Photogrammetry course for beginners

„ Photogrammetry 1

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Introduction

- **Photogrammetry part of the field of geodesy and cartography, which deals with the determination of geometric and positional information from image records, most often from photographic images**
- **RemoteSensing(RS) deals with non-contact detection of surface cover types and their condition**
- **Laser scanning3D scanning technology that uses laser measurements to determine the spatial coordinates of detailed points on an object.**

Introduction

Photogrammetry, RS and laser scanning are methods that provide mass localized information for technology

GIS

- Basic sources of primary information about the territory supply:
- **geodetic methods** accuracy in the order of **mm - cm**
- **photogrammetry** accuracy in **mm, cm - dm**
- **RS** accuracy in the order of **m - km**

History

Al-Hassan bin Al – Haithm
(965-1039)

*****

in 1032 he was the first to describe the*"camera obscura" –the central projection principe*

History

- Leonardo da Vinci (1452 1519) described the pinhole camera " for the
construction of central projections
- 1605 Galileo Galilei invents the telescope
- 1657 Schott Kasper builds the first portable box camera
- in 1777 the invention of the light sensitive compound , AgCl C.H.Scheele)
- theories of reconstruction of acquired perspective images : Taylor (1715) and J.H.Lambert (1759)
- invention of photography : Niepce and Daquerre (1839
- negative positive: Talbot 1841
- the title of the photo comes from J.Herschel
- the first aerial photographs were taken by the famous French photographer
G.F.Tournachon called Nadar) in 1858

- the first phototeodolite was constructed according to the design of A. Laussedat (1859); used in France for mapping in 1861
- photogrammetry " dates back to 1858, when the German A.Meydenbauer used the term
- G . Eastman , 1884 paper film) and its introduction in 1889 (celluloid film , first roll film camera
- C . Pulfrich Zeiss Jena) in 1901 constructed the first device for stereoscopic measurement , the stereocomparator
- E .Orel , 1909 1911 constructed the first Autostereograph since 1909 in the Carl Zeiss Jena works as " Stereoautograph "
- Th . Scheimpflug constructed in 1911 the first rectifier for transforming an inclined image of flat terrain to the scale of a map
- W .Wright was the first to take pictures from a plane in 1903
- imaging from aircraft found its application with the advent of World War I.
- In 1935, the first Kodakchrome colour film was released

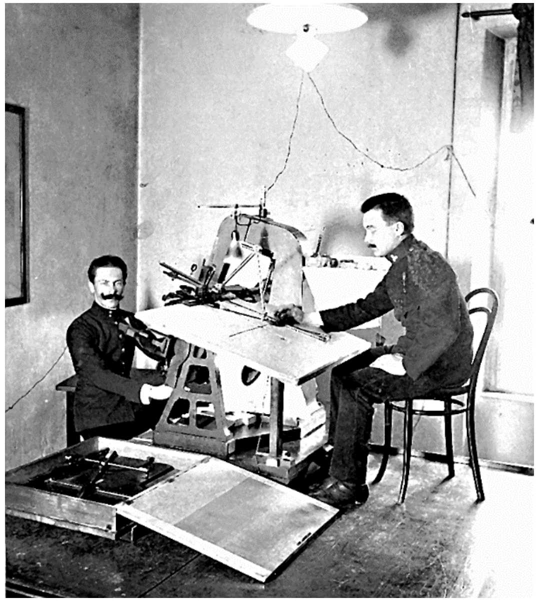
**- during the World War II. some new cameras and instruments were
constructed and methods of using photogrammetry were developed , but
mainly for military purposes**
**- further development of photogrammetry occurred again after 1945
analogue plotters**
- space technology
- Seventies analytical plotters used computers
- Eighties development of digital technologies
- 1990s full transition to digital technology

History

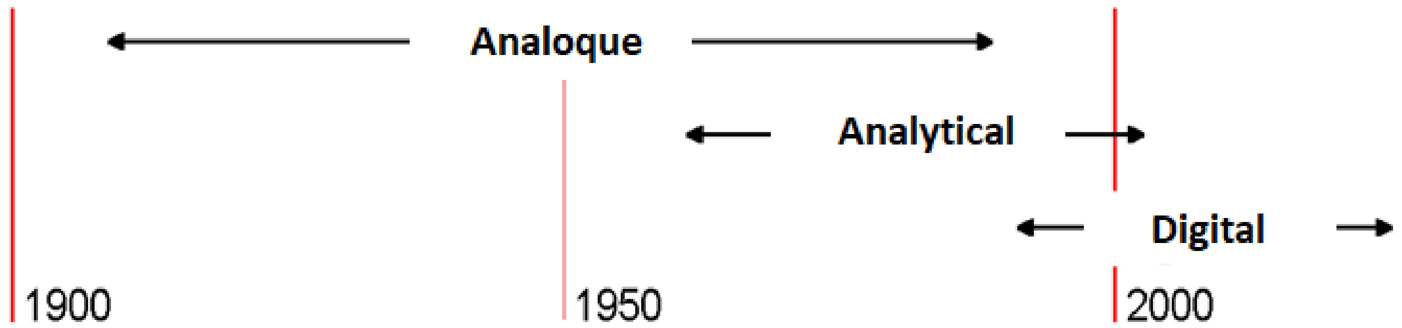
Karel Kořistka (1825 1906) professor of mathematics and geodesy , the first rector of the Royal Czech Polytechnic Institute in Prague in the school year 1864 65. Pioneer of photogrammetry in the Czech lands

K.Kořistka
got acquainted with photogrammetry on a study trip in 1862
directly with A.Laussedat and after his return from this trip he used it in Prague

World War I.



Photogrammetry subdivision
Basic stages of development: technology



Basic criteria for dividing photogrammetry
a) according to the position of the position
Terrestrial , air, satellite
b) by number and configuration of images
Single frame and multi frame : stereo or intersection
b) according to the processing technology
Analogue, analytical , digital

Use of photogrammetry

- **State map works in CZ, State Administration of Land Surveying and Cadastre (ČÚZK) topographic maps (1:10**
- **Military topographic maps (in CZ , Army of the Czech Republic)**
- **Information systems state administration , GIS information layers) , digital models DMR and DMP)**
- **Monument care (monument care workers , architects) documentation , documents**
- **Construction design and construction companies) documents , documentation , determination of deformations**
- **Environment (in CZ, administrations of National park, protected natural area , forestry maps (in CZ, Forest Management Institute ÚHÚL), vegetation delimitation**

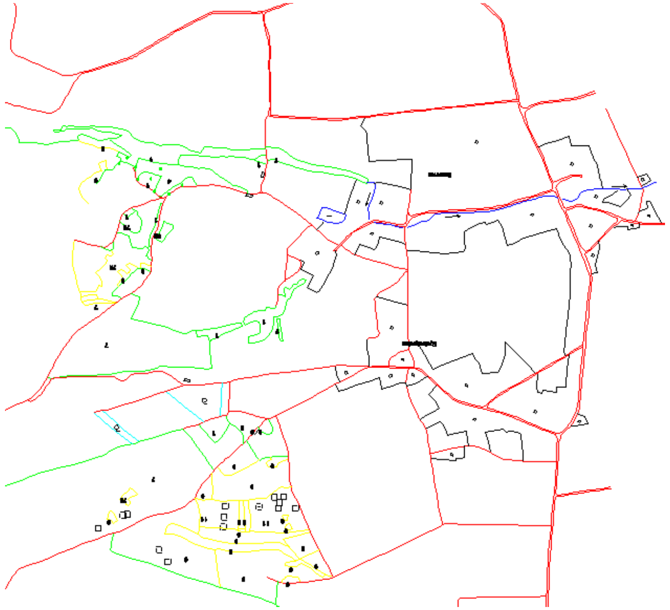
Next

Come in:

- **spatial planning 3D building models**
- **water management floods , Runoff profiles**
- **inventory and monitoring mines , quarries , landfills, landslides**
- **mechanical engineering precision control , deformation**
- **rehabilitation medicine , biomechanical applications etc.**

Use of photogrammetry in mapping

Original photogrammetric vertical image is similar to map



Reasons for using photogrammetry

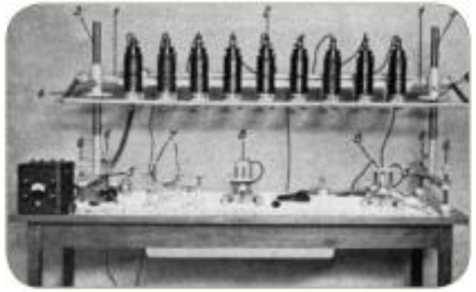
$\Rightarrow$ **minimising field work**

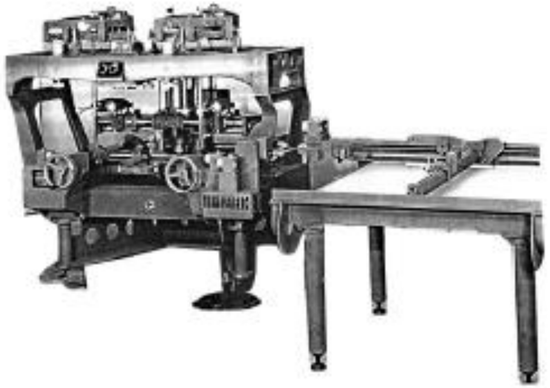
- **economics**

- **speed**

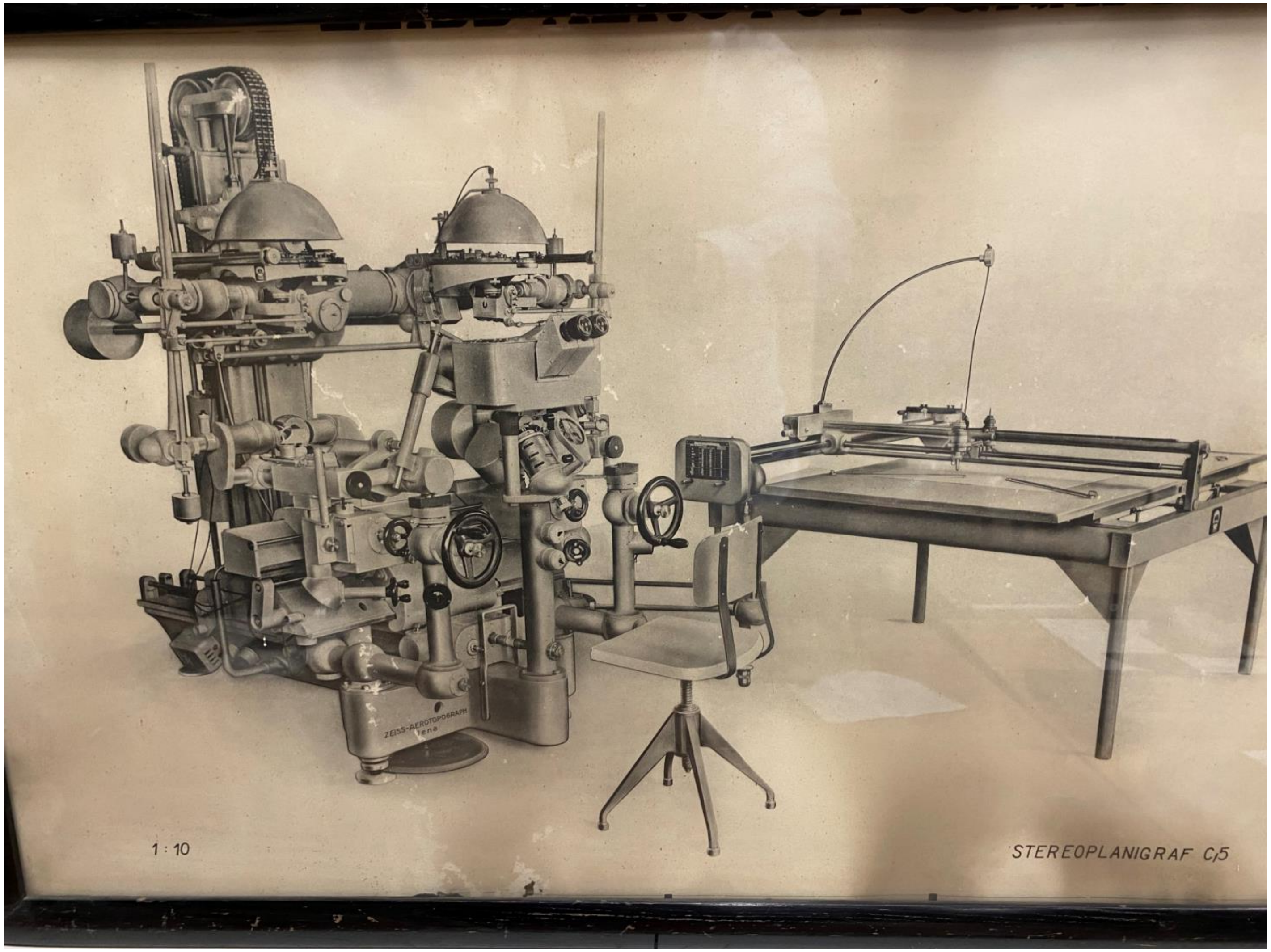
- **overall time saving**
- **cost saving**
- **documentary value of the images time series**
- **higher resolution of the images compared to the map digital orthophoto**

From analoque photogrammetry to the digital technology





Stereoplanigraph, 1930-1950



DPW (digital photogrammetric Workstation (1990- today)

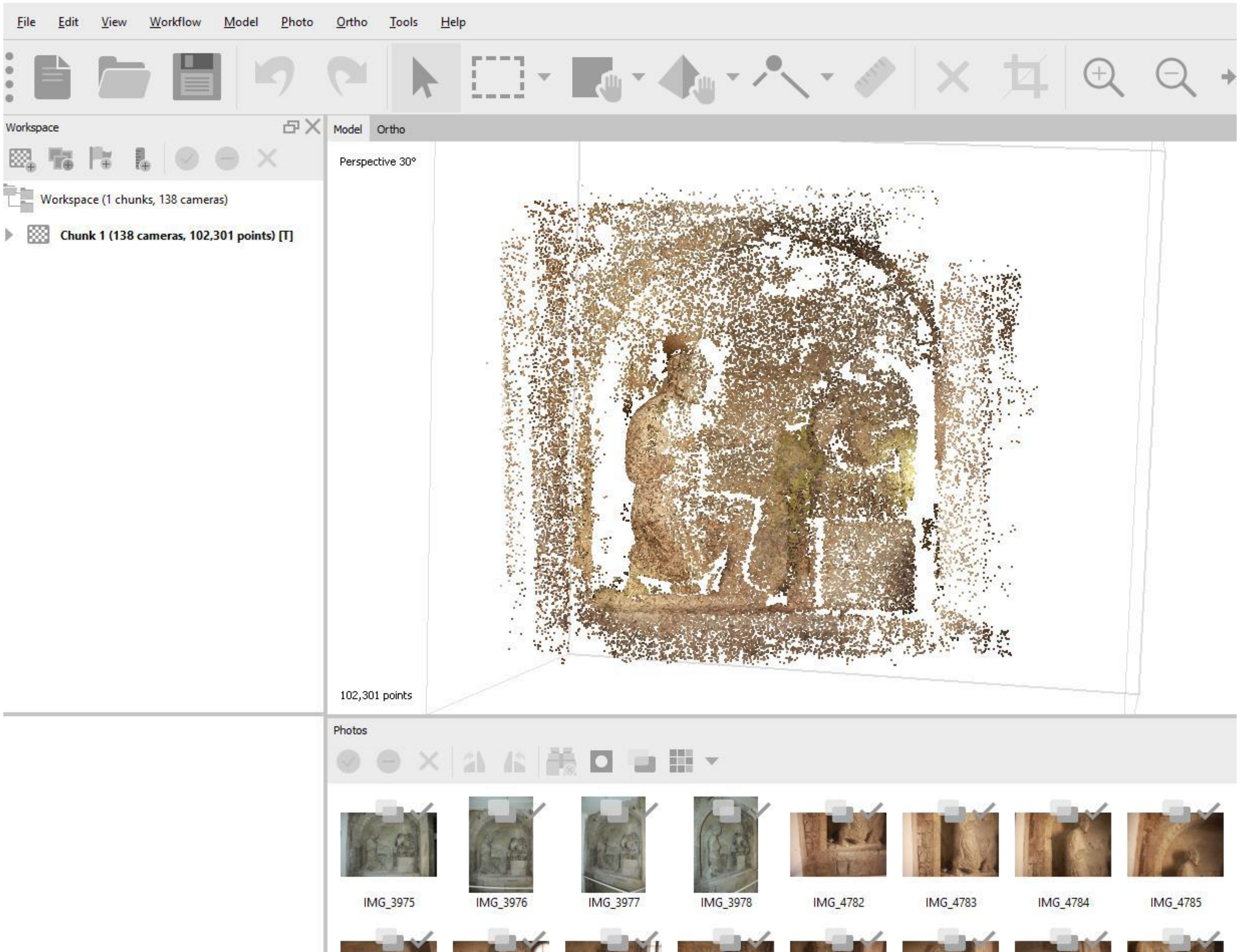
Stereo DWP

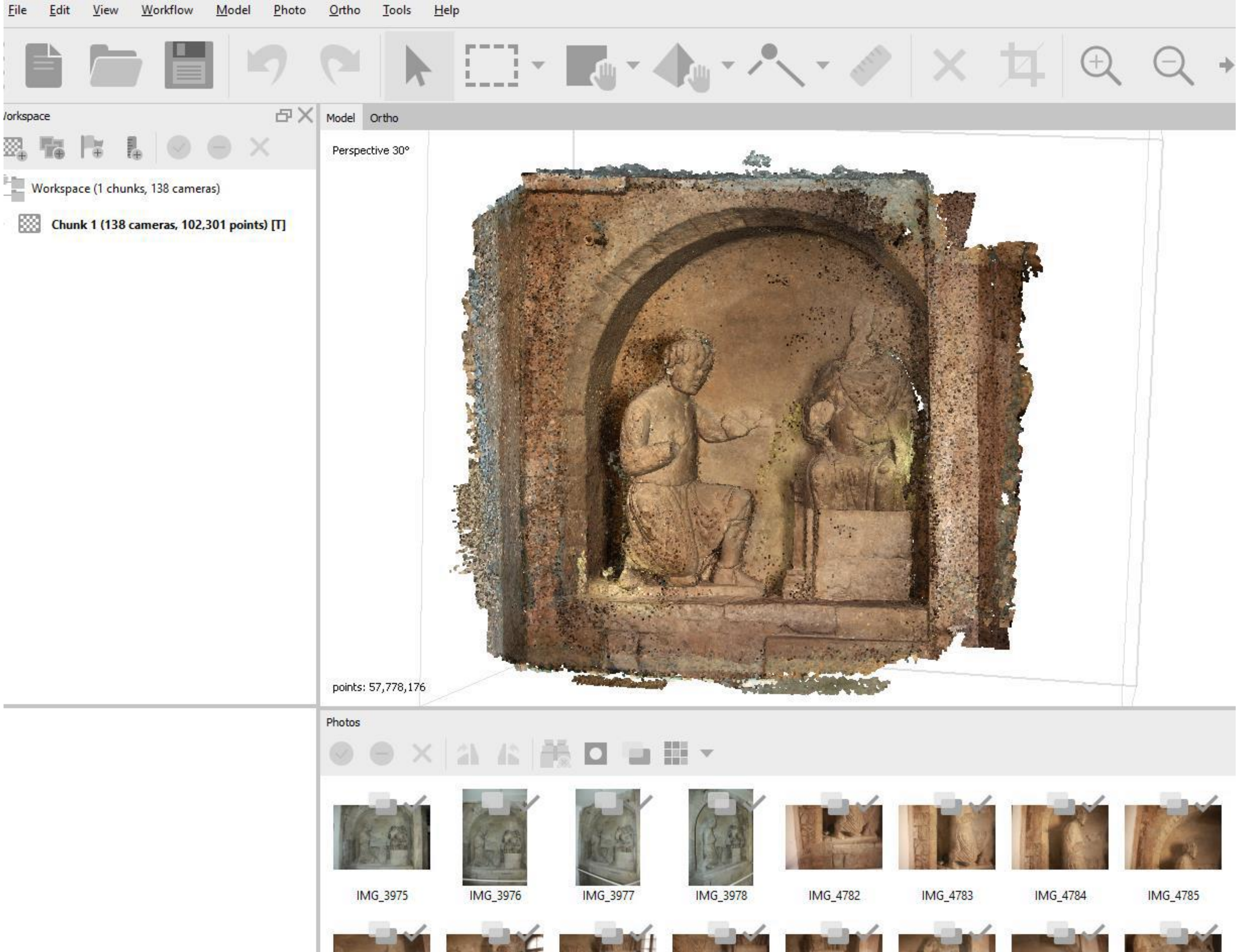
Digital (automated) photogrammetry (after 2010)

