in the two images:

$$p = x' - x''$$
$$q = z' - z'' \tag{7.2}$$

where p is the horizontal parallax and q is the vertical parallax.

The theoretical monocular resolution of the eye is about 60´´; this corresponds to a situation where at least one free cone is needed between two points to distinguish their images on the retina. The stereoscopic resolution of the spatial position of the points using the eyes ends around 500 m , where the magnitude of the error is already the same as the determined distance (but it depends on the observer).

If we replace the natural stereoscopic observation of the surrounding reality with the observation of specially made photographic images of the same object, we get the same stereoscopic perception as when observing the real object. For practical observation and to improve the stereovision, aids are used.

In order to produce an artificial stereoscopic perception, the following must be satisfied:

- The eyes must observe the images at the same moment, but at the same time with each eye separately.

- The images must both show at least partly the same location (there must be an overlapping area, i.e. the area shown in both images), the images must show horizontal parallax, i.e. they must be taken from two different points of the stereoscopic base.

- The directions of the observation rays to the corresponding points must intersect and the image must not show vertical parallax.

Aids for artificial stereovision

Due to the difficulties in inducing artificial stereoscopic perception using only the eyes and the short eye base, it is necessary to use aids to enhance or multiply the stereovision.

Stereoscopes

One of the best-known aids for forming stereovision is the stereoscope. The design of stereoscopes can be divided into lens, mirror and prism.

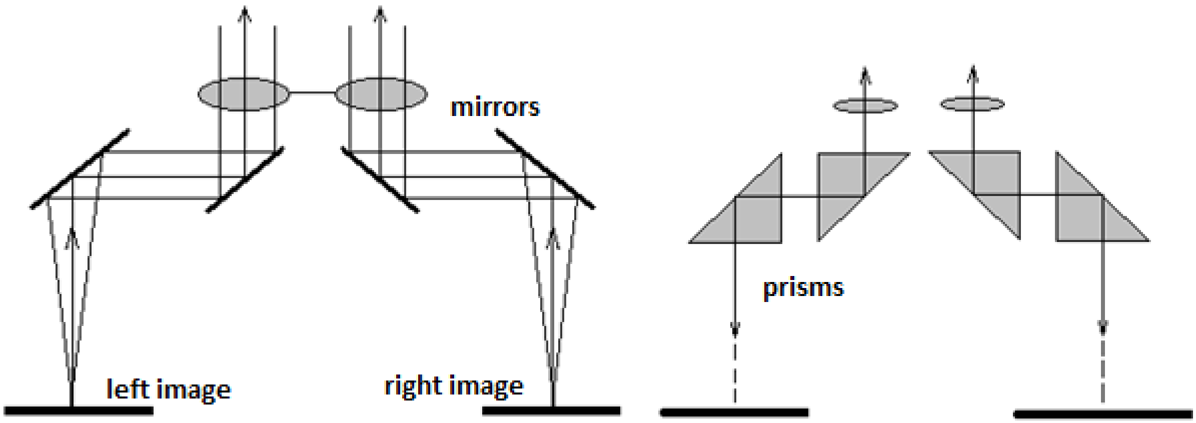


Fig.7.3: Mirror stereoscope (left), prism stereoscope (right)

Lens stereoscopes

The lens stereoscope is the simplest design. The base of the stereoscope is equal to the base of the eye, and at the centres of the eyes there are either apertures or viewing magnifiers only; in these stereoscopes it is not possible to refocus and change the eye spacing.

Mirror stereoscopes

In this type, the eye base is expanded k-times to about 25cm by means of optical mirrors (silvered from above, otherwise a double reflection is created). Eccentrically movable eyepieces are used for refocusing and changing the eye spacing and it is also possible to tilt the mirrors for corrections and changing the viewing base.

Prism stereoscopes

These are constructed on the same principle as mirror stereoscopes but use prisms for reflection. They are usually part of the optical system of photogrammetric plotters.