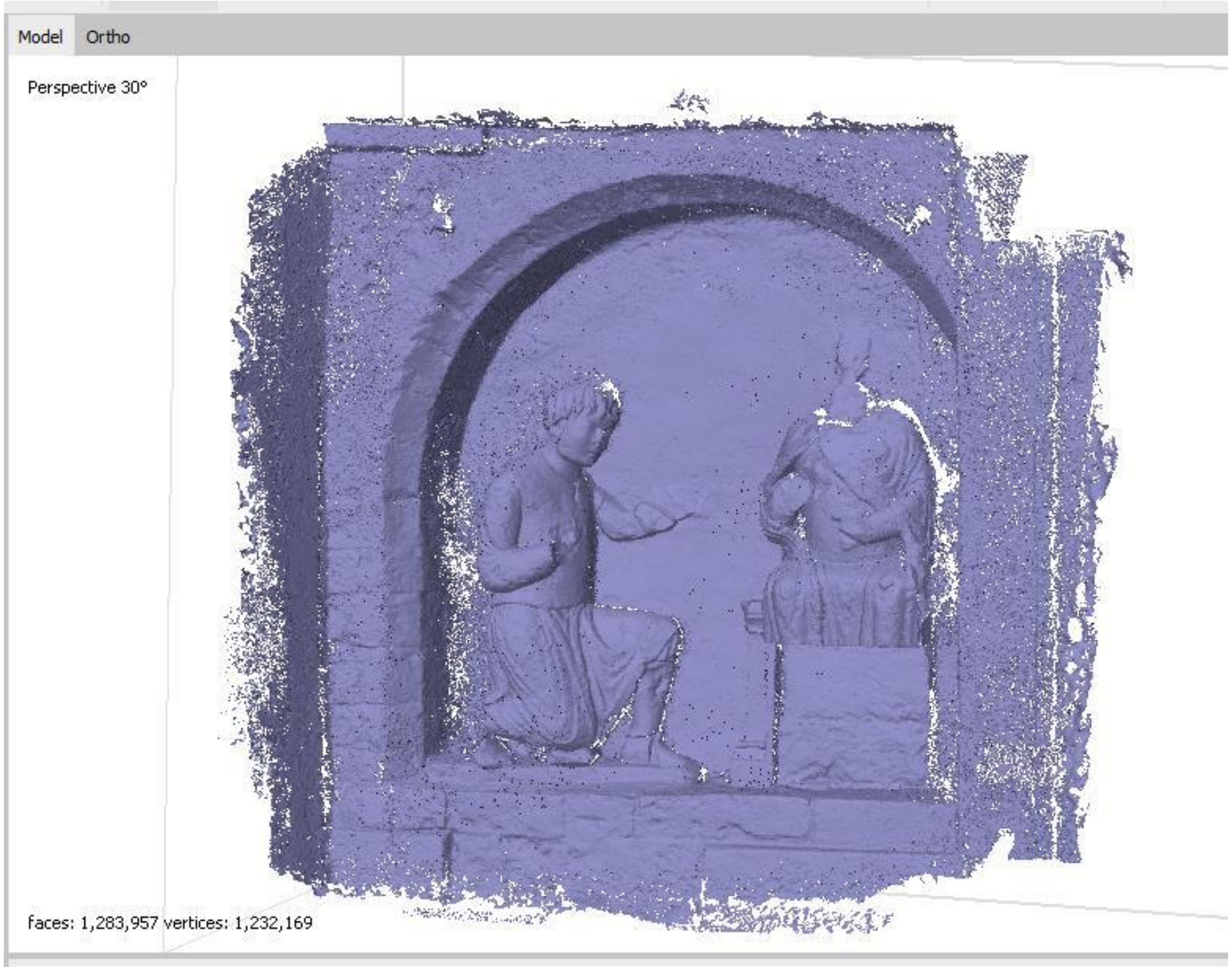
Image correlation principle
Agisoft Photoscan-Metashape,
Zephyr 3D, pix4D, 123catch, aj.)
SfM+ MVS (structure from motion
and multi view stereo)

Photogrammetry

terrestrial (hand – held)
aerial (aerial or using drones)
satellite

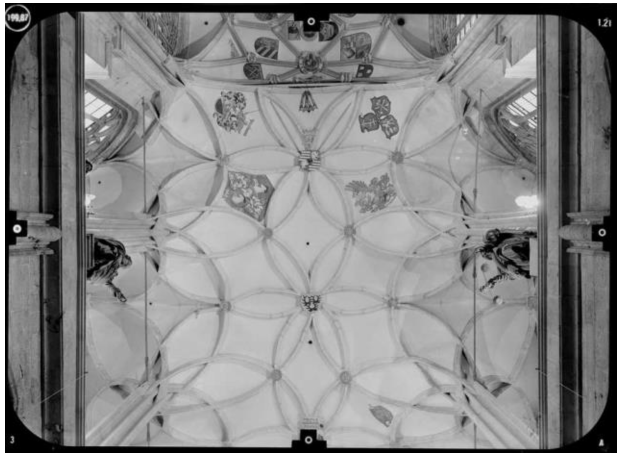

File
Edit
View
Workflow
Model
Photo
Ortho
Tools
Help
Workspace
Model
Ortho
Perspective 30°
Workspace (1 chunks, 138 cameras)
Chunk 1 (138 cameras, 102,301 points) [T]
102,301 points
Photos
IMG_3975
IMG_3976
IMG_3977
IMG_3978
IMG_4782
IMG_4783
IMG_4784
IMG_4785


File Edit View Workflow Model Photo Ortho Tools Help
Workspace
Workspace (1 chunks, 138 cameras)
Chunk 1 (138 cameras, 102,301 points) [T]
Model Ortho
Perspective 30°
points: 57,778,176
Photos
IMG_3975
IMG_3976
IMG_3977
IMG_3978
IMG_4782
IMG_4783
IMG_4784
IMG_4785


Model
Ortho
Perspective 30°
faces: 1,283,957 vertices: 1,232,169

Classical photogrammetric images

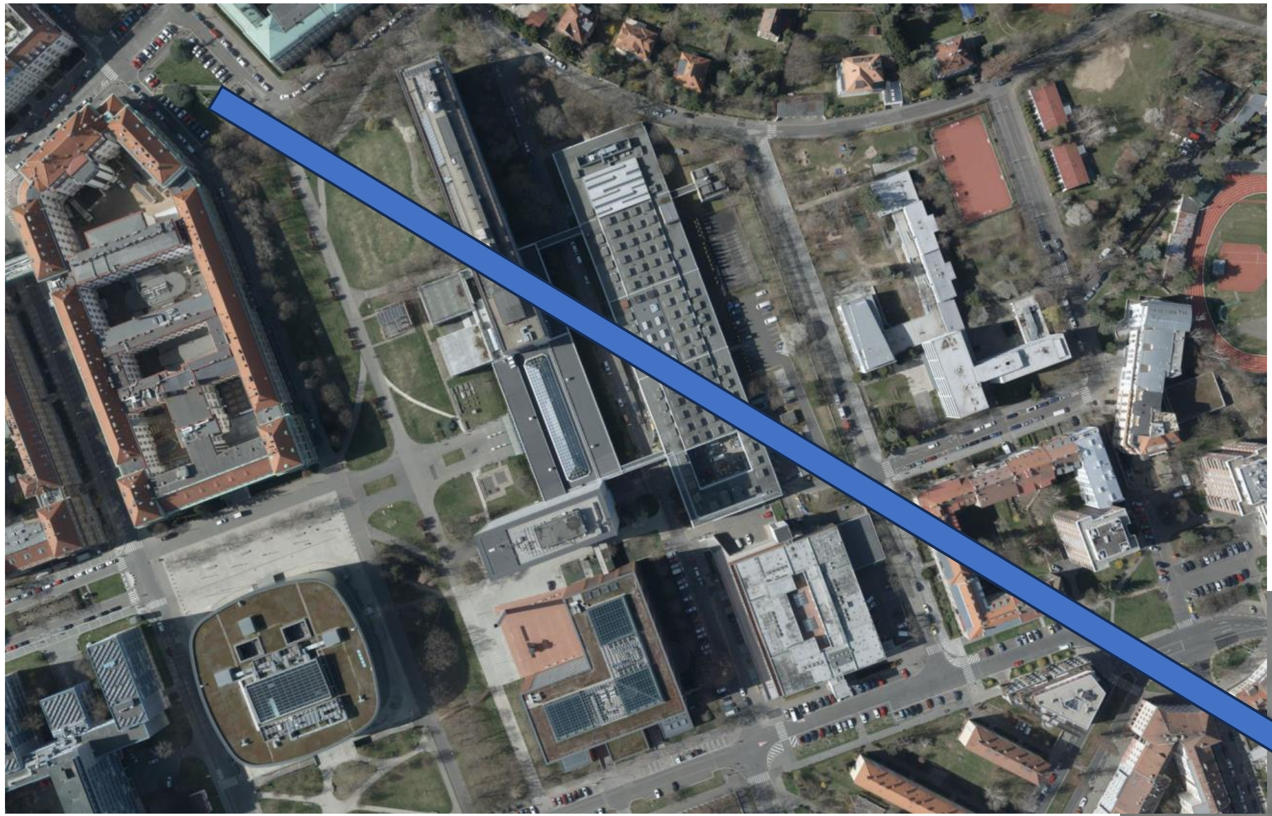
fiducials



2003, GSD 50 cm

2015, GSD 15cm

L-410FG



2021 GSD 2cm

Ikonos 1, 1999, 1m PAN, 4m MSS

Case study –combined documentation of a historical object

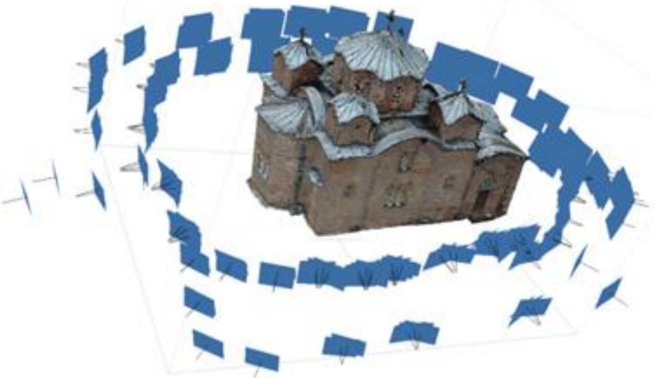
- photogrammetry (terrestrial, drone)
- laser scanning
- mobile laser scanning
- VR

Laser scanning (comparing 2002 and 2020

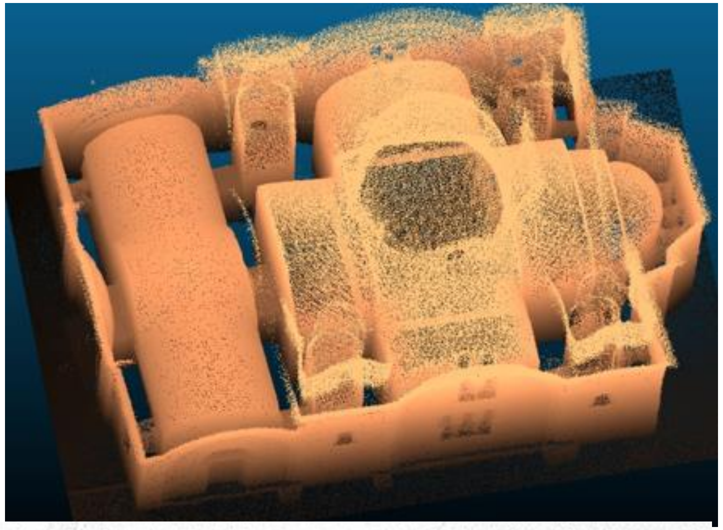
Used instruments

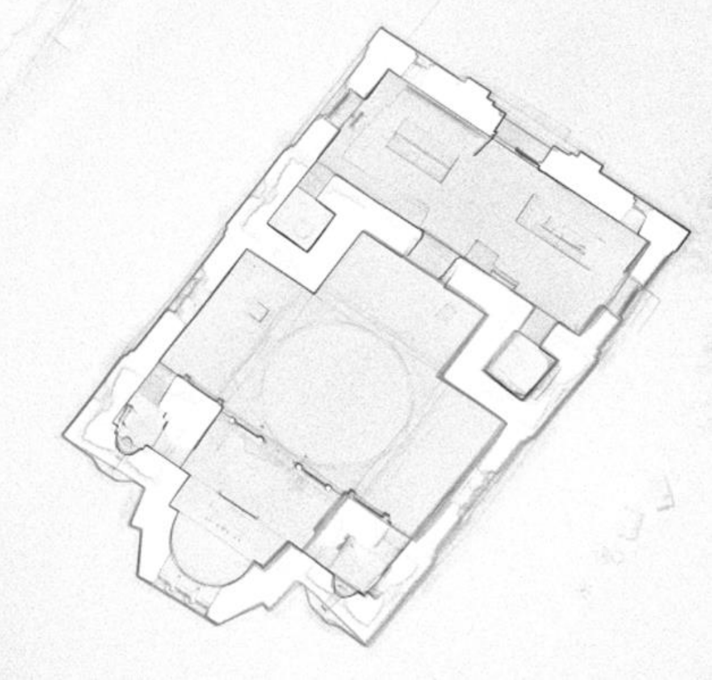

Perspective 30°
Snap: Axis, 2D
Z
X
Y
faces: 5,270,156 vertices: 2,648,118

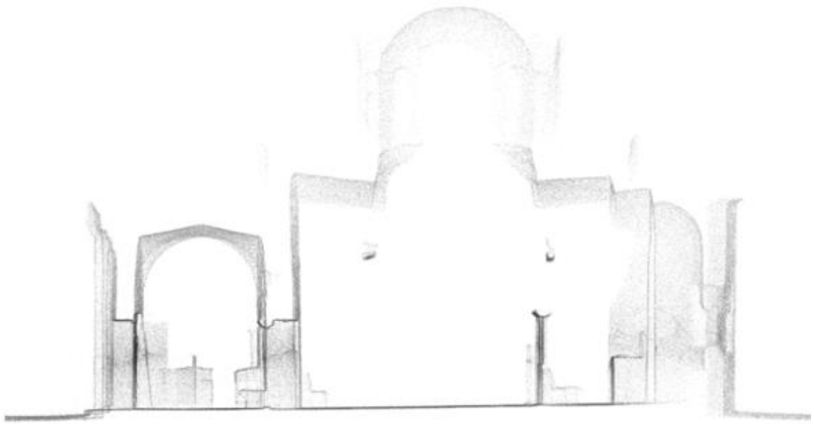
Sv. Panteleimon, Skopje

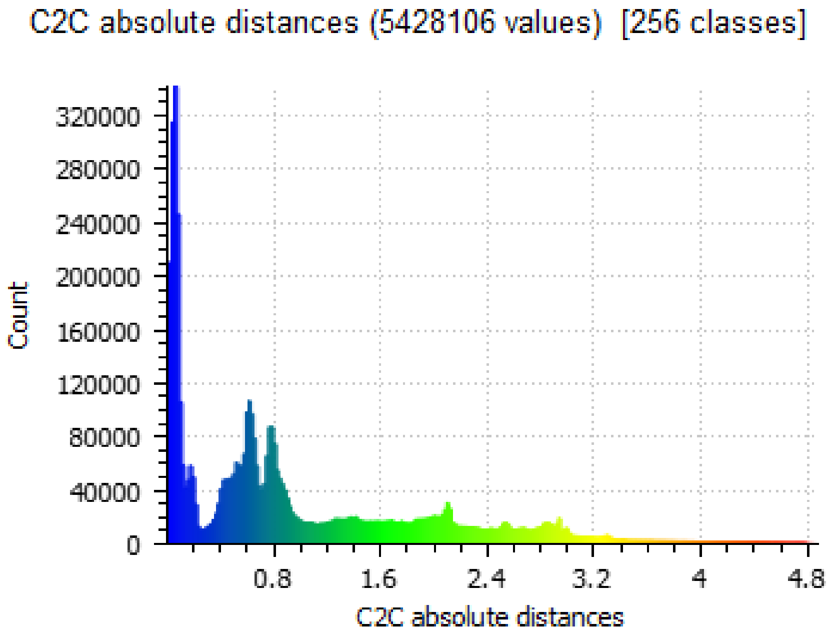


Point cloud from PLS ZEB REVO



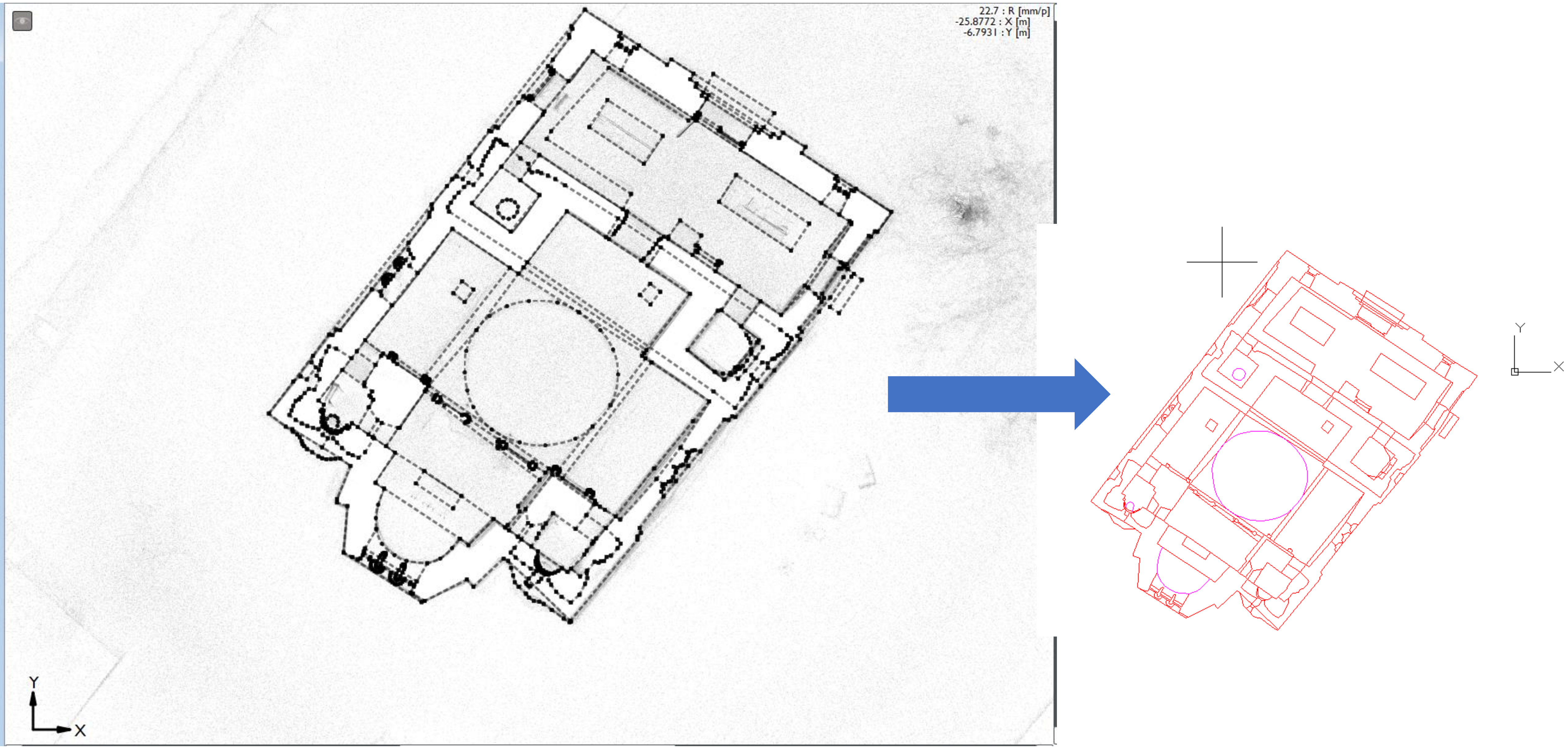


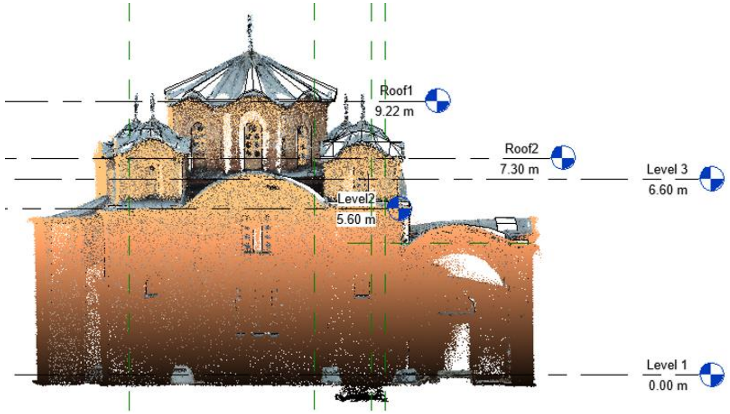




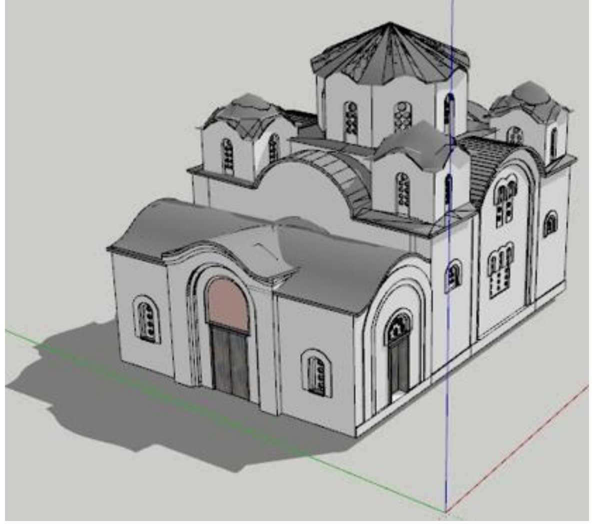
Differences between point cloud from combined terrestrial and aerial photogrammetry

Device	Accuracy [mm] on 10 m	Range [m]	Acquiring points/sec	Data size [MB]	Time [minutes]
Nikon D3200	5-20	10	~30 photos/minute	~10/photo	~30 photos/minute
DJI Mavic Pro	5-30	30	~20 photos/minute	~4/photo	~20 photos/minute
ZEB-REVO	10-30	30	43000	100/min	slow walking
BLK360	4	30	360000	600/scan	6/scan

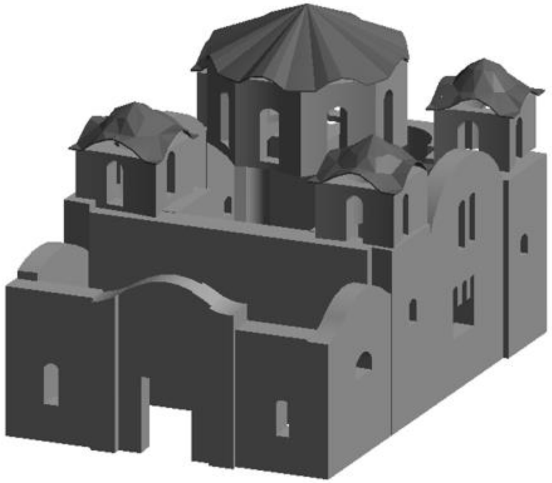

22.7 : R [mm/p]
-25.8772 : X [m]
-6.7931 : Y [m]
Y
X
Y
X



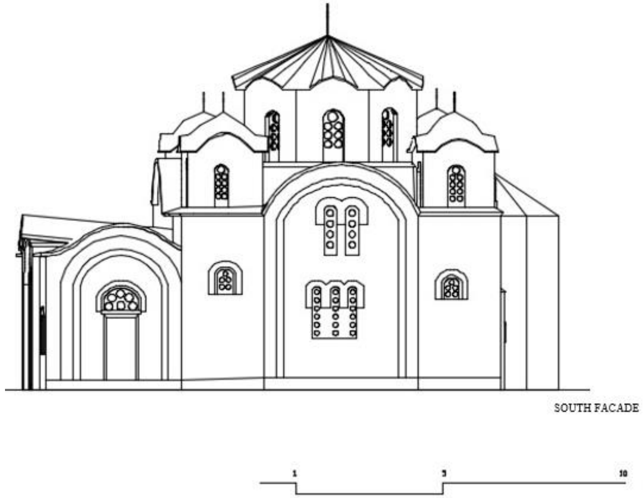
Defining of levels for creating of simplified model



Model from SketchUp



Model from Revit



Generated views from simplified models (SketchUp)

Generated VR model with texture (UnrealEngine)

VR model rendered in Lumion

Comparison of both 3D models: BIM geometrical basic model (grey), photogrammetrical model (blue)


Perspective 30°
Snap: Axis, 2D
Faces: 5,270,156 vertices: 2,648,118
Z
X
Y

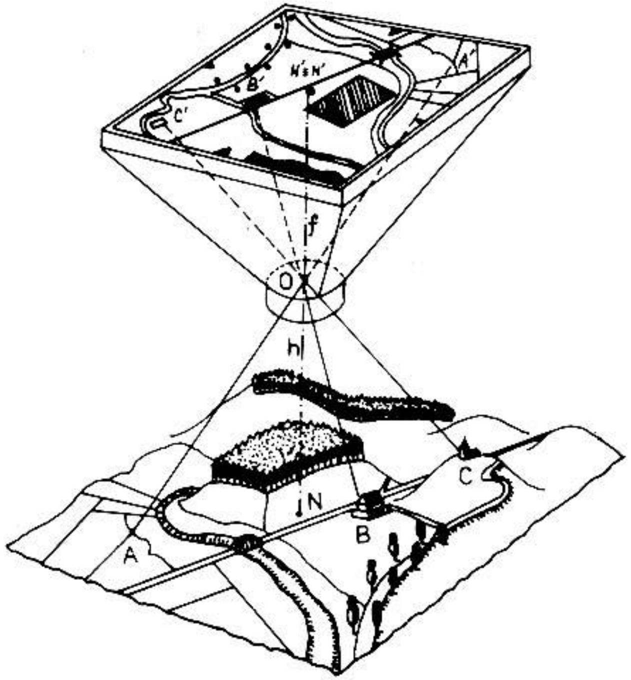
Basics of photogrammetry

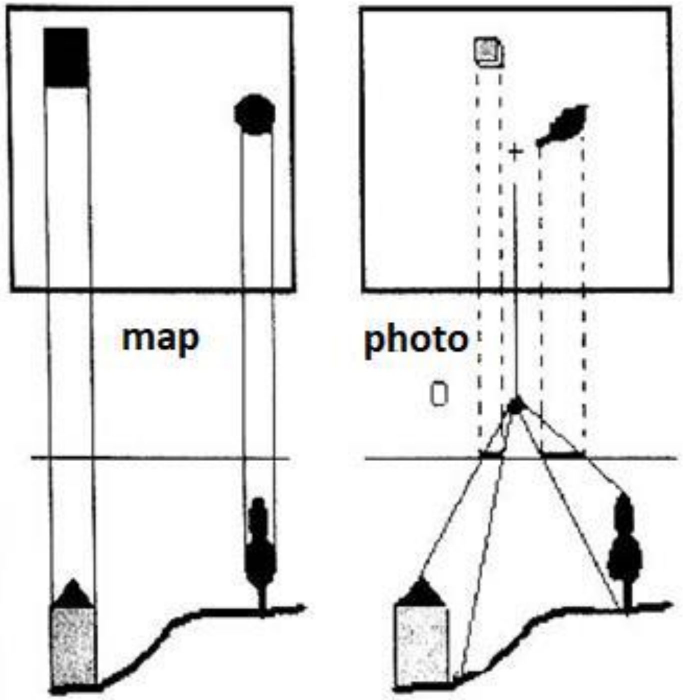
Basics of photogrammetry

Photographic image = central projection of the displayed object

Input data:

- **image source of information necessary**
- **additional data geodetic coordinates of the ground control points , elements of internal and external orientation**





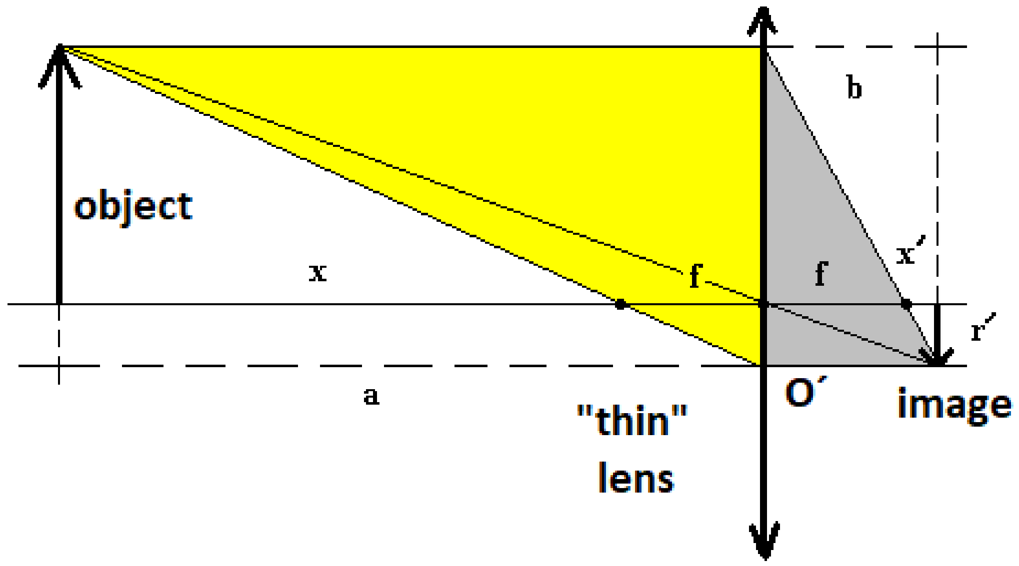
Theory - optics

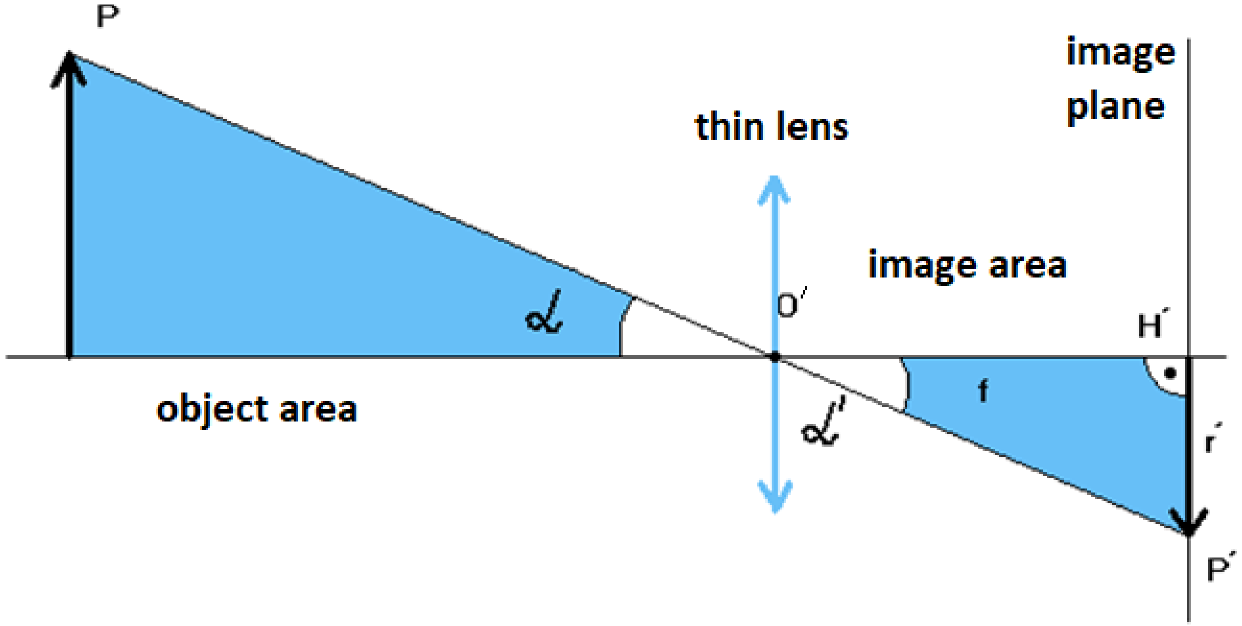
Ideal Lens Imaging
projection centre O´
main frame point H´
Focal length f

for an ideal projection:

$\alpha = \alpha'$ $\qquad r' = f \, . \, tg \, \alpha$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

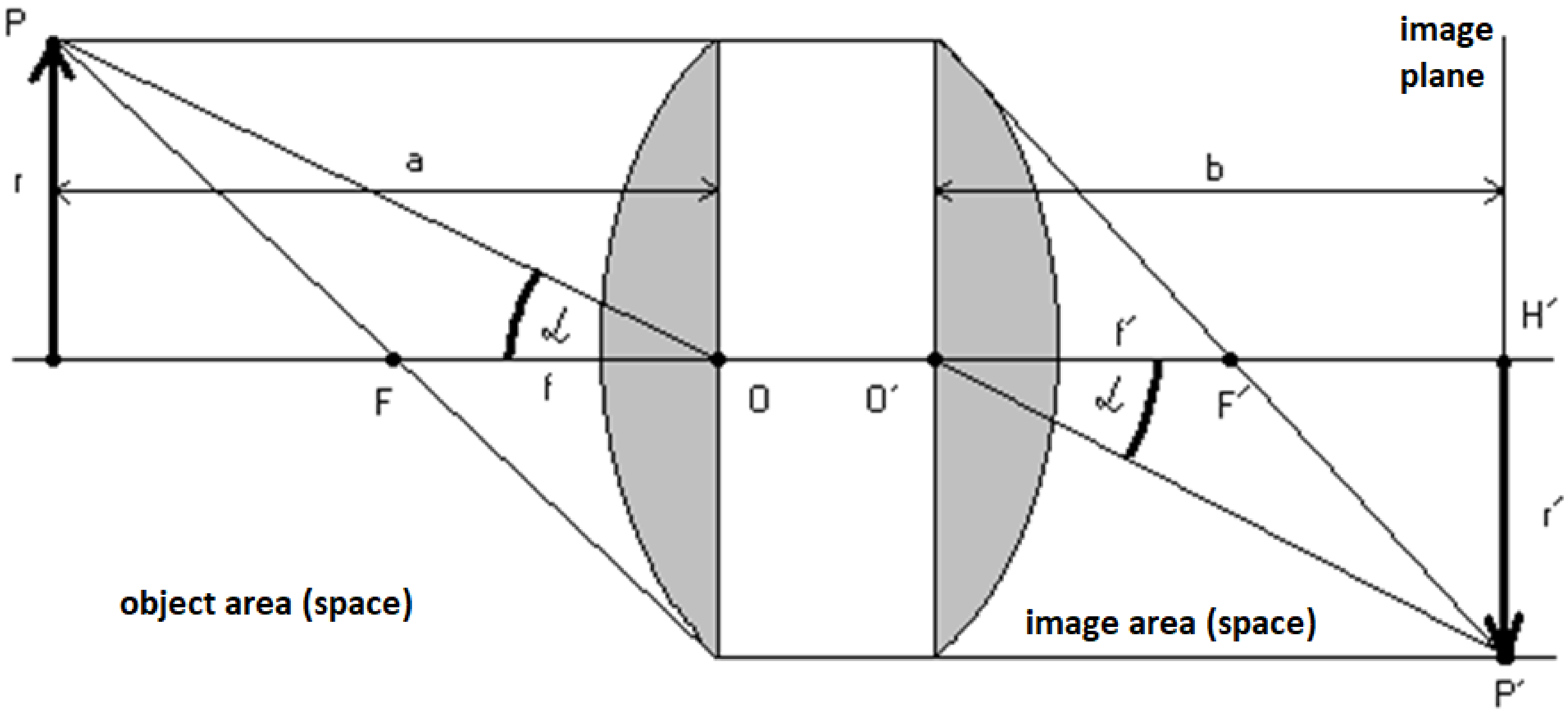




Theory - optics

A real lens aperture

- *reduces the number of rays that form an image*
- *O Input pupila image of the aperture in the subject space*
- *O´ output pupila image of the aperture in the image space*
- *f (camera lens) constant distance O ´ H ´*



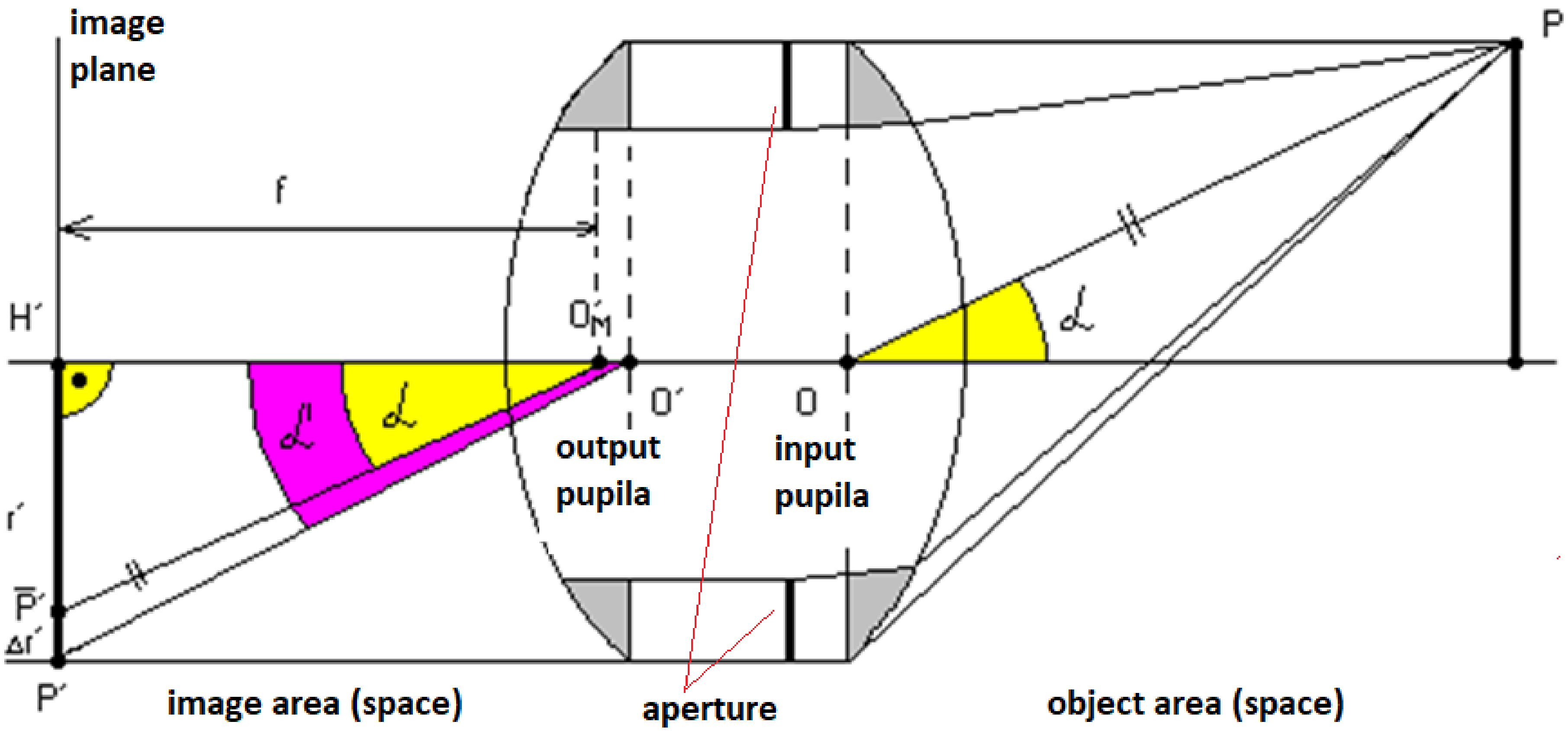
Theory - optics

$\alpha \neq \alpha'$

Real lens view

$r' = f \cdot tg\,\alpha + \Delta r'$

$\Delta r'$ - effect of lens distortion



K.Kraus, 1994, Photogrammetrie, Band I.,Dümmler/Bonn, ISBN 3-427-78645-5

Depth of field

$$y_{\min} = \frac{f^2}{\left(\frac{f}{A}\right) \cdot \Delta u} = \frac{f^2}{n \cdot \Delta u}$$

where n is the aperture number and u is the dispersion ring (unsharpness), which should not be larger than the diameter of the measuring mark in old analoque plotters (0.02-0.05mm); A is the diameter of the input pupila.