Working with a stereoscope

A stereoscope uses eyepieces or viewing magnifiers that convert the cone beams coming from images that are approximately in the focal planes of the eyepieces into parallel beams. The eye is then not forced to accommodate a nearby object, and looking into a stereoscope can be likened to looking at a distant object. When viewing through a stereoscope, the images that are placed under it must be oriented so that the corresponding points on the two images lie on parallel lines with the base of the eye.

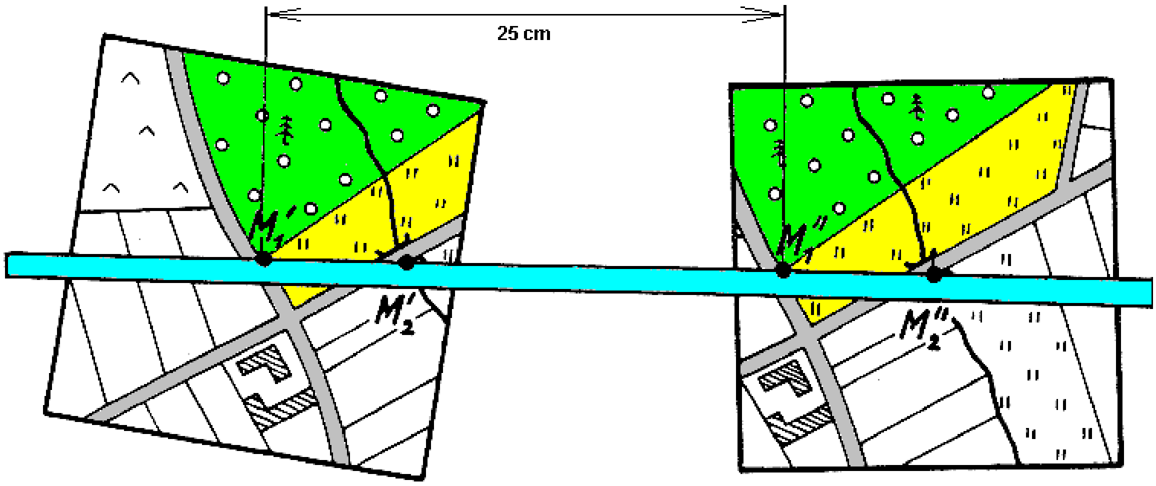


Fig.7.4: Orientation of images under the stereoscope

Anaglyphs

Anaglyphic observing systems are based on a simple principle - they use glasses with coloured lenses (usually blue and red - spectrally as far apart as possible, so that the images interfere as little as possible, which cannot be completely secured). For this reason, this method is rarely used professionally. The following is observed with coloured glasses:

- projected left and right images of a stereoscopic pair on a display in the colours of the viewing glasses

- a stereoscopic pair of images (in paper or film form), each of which is made in one colour of the viewing glasses

- a juxtaposition of red and blue stereoscopic images (often found on the Internet or in magazines)

Stereoscopic perception, made with anaglyphs, is not very high quality, but it is simple and inexpensive. It also allows observation by several people at the same time.

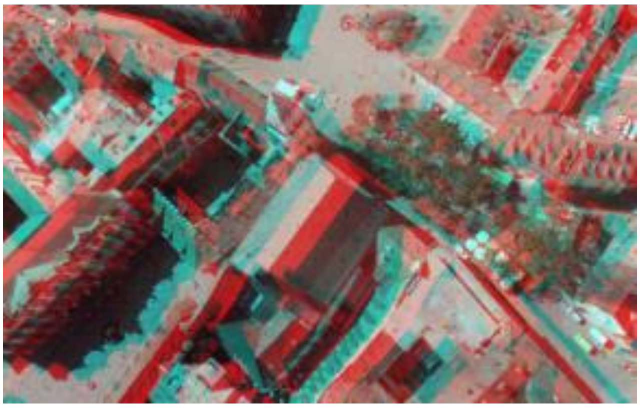



Fig.7.5: Anaglyphic image and glasses

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Use of polarizing filters

A widely used system of stereoscopic observation is one that uses light polarization and special construction components. It is used in digital stereophotogrammetry. A liquid crystal filter is placed in front of the monitor, changing the polarization in synchronization with the imaging of the right and left images. The result is observed through glasses with polarising glasses, one of which transmits vertically and the other horizontally polarised light. These glasses are inexpensive, lightweight, simple and do not need a power source.