Type of camera	f [mm]	n	y_{min} (Δu=0.05 a 0.02mm) [m]
PhoTheo19/1318	195	25	30; 76
UMK 10/1318	100	8, 16, 32	25, 13, 6; 63, 32, 16

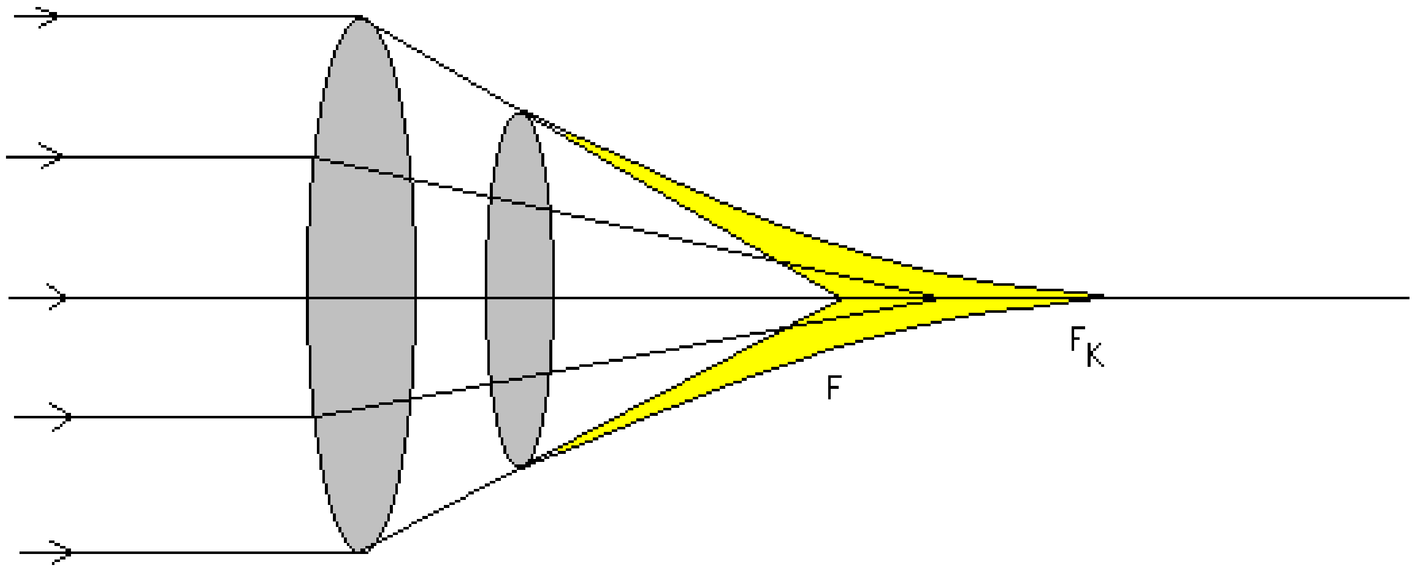
Parameters of old phototheodolites

Summary of effects on the geometry of the lens view

Aberrations
Optical defects can be sub divided into
a) monochrome
b) coloured

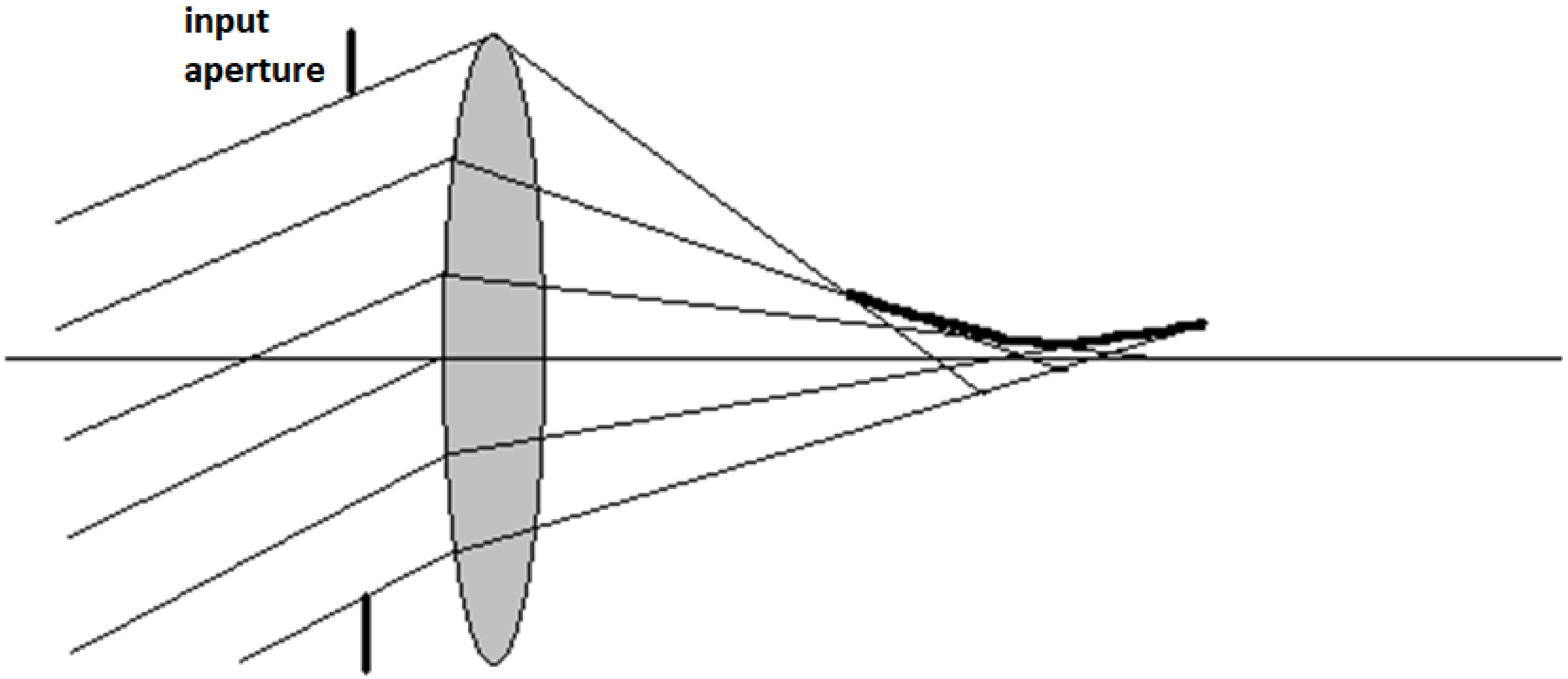
Furthermore,
- defects arising from
c) point imaging (spherical defect, astigmatism and coma)
d) imaging of the subject (field blurring and image distortion)

Lens defects



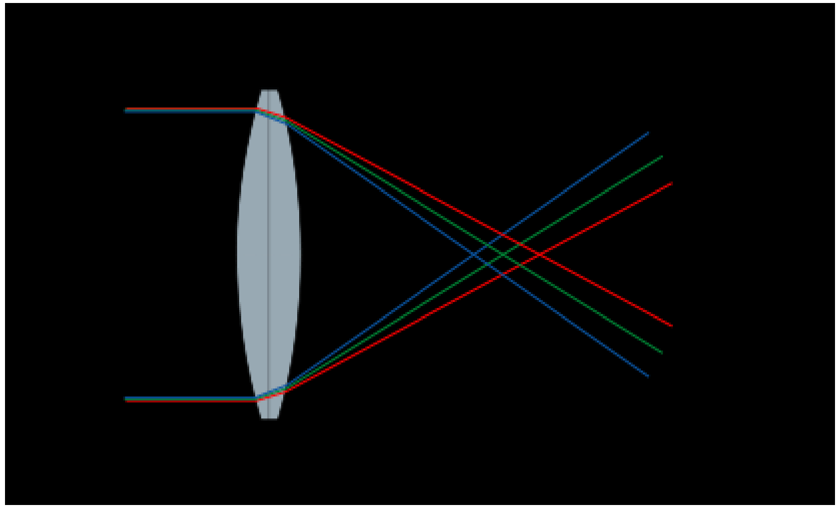
Lens view spherical defect (aberration)

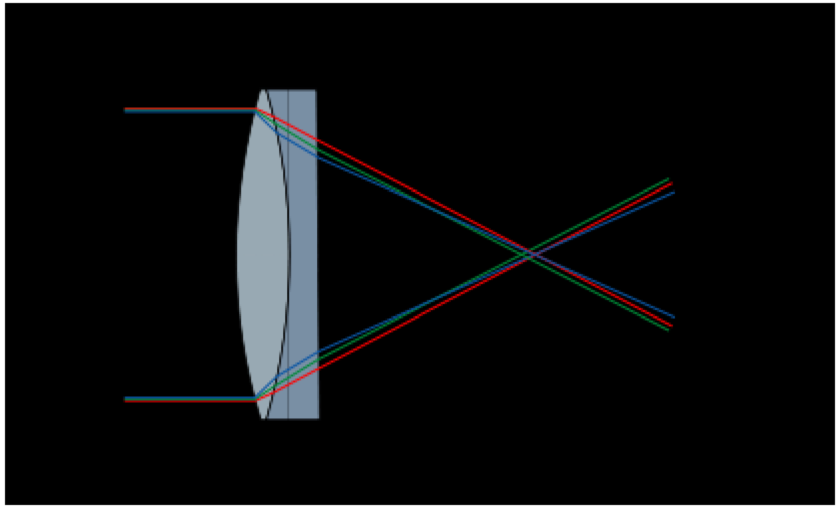
Lens defects



Lens view aspherical defect (aberration)

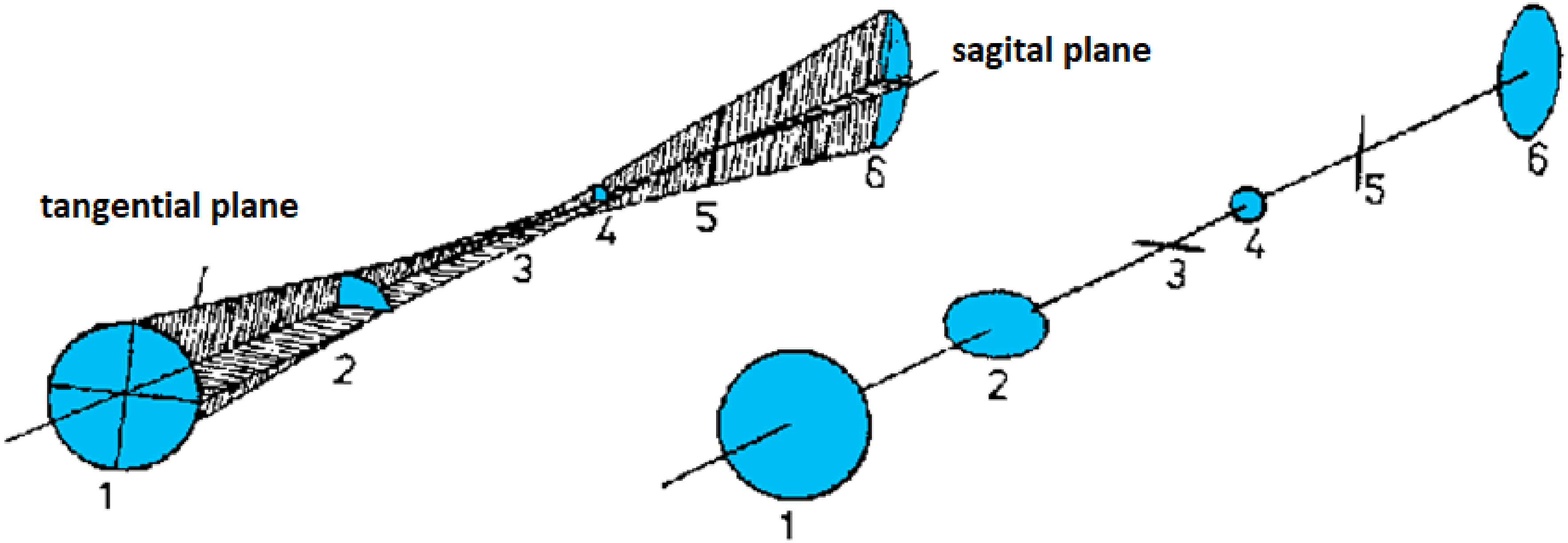
Lens defects





Lens view chromatic colour defect (aberration)
and its suppression

Lens defects



Lens View Astigmatism

Lens distortion

- **radial**
- **tangential**

$$x' = x'_{\text{measured}} + \Delta x'$$

$$y' = y'_{\text{measured}} + \Delta y'$$

Thus, in general, the effect of radial distortion can be expressed by a polynomial

$$\Delta x' = a_0 + a_1 x' + a_2 y' + \ldots$$
$$\Delta y' = b_0 + \ldots$$

$$r'^2 = x'^2 + y'^2$$

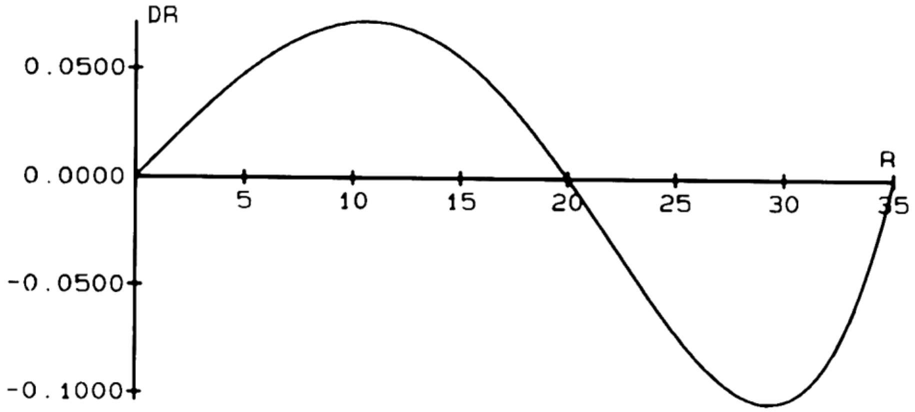
$$\Delta x' = x' \cdot (a_1 r'^2 + a_2 r'^4 + a_3 r'^6) + b_1(r'^2 + 2x'^2) + 2b_1 x' y'$$
$$\Delta y' = y' \cdot (a_1 r'^2 + a_2 r'^4 + a_3 r'^6) + b_2(r'^2 + 2y'^2) + 2b_2 x' y'$$

This form is too detailed, for most lenses one can make do with just the coefficients a1 , a2 .

Réseau system RolleiMetric

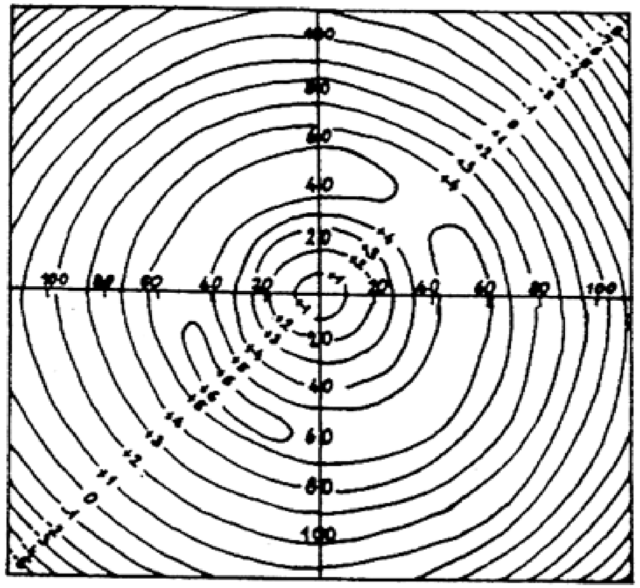
$$\Delta r' = a_1 \cdot r' \cdot \left(r'^2 - r_0'^2\right) + a_2 \cdot r' \cdot \left(r'^4 - r_0'^4\right)$$

Lens distortion



Lens projection radial distortion RolleiMetric 6006, 40mm lens)

Lens projection radial
distortion expressed by isolines



Deformation shrinking) of photographic material

The deformation of photographic material is sub divided into
a) regular over the whole image area this deformation can be easily detected by comparing known original dimensions
b) differential it is a regular deformation which is different in the direction of the x' and y' axis
c) irregular this deformation cannot be practically excluded without a special device réseau grid)

Material	average shrinking *s* = image size[mm]	values for image 13x18cm
Glass plate	max. 3-5μm	3-5μm
acetate pad	$4{\cdot}10^{-5}\ .s$	7μm
PET pad	$2.5{\cdot}10^{-5}\ .s$	4.5μm

Deformation of photographic material

Photographic material deflection

Pad	Note.	Thickness [mm]	Flatness [µm]
glass plates	flat ultraflat cut glass	1.3-3.0 1.3-3.0 6.0	30-50 25 5-10
PET film (polyester terephthalate)	mechanical pressure or vacuum pressing of material	0.06 ÷ 0.003 up to 0.18 ÷ 0.005	5-20 according to the type of adhesion of the material to the frame

Parameters of the photographic material

Coordinate systems

Generally, three types of coordinate systems are used in the photogrammetry

a) main coordinate systems (system of geodetic coordinates)

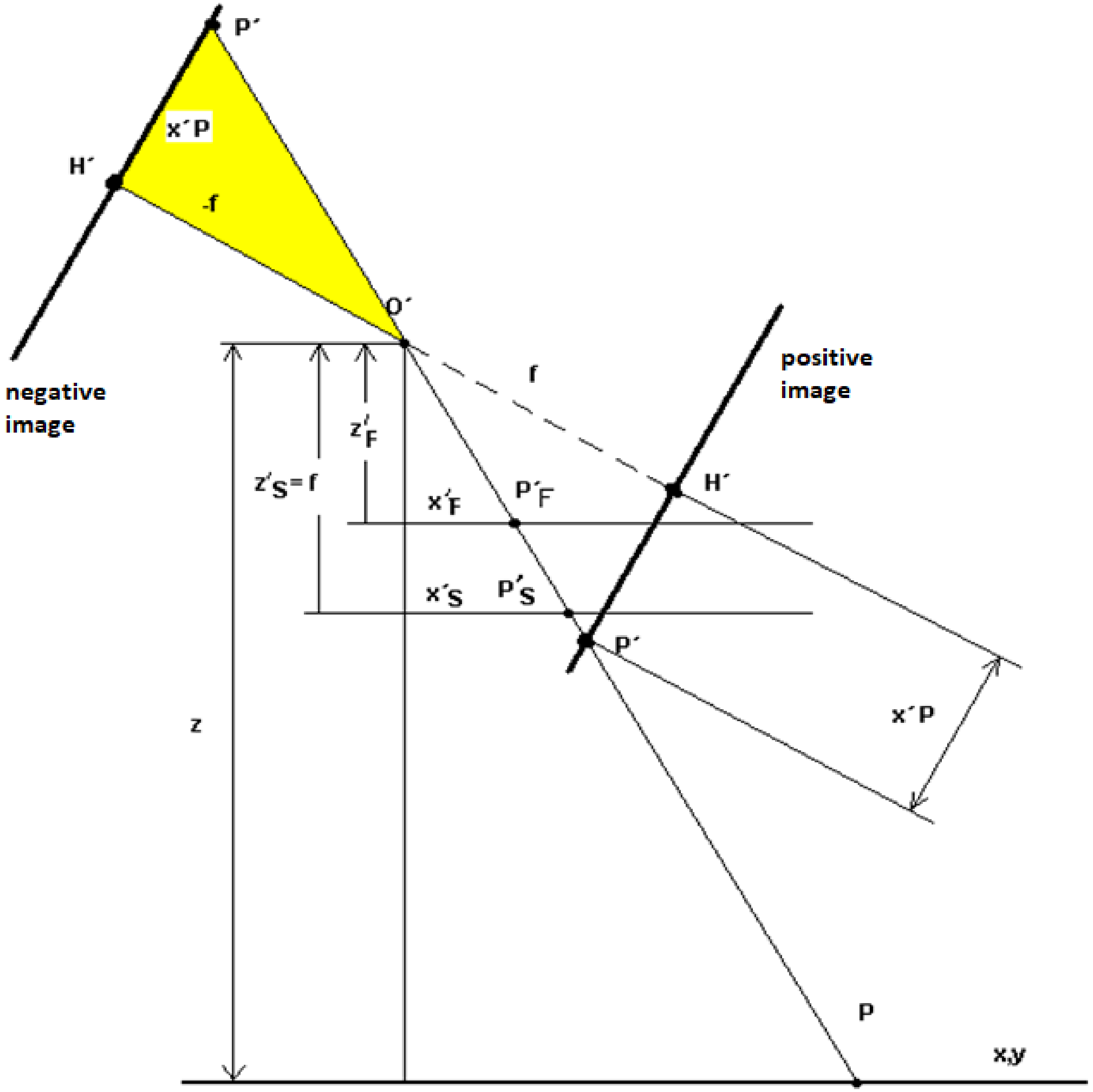
b) image coordinate system

c) model coordinate system

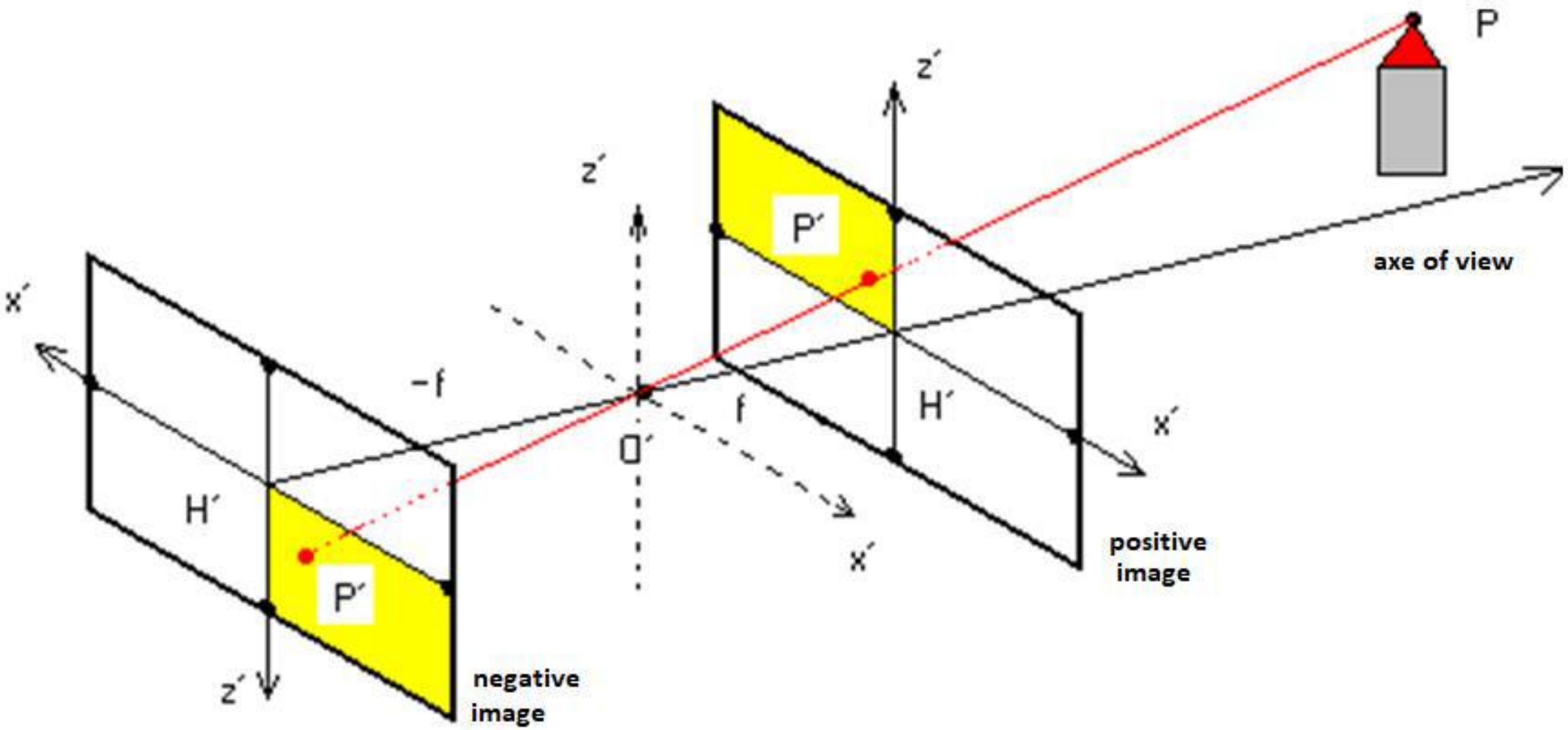
auxiliary coordinate systems

- fictitious image coordinate system (3 image coordinates)
- the image coordinate system of an exactly vertical image

Coordinate systems

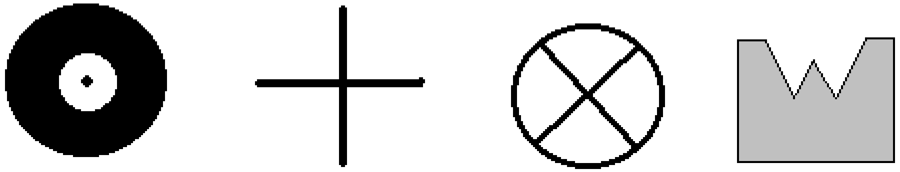


Elements of interior orientation

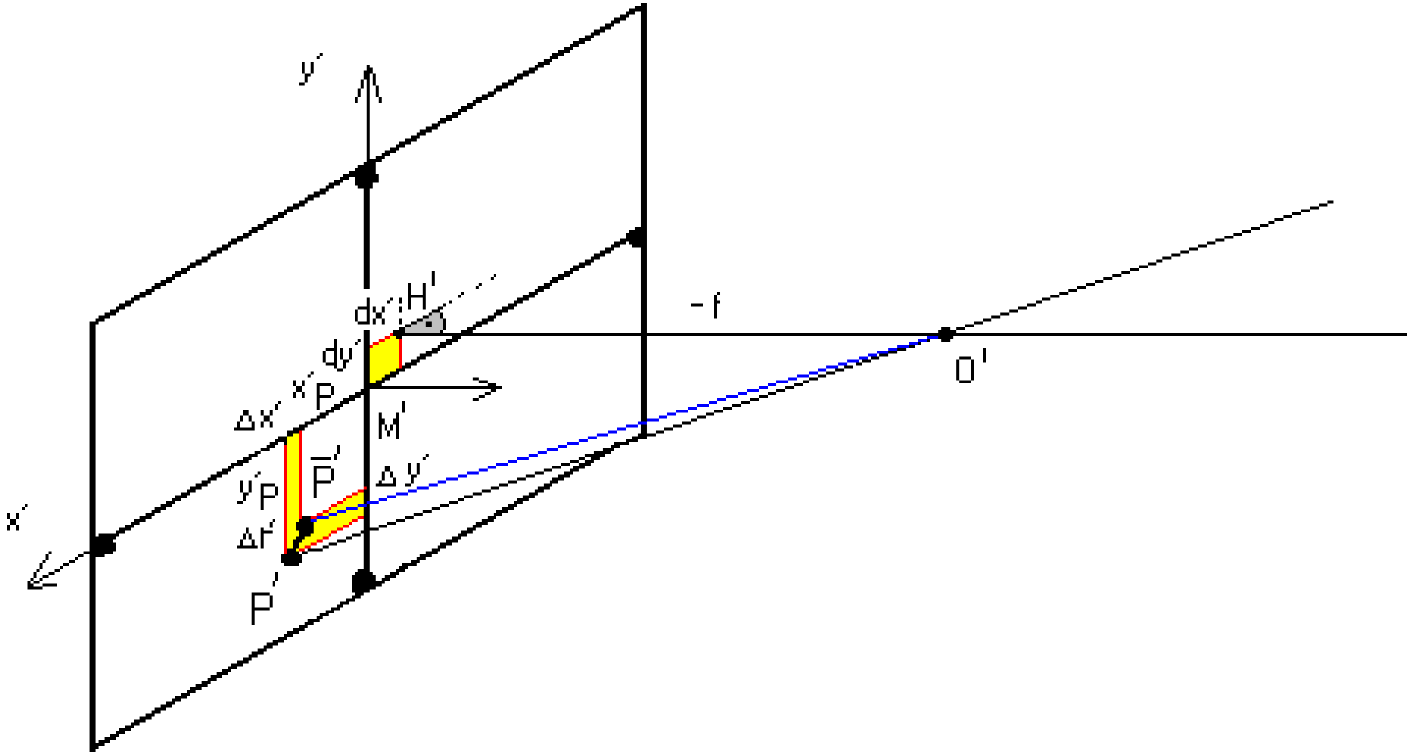


Camera constant , negative and positive, terrestrial configuration
Elements of interior orientation : x0 ´,y0 ´, f
(and known parametres of distortion)

Elements of internal orientation



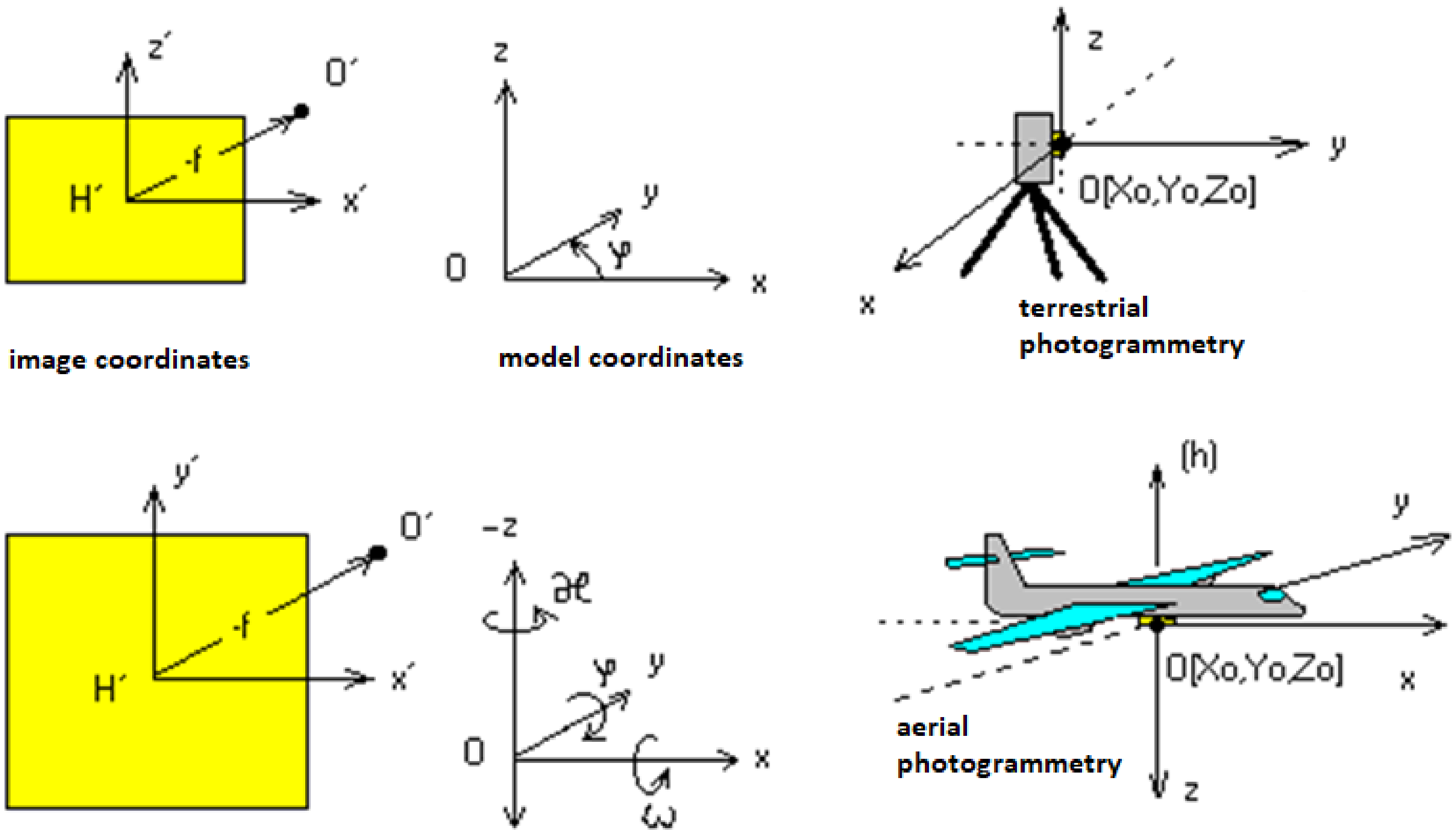
Different types of fiducial marks (on historical images or in historical cameras only)



Definition of the elements of the internal orientation in the general configuration (image coordinates x ´,y ´); The axis of view is the perpendicular to the image plane passing through the object projection centre.

Coordinate systems and conversions

Elements of internal and external orientation



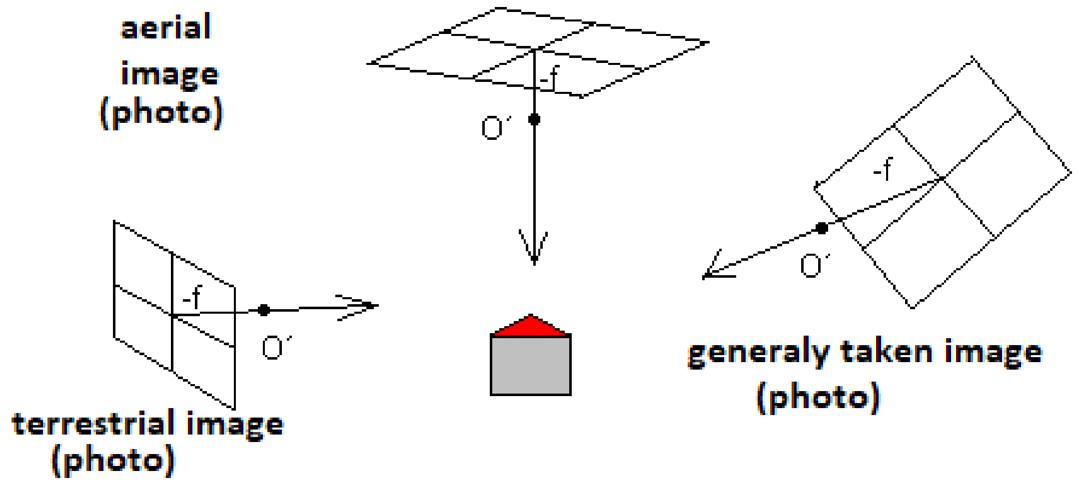
Internal and external orientation elements for terrestrial and aerial photogrammetry

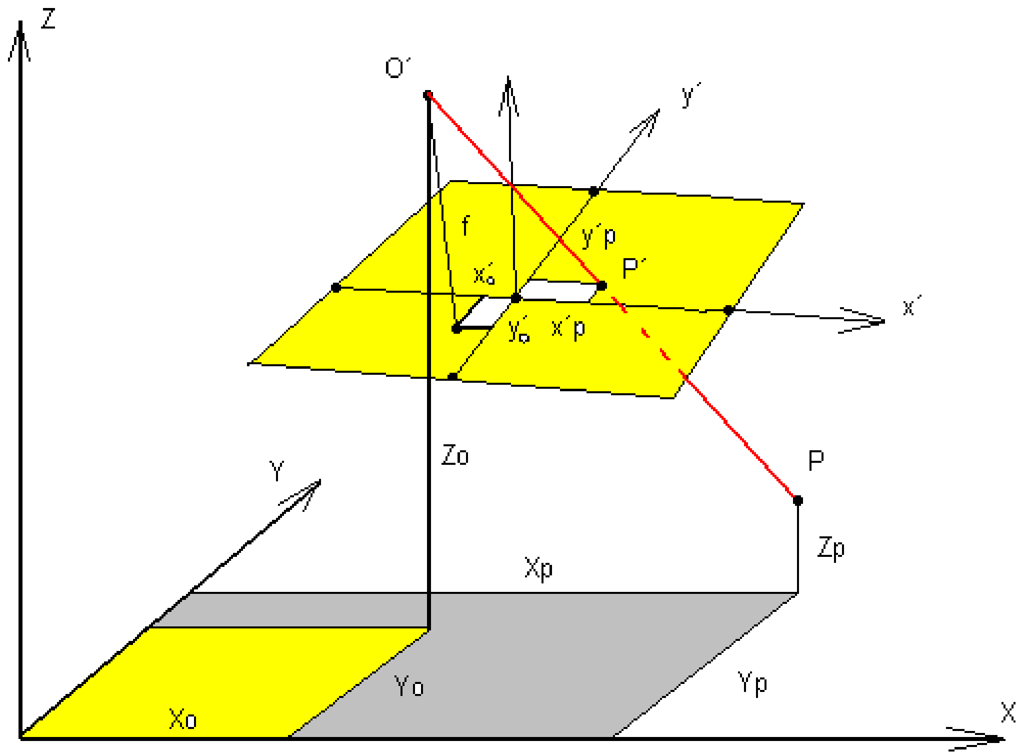
Coordinate systems

Image coordinate system label : x′ , y′ , (z′ = -f)
Model coordinate system label:: x , y , z
Geodetical system: X , Y , Z

Auxiliary systems
Fictional image coordinate system : x'_F , y'_F , z'_F
Vertical image coordinate system : x'_S , y'_S , z'_S

Classic terrestrial , aerial
and generally taken image





Coordinate systems in
aerial photogrammetry

Converting image information to geodetic systems

1) Restoration of internal orientation elements

2) External orientation

External orientations can be solved classically in two steps as:

1. **-relative orientation** (mutual orientation between the two stereo images, formation of an arbitrary spatially oriented stereo model)
2. **-absolute orientation** (rotation and displacement of the model into the geodetic reference system)

3. - in one step **using the Bundle** *Adjustment* **method** (*Bündelausgleichung*)

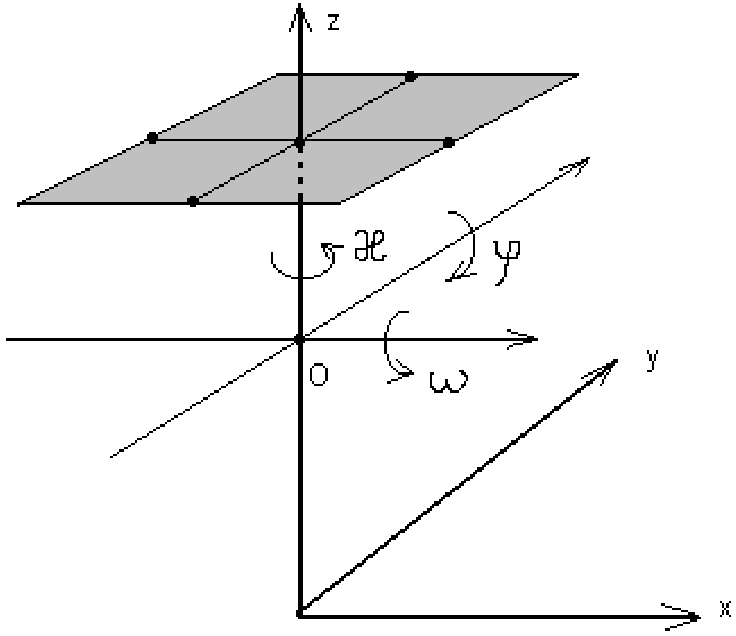
Rotation matrix

The interconversion of coordinate systems generally involves ***rotation, shifting*** and ***scaling***. While shifting and scaling are relatively simple operations, spatial rotation is more complex

Rotation in plane

$$X = x \cdot \cos\alpha - y \cdot \sin\alpha$$
$$Y = x \cdot \sin\alpha + y \cdot \cos\alpha$$

$$\mathbf{X} = \mathbf{R}\cdot\mathbf{x}.,\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\ \mathbf{R} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix}$$



Rotation in space

$$\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

(primary, secondary and tertiary rotation)