This system can be found, for example, in 3D cinema.

Fig.7.6 Polarizing glasses

Crystal glasses

Crystal glasses create a stereo image by synchronised aperture and projection of individual frames of a stereo doublet. These systems have been tried before with mechanical apertures but have failed to catch on - waiting for new technological features. These did not arrive until the mid-1980s under the name ***CrystalEyes©.***

The CrystalEyes© system is mainly used in digital stereophotogrammetry or virtual reality. Currently, the most widely used system works on the basis of active glasses, between whose glasses are liquid crystals that change the light transmission (closed or open). Left and right images are alternately transmitted to a monitor, and information about which image is being displayed is transmitted by an infrared transmitter above the monitor to the glasses; the glasses respond by closing the passage to the right eye with liquid crystals if the left image is transmitted, and vice versa. If the frequency of this change is high enough, at least 25Hz, the human eye perceives this change as a continuous image, similar to a conventional film. Crystal glasses are more expensive, larger and need a power source (battery). Today's systems have a frequency better than 120Hz.

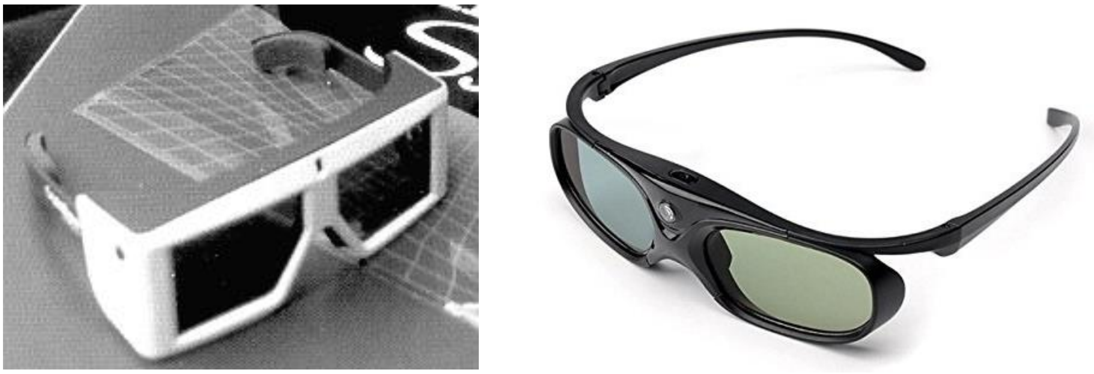

Fig. 7.7: Crystal glasses, on the left an older type around 2000, on the right today's product

Polarizing glasses and semi-transparent polarizing mirror

The two LCD monitors are positioned above each other at an angle of approximately 110°. A semi-transparent mirror is inserted in their axis, which also acts as a vertically polarizing filter. The right image is displayed on the upper monitor and its image is horizontally polarised when reflected by the mirror. The lower monitor displays the left image, whose image is polarised vertically when passing through the mirror. Conventional glasses with polarising filters are used for observation. The advantage is that there is no need to alternate two images on one monitor, therefore high-resolution monitors can be used, and the stereoscopic perception is of higher quality.

The display is high-resolution and free of some annoying flicker for a high-quality 3D stereo view. PluraView's innovative beam-splitting technology, which offers the highest possible stereo display quality, is the basis for accurate stereoscopic 3D displays. PluraView stereo displays can currently be up to 28-inch diagonal, with resolutions up to 4K (UHD) and a colour depth of 10 bits per pixel.