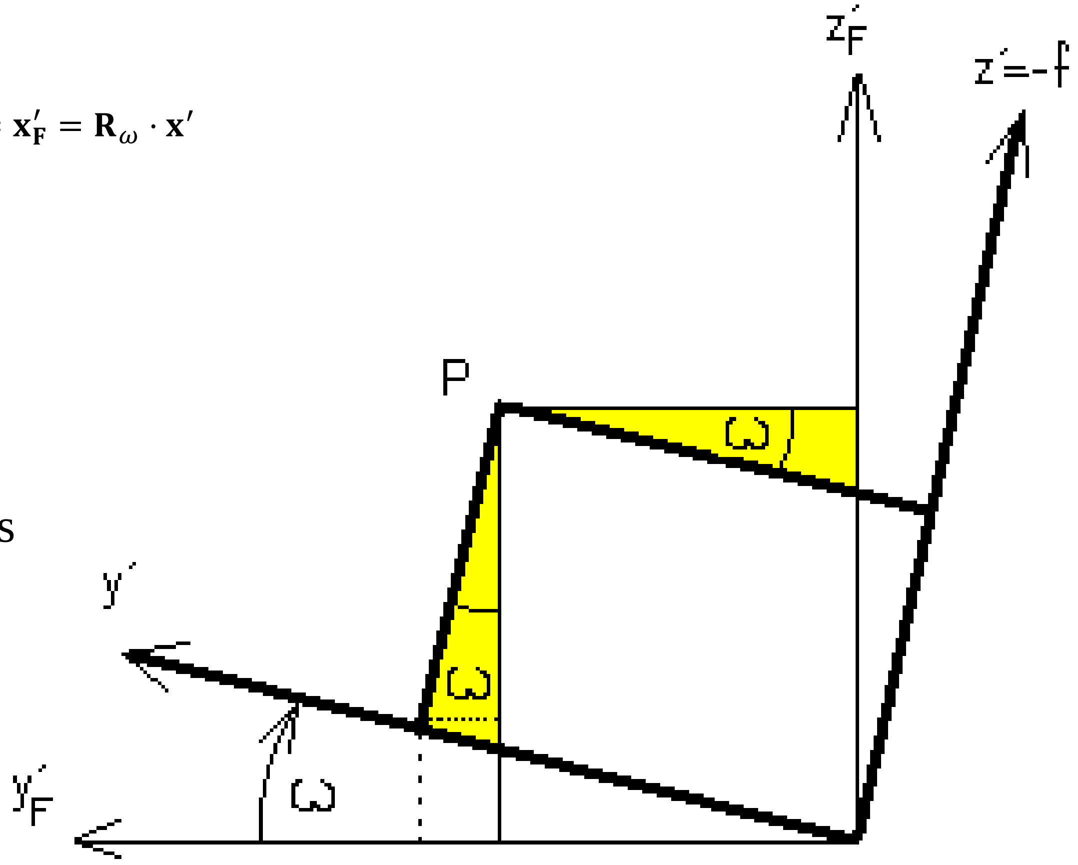
Rotation matrix

Rotation about the primary *x´* axis

$$x'_F = x'$$
$$y'_F = y' \cdot \cos\omega - z' \cdot \sin\omega$$
$$z'_F = y' \cdot \sin\omega + z' \cdot \cos\omega$$

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \mathbf{x}'_\mathbf{F} = \mathbf{R}_\omega \cdot \mathbf{x}'$$

Rotation of the system around the x´ axis by an angle



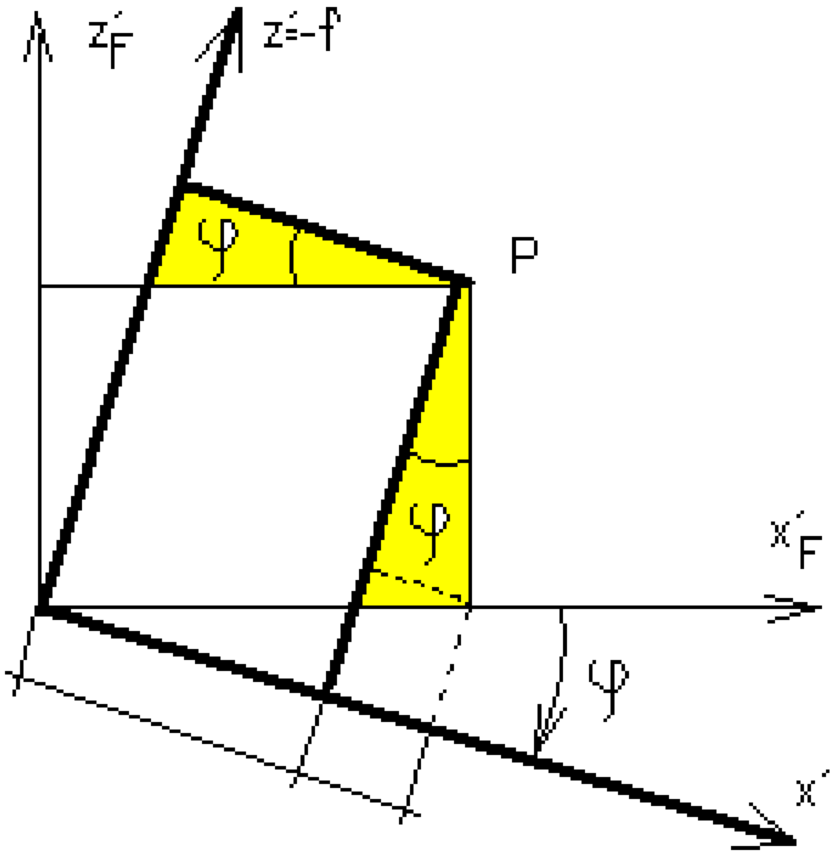
Rotation matrix

Rotation about the secondary y´ axis

$$x'_F = x' \cdot \cos\phi + z' \cdot \sin\phi$$
$$y'_F = y'$$
$$z'_F = -x' \cdot \sin\phi + z' \cdot \cos\phi$$

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \begin{pmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \mathbf{x}'_\mathbf{F} = \mathbf{R}_\phi \cdot \mathbf{x}'$$

Rotation of the system about the y´ axis
by an angle



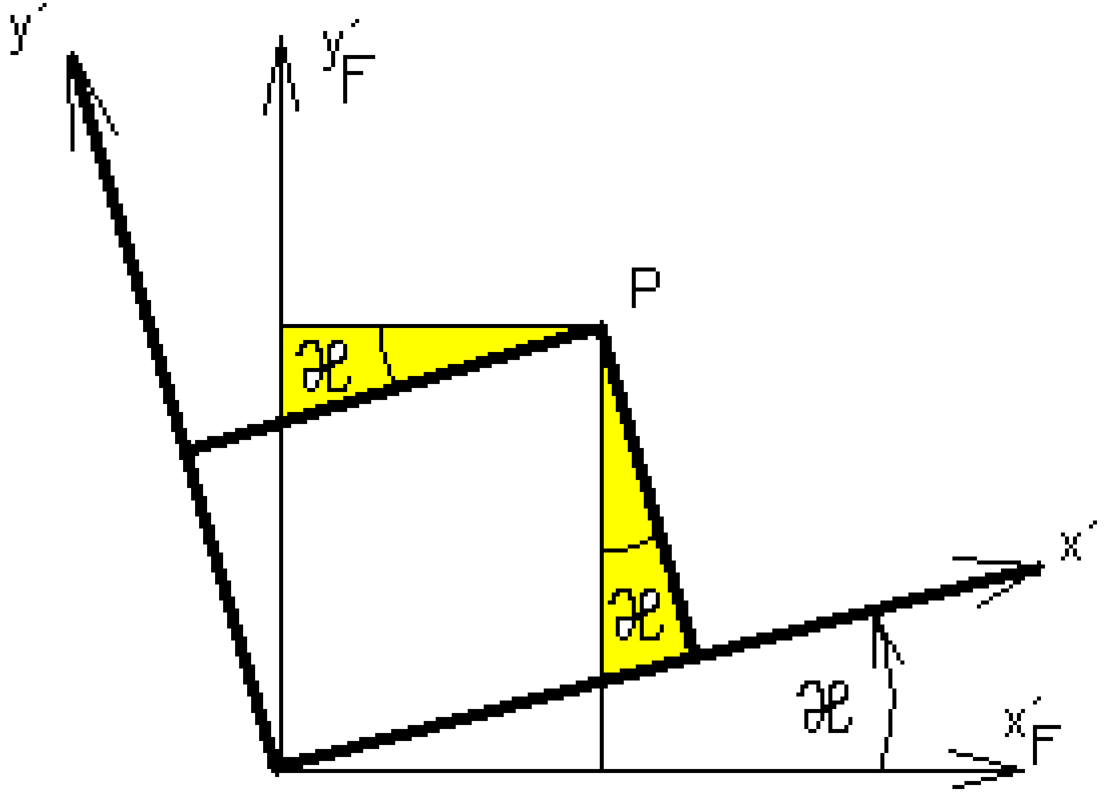
Rotation matrix

Rotation about the tertiary axis z´

$$x'_F = x' \cdot \cos\kappa - y' \cdot \sin\kappa$$
$$y'_F = x' \cdot \sin\kappa + y' \cdot \cos\kappa$$
$$z'_F = z'$$

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \begin{pmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \mathbf{x}'_\mathbf{F} = \mathbf{R}_\kappa \cdot \mathbf{x}'$$

Rotation of the system around the z' axis by an angle



Rotation matrix

Resulting rotation matrix *R*

$$\mathbf{R}_{\omega\phi} = \mathbf{R}_{\omega} \cdot \mathbf{R}_{\phi}$$

$$\mathbf{R}_{\omega\phi\kappa} = \mathbf{R}_{\omega\phi} \cdot \mathbf{R}_{\kappa}$$

$$\mathbf{R}_{\omega\phi\kappa} = \begin{pmatrix} \cos\phi\cos\kappa & -\cos\phi\sin\kappa & \sin\phi \\ \sin\omega\sin\phi\cos\kappa + \cos\omega\sin\kappa & -\sin\omega\sin\phi\sin\kappa + \cos\omega\cos\kappa & -\sin\omega\cos\phi \\ -\cos\omega\sin\phi\cos\kappa + \sin\omega\sin\kappa & \cos\omega\sin\phi\sin\kappa + \sin\omega\cos\kappa & \cos\omega\cos\phi \end{pmatrix}$$

$$\tan\omega = -\frac{r_{23}}{r_{33}}, \sin\phi = r_{13}, \tan\kappa = -\frac{r_{12}}{r_{11}}$$

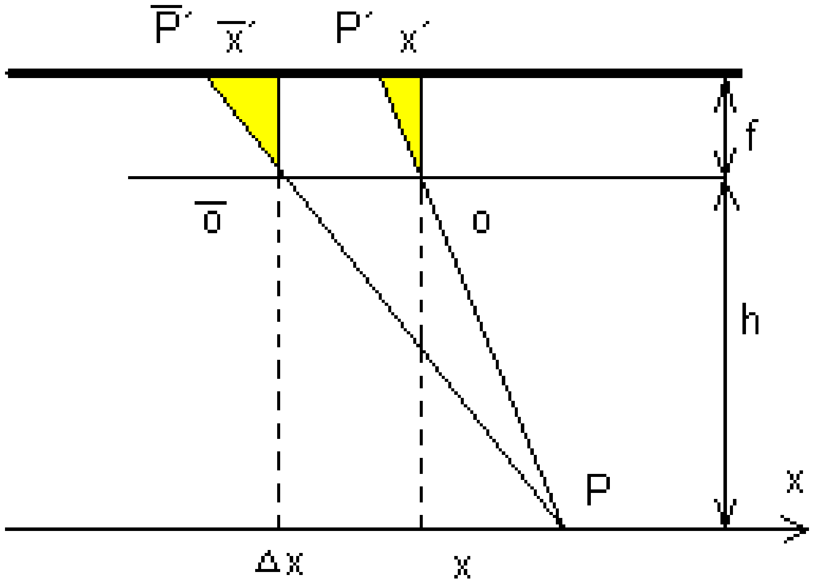
$$\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

Here it is important to note that r_{13} = sin > 0 in the 1st and 2nd quadrants and further that r_{13} = sin < 0 in the 3rd and 4th quadrants. Thus, the rotation is not uniquely determined. The quadrants of the other two rotations , are uniquely determined given the expressions from which we calculate them. ***Thus we get a total of two sets of rotations ,, to a single rotation matrix R****.*

Shift in space

- ***linear*** changes

Effect of changing the x-coordinate



$$x = \frac{z}{z'} x' = \frac{h}{f} x'$$

$$x' = \frac{f}{h} x$$

$$\Delta x' = \frac{f}{h} \cdot \Delta x, \quad \Delta y' = 0, \quad \Delta z' = \Delta f = 0$$

In this case, *f/h* is the scale of the image. Changes of *x*, *y*, *z* in the scale of the image are often referred to traditionally as $\boldsymbol{db_x}$ **,** $\boldsymbol{db_y}$ **,** $\boldsymbol{db_z}$, because of the same meaning on old analog machines. The expression goes to the shape:

$$\Delta x' = db_x, \Delta y' = 0, \Delta z' = \Delta f = 0$$

Shift in space

Effect of y-coordinate change (same as for x-coordinate)

$$\Delta x' = 0, \qquad \Delta y' = \frac{f}{h} y, \qquad \Delta z' = \Delta f = 0$$

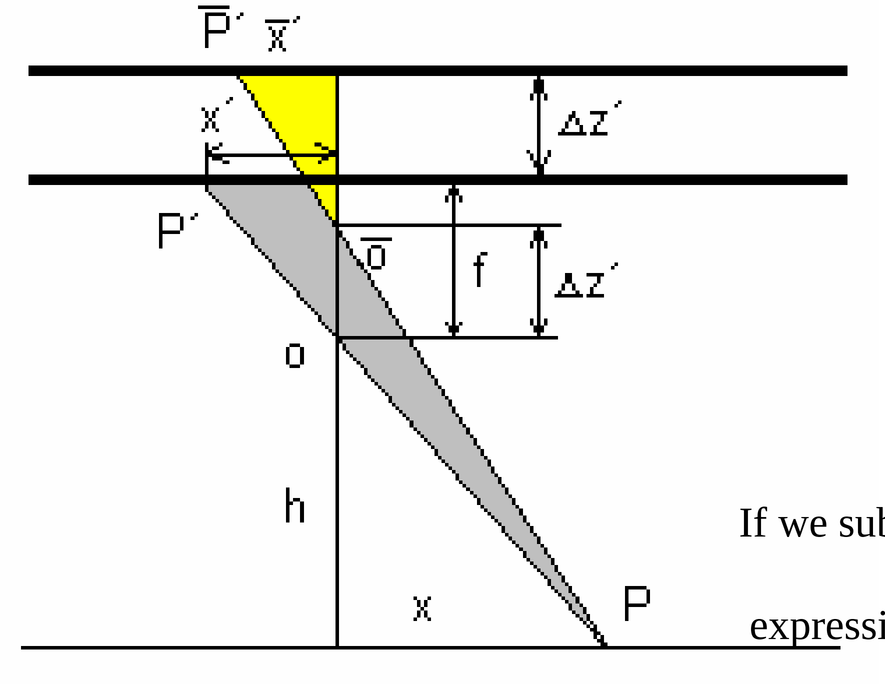
In this case, *f/h* is the scale of the image. Changes of *x*, *y*, *z* in the scale of the image are often referred to traditionally as $\boldsymbol{db_x}$, $\boldsymbol{db_y}$, $\boldsymbol{db_z}$, because of the same meaning on old analog machines. The expression transitions to the shape:

$$\Delta x' = 0, \qquad \Delta y' = db_y, \qquad \Delta z' = \Delta f = 0$$

Shift in space

The effect of changing the z coordinate

- However, when the flight altitude changes, there are changes in both image coordinates



$$x = \frac{z}{z'}x' = \frac{h}{f}x', y = \frac{z}{z'}y' = \frac{h}{f}y'$$

$$x' = \frac{f}{h}x, y' = \frac{f}{h}y$$

$$x^{\bar{}'}\frac{f}{h+\Delta z}{}^{\bar{}'\frac{f}{h+\Delta z}}$$

If we subtract the equations, we get after adjustment $\frac{1}{a \pm x} \cong \frac{1}{a}\left(1 \mp \frac{x}{a}\right)$

expression for differences in frame coordinates :

$$\Delta x' = x' - x^{\bar{}'\frac{x'}{h}'''\bar{}\frac{y'}{h}}$$

$$\Delta z = \Delta z' \cdot \frac{h}{f}$$

$$\Delta x' = \frac{x'}{f}db_z, \quad \Delta y' = \frac{y'}{f}db_z, \quad \Delta z' = \Delta f = 0$$

Scale change

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = m \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathbf{M} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad kde \quad \mathbf{M} = \begin{pmatrix} m_X & 0 & 0 \\ 0 & m_Y & 0 \\ 0 & 0 & m_Z \end{pmatrix}$$

Derivation of mathematical relations

Coordinate system conversions

x , y , z=(-f) x$_F$, y$_F$, z$_F$ x , y , z X , Y , Z

$$\begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix} = \mathbf{R} \cdot \begin{pmatrix} x' \\ y' \\ z' = -f \end{pmatrix},$$

$$\begin{pmatrix} x' \\ y' \\ z' = -f \end{pmatrix} = \mathbf{R}^T \cdot \begin{pmatrix} x'_F \\ y'_F \\ z'_F \end{pmatrix}$$

Further, the similarity applies:

$$\frac{x'_F}{z'_F} = \frac{x'_S}{z'_S} = \frac{x'_S}{-f} = \frac{x - x_0}{z - z_0}, \frac{y'_F}{z'_F} = \frac{y'_S}{z'_S} = \frac{y'_S}{-f} = \frac{y - y_0}{z - z_0}$$

By substitution we get a **collinear relation**:

$$x'_s = -f \frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f}, y'_s = -f \frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f}$$

Coordinate system conversions

$$x = x_0 + (z - z_0)\frac{r_{11}(x' - x'_0) + r_{12}(y' - y'_0) - r_{13}f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33}f}$$

$$y = y_0 + (z - z_0)\frac{r_{21}(x' - x'_0) + r_{22}(y' - y'_0) - r_{23}f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33}f}$$

A more common form because we can conveniently assign corrections to the frame coordinates and linearize the relationship :

$$x' = x'_0 - f\frac{r_{11}(x - x_0) + r_{21}(y - y_0) + r_{31}(z - z_0)}{r_{13}(x - x_0) + r_{23}(y - y_0) + r_{33}(z - z_0)}$$

$$y' = y'_0 - f\frac{r_{12}(x - x_0) + r_{22}(y - y_0) + r_{32}(z - z_0)}{r_{13}(x - x_0) + r_{23}(y - y_0) + r_{33}(z - z_0)}$$

Direct relationship between image and geodetic coordinates

collinear frame-model relationship

$$\frac{x' - x'_0}{-f} = \frac{x - x_0}{z - z_0}, \quad \frac{y' - y'_0}{-f} = \frac{y - y_0}{z - z_0}$$

$$\begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} = \mathbf{R} \cdot \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}, \qquad \mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

the model coordinate system can be converted to a geodetic system by rotations around three axes

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} = \mathbf{R}^T \cdot \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix}$$

<u>the direct relationship between image and geodetic coordinates, which is the basis of all contemporary photogrammetry</u>:

$$x' = x'_0 - f\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

$$y' = y'_0 - f\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

Direct relationship - transformation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} + m \cdot R \cdot \begin{bmatrix} x' - x'_o \\ y' - y'_o \\ -f \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} + m \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \cdot \begin{bmatrix} x' - x'_o \\ y' - y'_o \\ -f \end{bmatrix}$$

$$X = X_o + m \cdot (r_{11} \cdot (x' - x'_o) + r_{12} \cdot (y' - y') - r_{13} \cdot f)$$
$$Y = Y_o + m \cdot (r_{21} \cdot (x' - x'_o) + r_{22} \cdot (y' - y') - r_{23} \cdot f)$$
$$Z = Z_o + m \cdot (r_{31} \cdot (x' - x'_o) + r_{32} \cdot (y' - y') - r_{33} \cdot f)$$

$$m = \frac{Z - Z_o}{r_{31} \cdot (x' - x'_o) + r_{32} \cdot (y' - y') - r_{33} \cdot f}$$

$$X = X_o + (Z - Z_o) \cdot \frac{r_{11} \cdot (x' - x'_o) + r_{12} \cdot (y' - y') - r_{13} \cdot f}{r_{31} \cdot (x' - x'_o) + r_{32} \cdot (y' - y') - r_{33} \cdot f}$$
$$Y = Y_o + (Z - Z_o) \cdot \frac{r_{21} \cdot (x' - x'_o) + r_{22} \cdot (y' - y') - r_{23} \cdot f}{r_{31} \cdot (x' - x'_o) + r_{32} \cdot (y' - y') - r_{33} \cdot f}$$

Photogrammetric series

Definition:
Photogrammetric series are expressions which, with a degree of precision given by the linearization of a complete relation, express the effect of elements of exterior orientation on image coordinates.