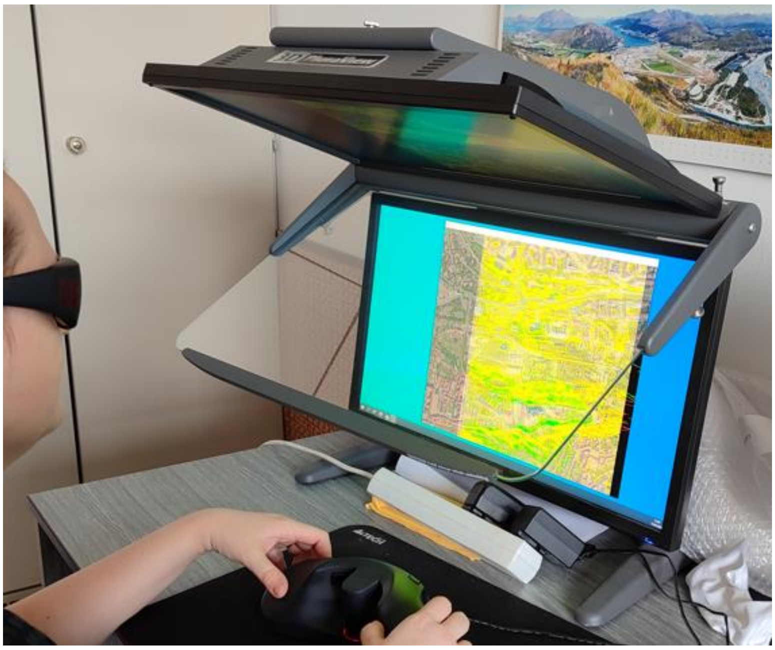

Fig.7.8 3D Plura View stereo monitor

8 Stereophotogrammetry

The stereophotogrammetry method uses artificial stereo vision and binocular observation of spatial points. The image pairs can be taken in different ways, but for a good evaluation it is necessary to keep a suitable base length, approximately equal heights of both shooting positions and preferably parallel axes of view. Current evaluation systems allow the evaluation of almost all image material taken in different ways. It is usually not necessary to follow old, well-defined procedures that were based on limited evaluation possibilities. Nevertheless, even in today's digital and computer age, it is still advisable to follow the following guidelines for quality stereo imaging. In the **classical** concept of terrestrial analogue photogrammetry, we classify:

- normal case (axes of view are perpendicular to the photogrammetric base, $\omega,\kappa=0$, $\varphi=100^{gon}$)
- twisted case (axes of view are twisted by a certain angle to the base, $\omega,\kappa=0$, φ=general)
- tilt case (axes of view are tilted by a certain angle. $\varphi=100^{gon},\kappa=0$, ω=general)
- general oriented case (ω, φ, κ are general)
- convergent images
- divergent images

8.2 Mathematical background

The following text describes the classical procedures, especially applicable to analogue photogrammetry; it is useful to describe them in order to understand the whole legacy technology. Current digital technology no longer proceeds in this way; everything is solved analytically from the basic photogrammetry equation (4.43). The procedures had to be adapted to analogue evaluation machines, which were only able to evaluate certain configurations of images; today, although this is not necessary, the rules of stereophotogrammetry still apply.

8.2.1 The normal case

The basis of terrestrial photogrammetry is the so-called **normal case**, i.e. the situation where the axes of the image are perpendicular to the base. According to Fig. 8.1, a model system is chosen with the *x-axis* parallel to the base *b*, the *z-axis* perpendicular to the *x-axis* and pointing upwards, and the spatial *y-axis* (distance). Often, the centre of the input pupil of the left position is chosen as the origin of the model system. At the same time, the frame coordinate system is defined.