Rotation matrix linearization (up to 2-3)

$cos\,(\alpha) \cong 1$ and $sin\,(\alpha) \cong \mathrm{d}\alpha$

$$\mathbf{dR} = \begin{pmatrix} 1 & -d\kappa & d\varphi \\ d\kappa & 1 & -d\omega \\ -d\varphi & d\omega & 1 \end{pmatrix}$$

$$x = x_0 + (z - z_0)\frac{r_{11}(x' - x'_0) + r_{12}(y' - y'_0) - r_{13}f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33}f}$$

$$y = y_0 + (z - z_0)\frac{r_{21}(x' - x'_0) + r_{22}(y' - y'_0) - r_{23}f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33}f}$$

Photogrammetric series

$$x'_s = -f \frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f}, \quad y'_s = -f \frac{r_{21}x' + r_{22}y' - r_{23}f}{r_{31}x' + r_{32}y' - r_{33}f}$$

for small angles using a linearized rotation matrix

$$x'_s = -f \frac{x' - y'd\kappa' - f d\phi'}{-x'd\phi' + y'd\omega' - f} y'_s = -f \frac{x'd\kappa' + y' + f d\omega'}{-x'd\phi' + y'd\omega' - f}$$

The next procedure is analogous for x'_s, y'_s

$$\left(-x'd\varphi' + y'd\omega' - f\right)x'_s = -f\left(x' - y'd\kappa' - f\,d\varphi'\right)$$

after division by -*f*

$$\left(1 - \frac{y'd\omega'}{f} + \frac{x'd\phi'}{f}\right)x'_s = (x' - y'd\kappa' - f d\phi')$$

Photogrammetric series

$$x'_s = (x' - y'd\kappa' - f d\phi')\left[1 - \left(\frac{y'd\omega'}{f} - \frac{x'd\phi'}{f}\right)\right]^{-1}$$

$$x'_s = A \cdot [1 - B]^{-1} \approx A \cdot (1 + B) = A + AB$$

$$\Delta x' = x'_s - x' = -y'd\kappa' - \left(f + \frac{x'^2}{f}\right)d\phi' + \frac{x'y'}{f}d\omega'$$

$$\Delta y' = y'_s - y' = x'd\kappa' - \frac{x'y'}{f}d\phi' + \left(f + \frac{y'^2}{f}\right)d\omega'$$

Photogrammetric series

Finally, these expressions need to be supplemented by the effect of translation

$$\Delta x' = -y' d\kappa' - \left(f + \frac{x'^2}{f}\right) d\phi' + \frac{x'y'}{f} d\omega' + db'_x + \frac{x'}{f} db'_z$$

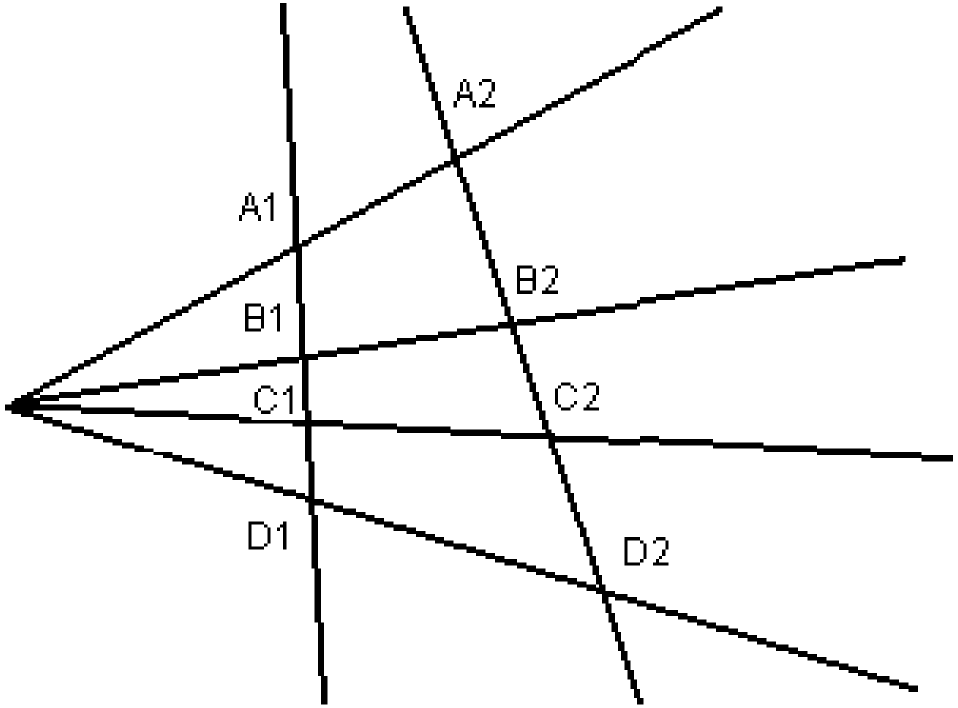
$$\Delta y' = x' d\kappa' - \frac{x'y'}{f} d\phi' + \left(f + \frac{y'^2}{f}\right) d\omega' + db'_y + \frac{y'}{f} db'_z$$

The term is called "**complete photogrammetric series**", or historically Gruber series, and is used in simplified theoretical derivations. The meaning and use of series was quite fundamental, especially in the era of analogue photogrammetry.

Methods of photogrammetry

Single-shot photogrammetry

- relationship between two planes



Papp's theorem:
The point or ray quadrature binary ratio is preserved in the map and image planes.

$$\frac{\dfrac{A_1C_1}{B_1C_1}}{\dfrac{A_1D_1}{B_1D_1}} = \frac{\dfrac{A_2C_2}{B_1C_2}}{\dfrac{A_2D_2}{B_2D_2}}$$

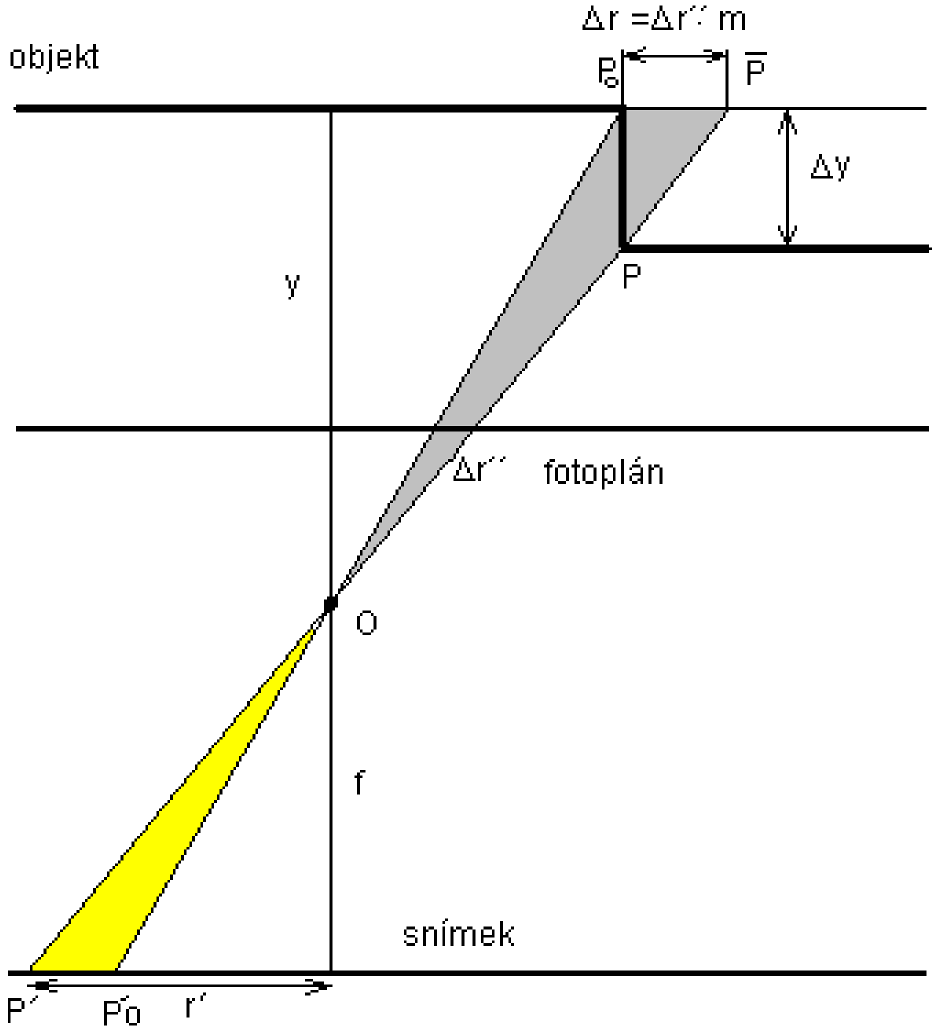
Mathematical expression
in fact, the **collinear transformation**

$$X = \frac{a_1x' + a_2y' + a_3}{c_1x' + c_2y' + 1}$$

$$Y = \frac{b_1x' + b_2y' + b_3}{c_1x' + c_2y' + 1}$$

Effect of depth zoning

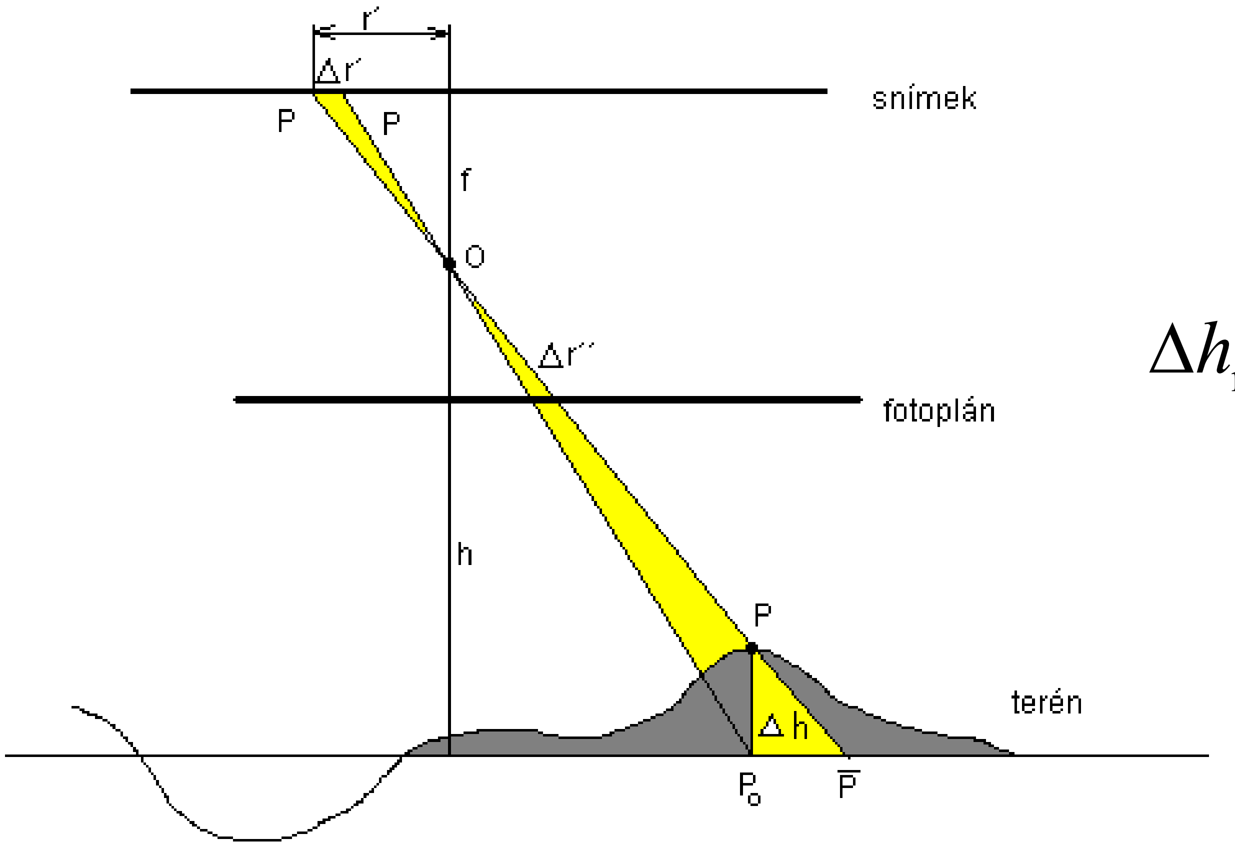
$$\Delta r = \Delta r'' \cdot m_F, \qquad \frac{\Delta r}{\Delta y} = \frac{r'}{f}, \qquad \Delta r'' = \frac{\Delta y \cdot r'}{f \cdot m_F}$$



$$\Delta y_{\max} = \frac{f \cdot m_F \cdot \Delta r''_{\max}}{r'}$$

Effect of height zoning

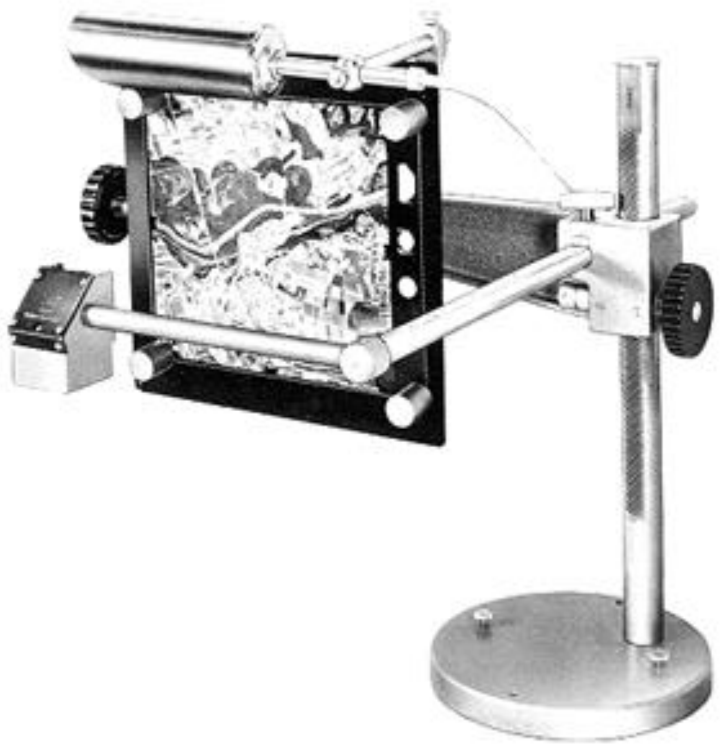
$$\Delta r'' = \frac{\Delta h \cdot r'}{f \cdot m_F}$$



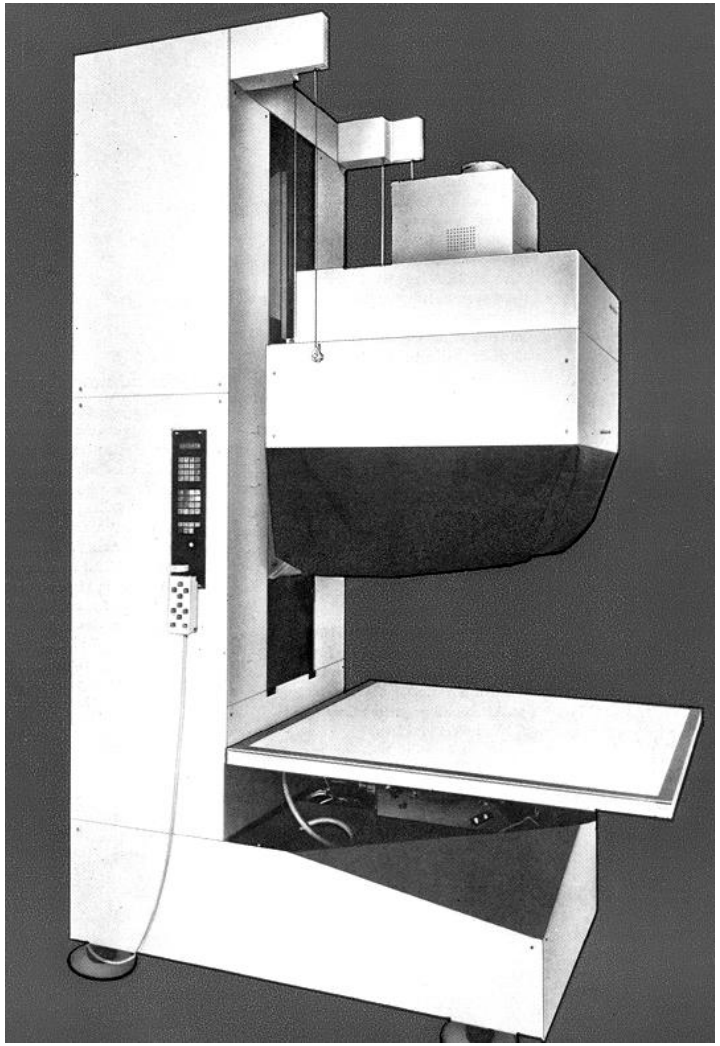
$$\Delta h_{\max} = \frac{f \cdot m_F \cdot \Delta r''_{\max}}{r'}$$

Methods of evaluation of single-frame photogrammetry

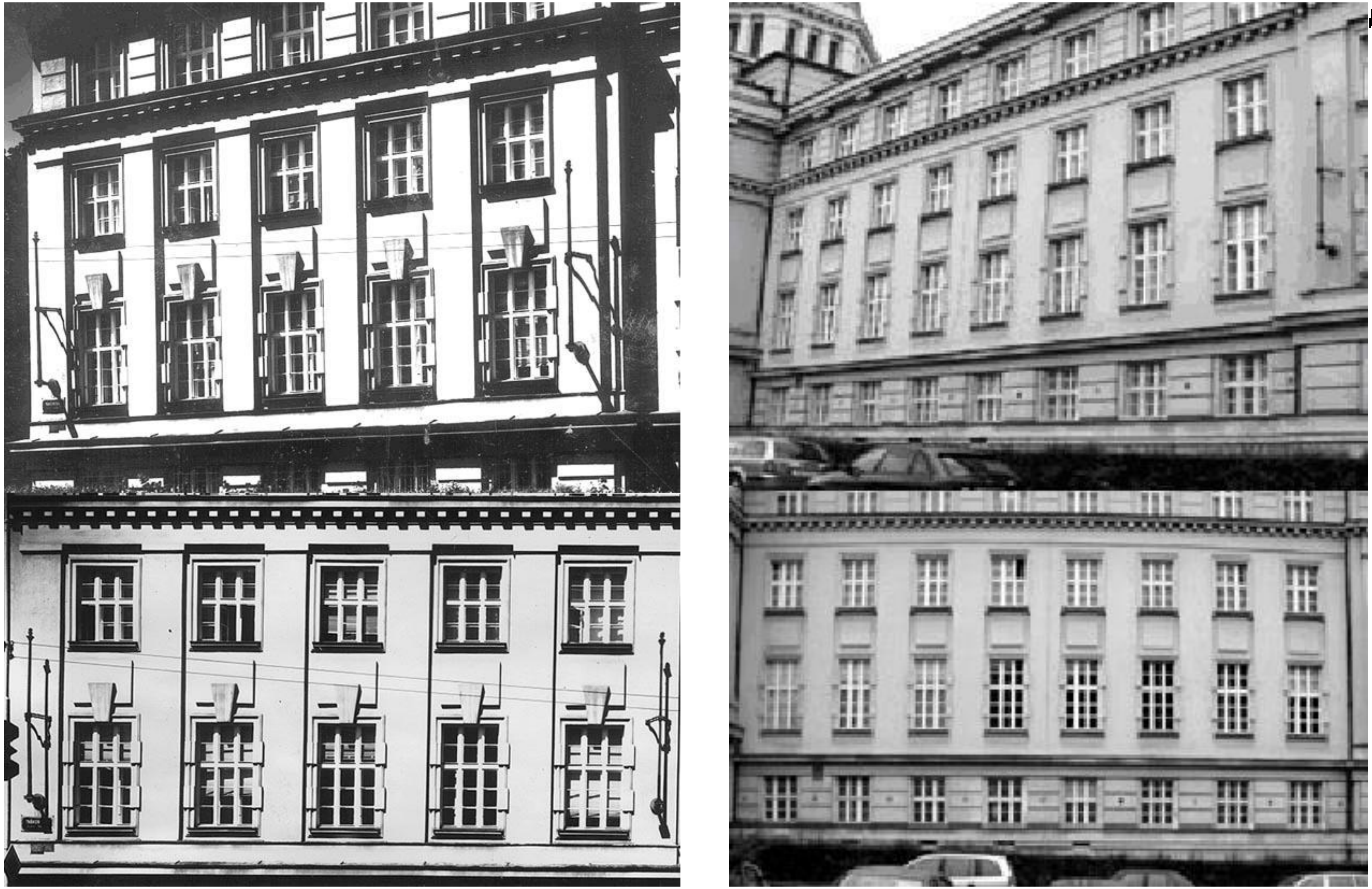
Circularizer



Transliterator



Digital redrawing



Original image and photo plan, taken by the measuring camera (left),
an image taken with an ordinary camera and its digitally redrawn form - an image vault,
due to uncorrected radial distortion is clearly visible (right)

Solution

$$X = \frac{a_1 x' + a_2 y' + a_3}{c_1 x' + c_2 y' + 1}$$

$$Y = \frac{b_1 x' + b_2 y' + b_3}{c_1 x' + c_2 y' + 1}$$

$$\begin{aligned} X &= a_1 x' + a_2 y' + a_3 \qquad\qquad\qquad\qquad - c_1 x' X - c_2 y' X \\ Y &= \qquad\qquad\qquad\qquad b_1 x' + b_2 y' + b_3 - c_1 x' Y - c_2 y' Y \end{aligned}$$