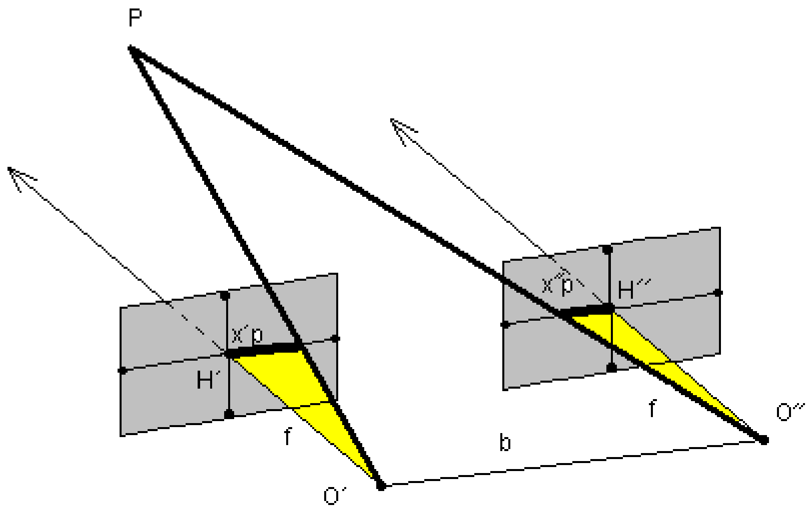

Fig.8.1: Photogrammetrical normal case

The derivation of the relationships between the image coordinates and the model system is simple:

$$\frac{y}{f} = \frac{b}{p} \tag{8.1}$$

$$x = \frac{b \cdot x'}{p} \qquad y = \frac{b \cdot f}{p} \qquad z = \frac{b \cdot z'}{p}$$
$$x = y\frac{x'}{f} \qquad\qquad z = y\frac{z'}{f} \tag{8.2}$$

For practical purposes, it is necessary to derive the theoretical accuracy of the determination of individual model coordinates. For the equation describing y, we form a total differential:

$$y = \frac{b \cdot f}{p}, dy = \frac{f}{p}db + \frac{b}{p}df - \frac{bf}{p^2}dp \tag{8.3}$$

Equation (8.3) describes the effect of all three variables; *db* expresses the effect of the inaccurately determined base length. It is true that *dy/y=db/b* and the error in the base will result in an equally large relative error in the *y* coordinate. For larger base lengths over 3m, maintaining the required accuracy will not be a problem (just measure it several times with a band to mm), but for short bases, the base length needs to be determined more accurately than we normally can. In order to maintain the accuracy of the short base length, double measuring chambers have been constructed where fixed or forced centering is used. In normal cases, therefore, the error from an incorrectly determined base can be neglected. A similar reasoning applies to *df*, i.e. the inaccurately determined size of the camera constant. However, this is determined in the laboratory or numerically to an accuracy of 0.01mm and the relative error of *df/f* will be negligible if it is maintained that the size of the camera constant has not changed (not refocused!) during imaging at both sites.

The accuracy in the spatial component of coordinates is the smallest, so the relationship for the accuracy of photogrammetry is derived as follows. A completely decisive influence on the accuracy of the y determination will be dp - an incorrect determination of the horizontal parallax. In most error analyses, a simplified relation will be enough:

$$dy = -\frac{b \cdot f}{p^2} dp, p^2 = \left(\frac{b \cdot f}{y}\right)^2$$

$$dy = -\frac{y \cdot y}{b \cdot f} dp \quad (8.5)$$

After moving to the mean errors, we arrive at the formula used:

$$m_y = \pm \frac{y \cdot y}{b \cdot f} m_P \quad (8.6)$$

Several important considerations follow from this formula:

- the accuracy of distance determination decreases with the square of the distance

- the accuracy of *y* can be improved by reducing the shooting distance, increasing *b* or *f*

- the accuracy of parallax determination is of great importance and is usually determined by the capabilities of the machine (0.01-0.001mm)

- in the formula the first term is the inverse of the base ratio, which is the carrier of the accuracy - these are freely selectable variables, m_P and *f* are determined by the equipment used

A simplified relation is used for other coordinates

$$m_x = \pm \frac{x'}{f} m_y, m_z = \pm \frac{z'}{f} m_y \quad (8.7)$$

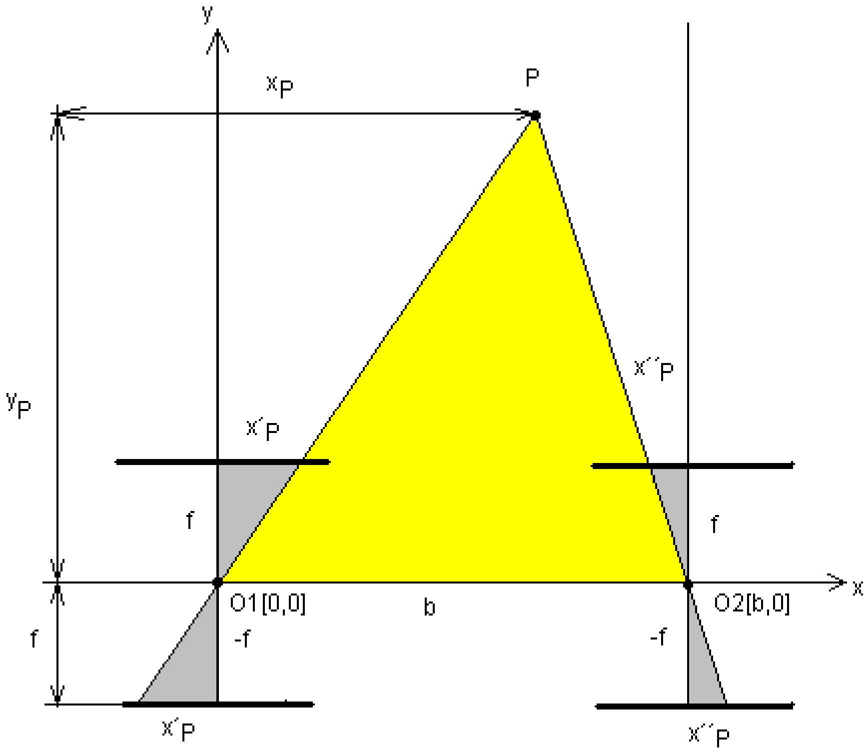

Fig.8.2: Normal case

A different derivation follows from the following figure and reasoning: we start from the basic equation (4.47) and the rotation matrix. The rotation matrix will be a unitary matrix for the normal case (i.e. ω=0, φ=100^{gon}, κ=0).

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{8.8}$$

We rewrite equation (3.6) for the case of terrestrial photogrammetry

$$\begin{aligned} x &= x_0 + \left(y - y_0\right) \frac{r_{11}\left(x' - x'_0\right) + r_{12}\left(z' - z'_0\right) - r_{13} f}{r_{31}\left(x' - x'_0\right) + r_{32}\left(z' - z'_0\right) - r_{33} f} \\ z &= z_0 + \left(y - y_0\right) \frac{r_{21}\left(x' - x'_0\right) + r_{22}\left(z' - z'_0\right) - r_{23} f}{r_{31}\left(x' - x'_0\right) + r_{32}\left(z' - z'_0\right) - r_{33} f} \end{aligned} \tag{8.9}$$

After substituting from (8.8) into (8.9) we get:

$$x = x_0 + (y - y_0)\frac{(x' - x_0')}{f}$$
$$z = z_0 + (y - y_0)\frac{(z' - z_0')}{f} \qquad (8.10)$$

According to Figure 8.2, we express the projection centres in the model coordinate system: O1=[0,0], O2=[b,0]. For the two images of the stereoscopic pair, we get the expressions:

$$x = y\frac{x'}{f}, \qquad z = y\frac{z'}{f} \qquad 1.\,\textit{snímek}$$
$$x = b + y\frac{x''}{f}, \quad z = y\frac{z''}{f} \qquad 2.\,\textit{snímek} \qquad (8.11)$$

We solve the system of equations:

$$y\frac{x'}{f} = b + y\frac{x''}{f} \qquad (8.12)$$

$$y\frac{x'}{f} - y\frac{x''}{f} = b, \qquad \frac{y}{f}(x' - x'') = b, \qquad (x' - x'') = p \qquad (8.13)$$

$$y = \frac{b \cdot f}{p}, \quad z = y\frac{z'}{f} = y\frac{z''}{f}, \quad x = y\frac{x'}{f} \qquad (8.14)$$

The coordinate z is obtained twice with the possibility of checking if the condition *z´= z´´* (*z´- z´´=0* or *q=0, the* condition of zero vertical parallaxes) is fulfilled.

8.2 Photogrammetric base

Among the basic parameters in stereophotogrammetric imaging is the determination of the most suitable length of the photogrammetric base. In stereophotogrammetry, we define the so-called *base ratio (b/y)* as the carrier of the accuracy of coordinate determination. The length of the base depends on the distance of the nearest and farthest points that will be evaluated from the considered base with the desired accuracy. The determination of the most suitable length of the photogrammetric base is based on the normal case of stereophotogrammetry.

The relationships for the maximum and minimum thrust base can be derived, the resulting modified formulas are used:

$$b_{\max} = y_{\min} \cdot \frac{p_{\max}}{f} \tag{8.15}$$

$$bmin = ymax \; \frac{10mm}{f[mm]}$$

So far we have assumed that the elements of the exterior and interior orientation were set up perfectly accurately in the terrestrial photogrammetry. Since each method has its own technical possibilities, it was necessary to define, especially before, how precisely the elements of the inner and outer orientation had to be set. For aerial photogrammetry the situation was different, the elements of the internal orientation were usually defined and the elements of the external orientation had to be calculated (due to the impossibility of obtaining directly accurate vertical images). For terrestrial photogrammetry, it was then necessary to define:

- the elements of internal orientation; *f*, d_x, d_z are usually known
- the elements of the external orientation; the angles ω, φ, κ are set using libels and an orientation device, the position of the projection centre is determined by geodetically locating the left photographic position (the right one can be calculated if the base length and orientation are known)

Next, it is necessary to know how exactly these operations are to be carried out. An accuracy analysis can be made on the basis of photogrammetric series, where the effects of the individual elements of the exterior orientation on the image coordinates will be considered separately.

At present, the solution is the same for terrestrial and aerial stereo-photogrammetry; everything is solved on a computer that does not care whether it calculates with a unit matrix or a general matrix (this is usually the approximately normal case). It must be remembered that this was not the case before and the calculations of the exterior orientation were problematic and lengthy; for this reason, the procedures of terrestrial photogrammetry (where the viewpoint and the object do not move and the measurement parameters can be standardised) have been technologically simplified to the methods described. Simpler relations and photogrammetric plotters could be used to solve them.

Performing photogrammetric surveying

Nowadays, field work has become much easier. Here, of course, is the procedure for ground photogrammetry. Practically, after the object has been recognised, it is necessary to take a good image of the object and to locate the control points for transformation into the geodetic system. The control points are usually signalled with targets or other suitable markers so that they are visible on the images (their size can be calculated). The control points must also be suitably spaced around the object, must not lie in a straight line and should be spatially distributed, the minimum number for spatial transformation being 3 points (see below). An excessive number of control points shall always be targeted due to alignment, control and the possibility that some control points may not be suitable or show significant variation despite best efforts to locate them.

Their measurement is nowadays simple, using a total station or GNSS equipment. The actual imaging is done with a good quality digital camera from a tripod or handheld, but it is advisable to follow the basic photogrammetric rules, i.e. to shoot for stereophotogrammetry with a large overlap between images and with approximately parallel axes of view. Modern SfM (Structure from Motion) methods require a large number of overlapping images in contrast with converging shot axes complemented by a set of images with parallel shot axes for MVS (Multi View Stereo). The processing of such data is then done almost automatically in the software.

The actual imaging procedure depends on the type of camera and the possibilities of the method used, but also on the time available. It is advisable to keep a sketch of where the photo was taken from, record the time and the weather. Because of possible shadows in the photographs, which are very distracting, it is advisable to find out the right time of day to shoot or wait for a compact cloud cover, which is best for technical photography - there are no shadows, and the lighting is constant). Furthermore, it is necessary to arrange access to the building and, if necessary, access to surrounding buildings, to provide a platform or at least a ladder for smaller buildings. Sufficient capacity of the storage medium is a matter of course, as well as the condition of the batteries and, if necessary, the installation of lighting of the object with photographic lights. In general, bad images are difficult to process. Especially when shooting indoors, this matter is highly problematic and it is advisable to consult, for example, a professional photographer.

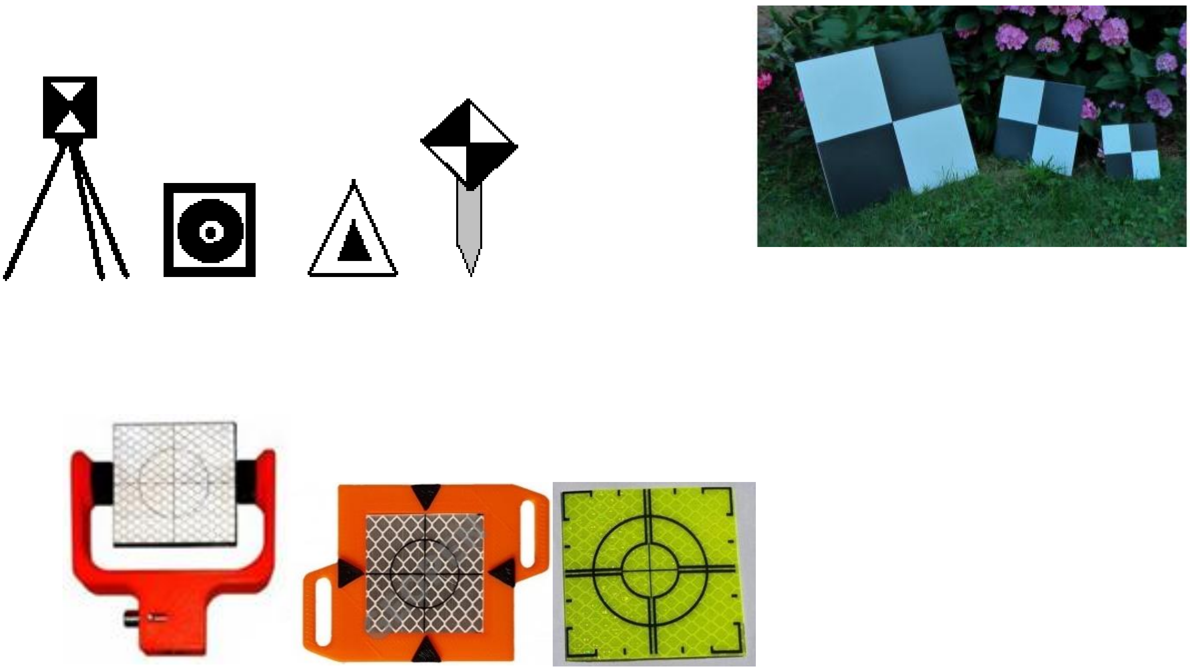

Fig.8.3 *Fig.8.4: Signalling of the control points*