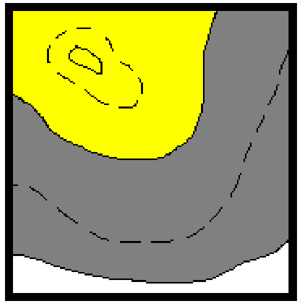
$$\begin{pmatrix} x_1' & y_1' & 1 & 0 & 0 & 0 & -x_1' X_1 & -y_1' X_1 \\ 0 & 0 & 0 & x_1' & y_1' & 1 & -x_1' Y_1 & -y_1' Y_1 \\ x_2' & y_2' & 1 & 0 & 0 & 0 & -x_2' X_2 & -y_2' X_2 \\ 0 & 0 & 0 & x_2' & y_2' & 1 & -x_2' Y_2 & -y_2' Y_2 \\ x_3' & y_3' & 1 & 0 & 0 & 0 & -x_3' X_3 & -y_3' X_3 \\ 0 & 0 & 0 & x_3' & y_3' & 1 & -x_3' Y_3 & -y_3' Y_3 \\ x_4' & y_4' & 1 & 0 & 0 & 0 & -x_4' X_4 & -y_4' X_4 \\ 0 & 0 & 0 & x_4' & y_4' & 1 & -x_4' Y_4 & -y_4' Y_4 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ b_1 \\ b_2 \\ b_3 \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \\ X_4 \\ Y_4 \end{pmatrix}$$

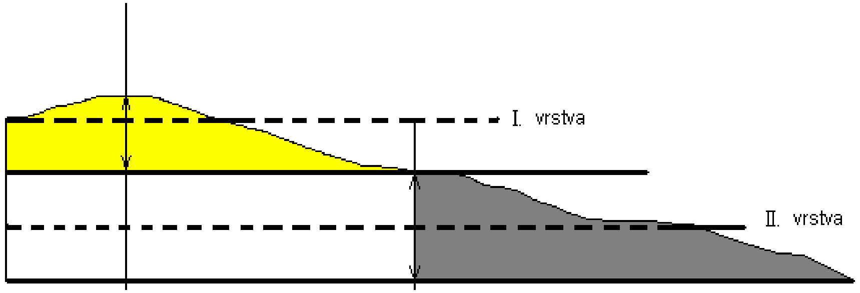
$$\mathbf{A} \cdot \mathbf{a} = \mathbf{X}$$

$$\mathbf{a} = \mathbf{A}^{-1} \cdot \mathbf{X}$$

Options

Redrawing by layers



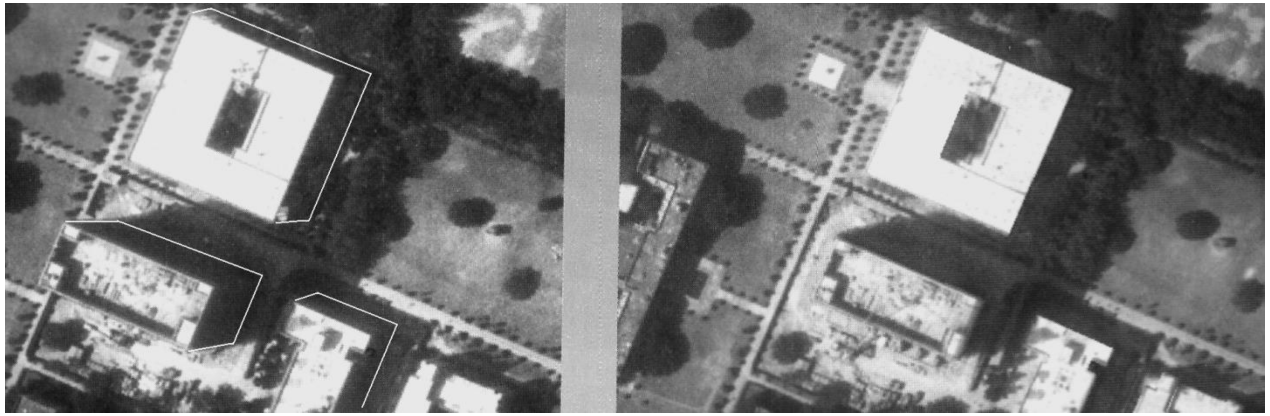


Differential redrawing (historical approach)

Digital orthophoto

Digital orthophoto

Digital orthophoto is a photogrammetric product - image conversion with central projection to orthogonal projection

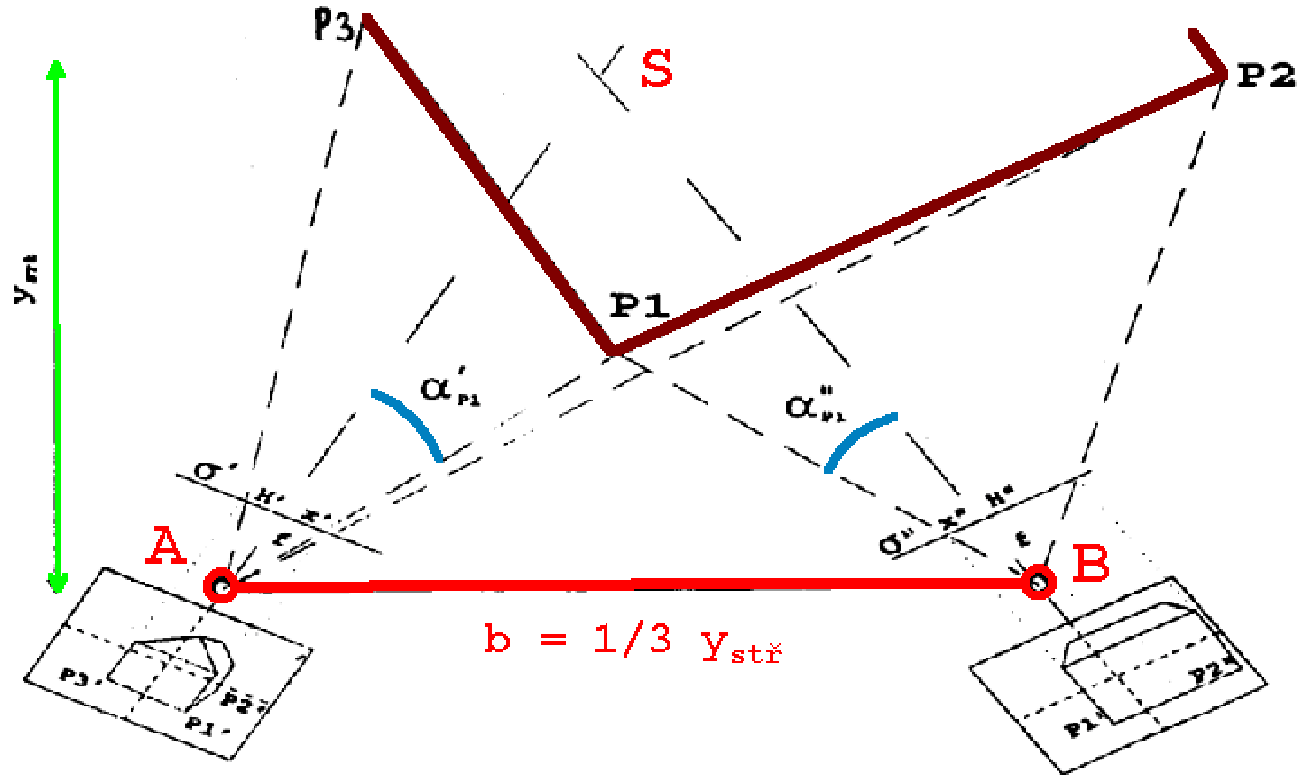


For creation:
Required image with known elements of internal and external orientation and DMT

Ortophoto and true ortophoto

The problem of mosaicking
- seamless orthophoto

Multi-frame photogrammetry
- cross-sectional



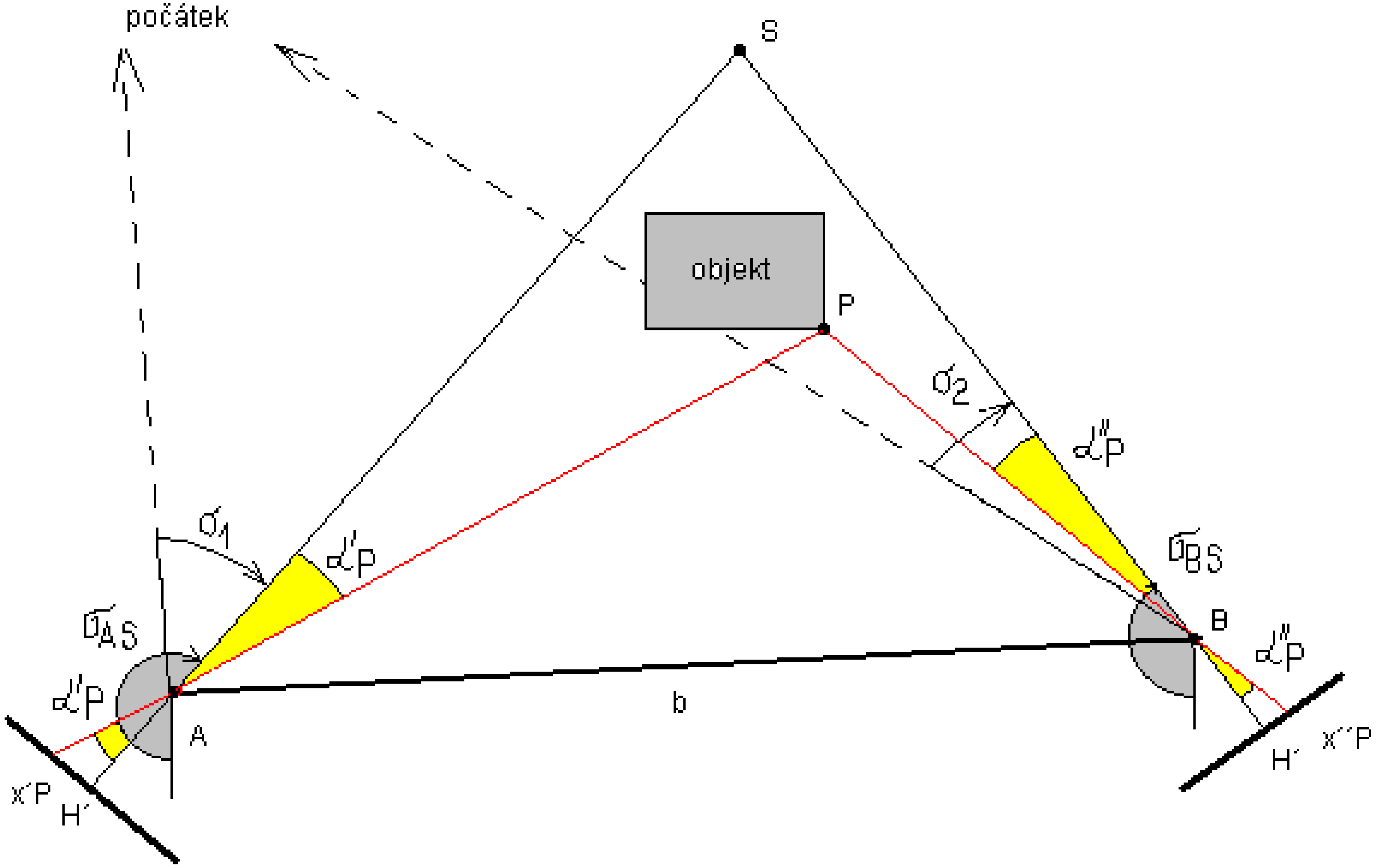
The oldest photogrammetric method
- based on intersection of rays
- today only in digital form

Historical solution

$$tg\alpha' = \frac{x'}{f}, \quad tg\beta' = \frac{z'}{\sqrt{\left(f^2 + x'^2\right)}} = \frac{z'}{f}\cos\alpha'$$

$$tg\alpha' = \frac{x'}{f}, \quad tg\beta' = \frac{z'}{\sqrt{\left(f^2 + x'^2\right)}} = \frac{z'}{f}\cos\alpha'$$

$$x' = f\frac{x'_F}{f\cdot\cos\omega - z'_F\cdot\sin\omega}, \quad z' = f\frac{f\cdot\sin\omega + z'_F\cdot\cos\omega}{f\cdot\cos\omega - z'_F\cdot\sin\omega}$$



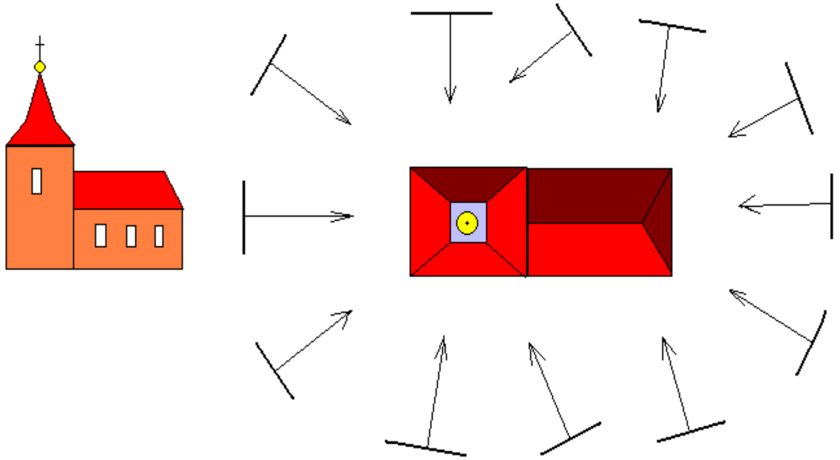
Today's solution

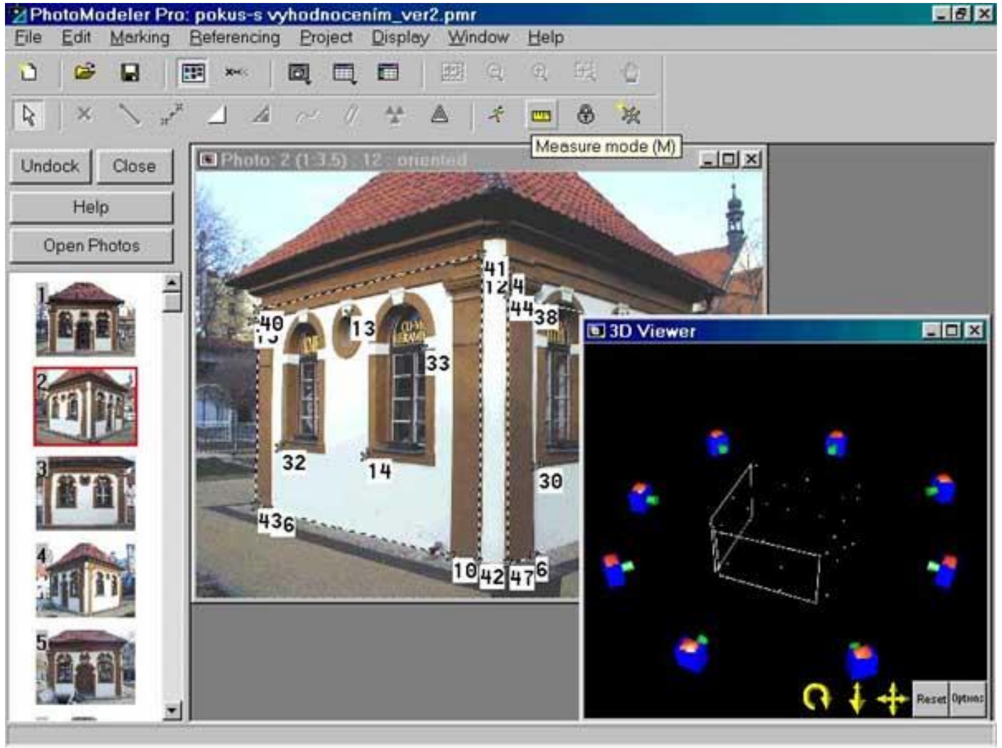
$$X = X_0 + (Z - Z_0)\frac{r_{11}(x' - x_0') + r_{12}(y' - y_0') - r_{13} f}{r_{31}(x' - x_0') + r_{32}(y' - y_0') - r_{33} f}$$

$$Y = Y_0 + (Z - Z_0)\frac{r_{21}(x' - x_0') + r_{22}(y' - y_0') - r_{23} f}{r_{31}(x' - x_0') + r_{32}(y' - y_0') - r_{33} f}$$

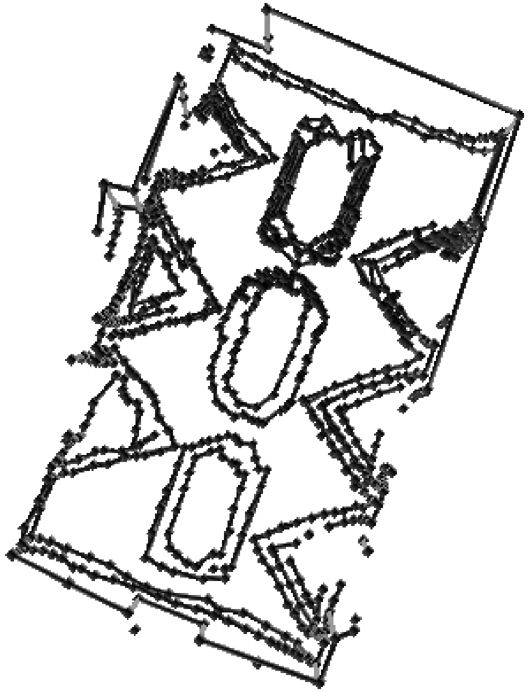
$$\begin{pmatrix} x' - x_0' + \Delta x' \\ y' - y_0' + \Delta y' \\ -f \end{pmatrix} = m \cdot \boldsymbol{R}^T \cdot \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix}$$

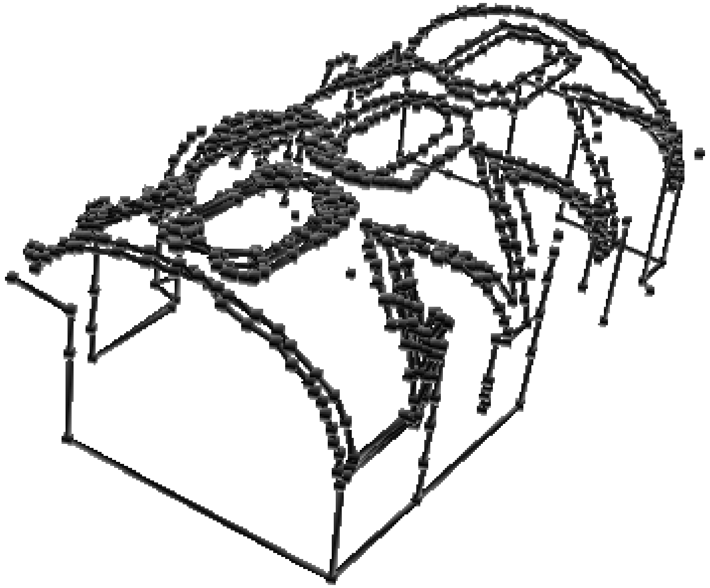
Cross-sectional photogrammetry





Documentation of the spatially articulated vault in the historical building on Nerudova Street by intersection photogrammetry (sw Photomodeler)





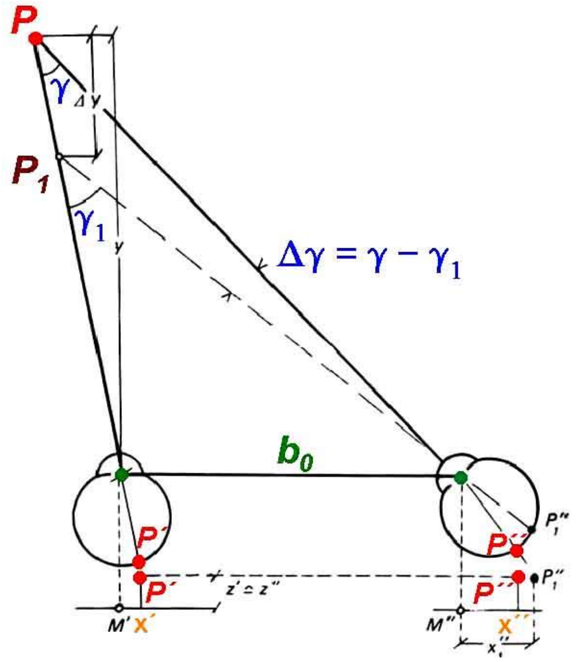
Stereophotogrammetry

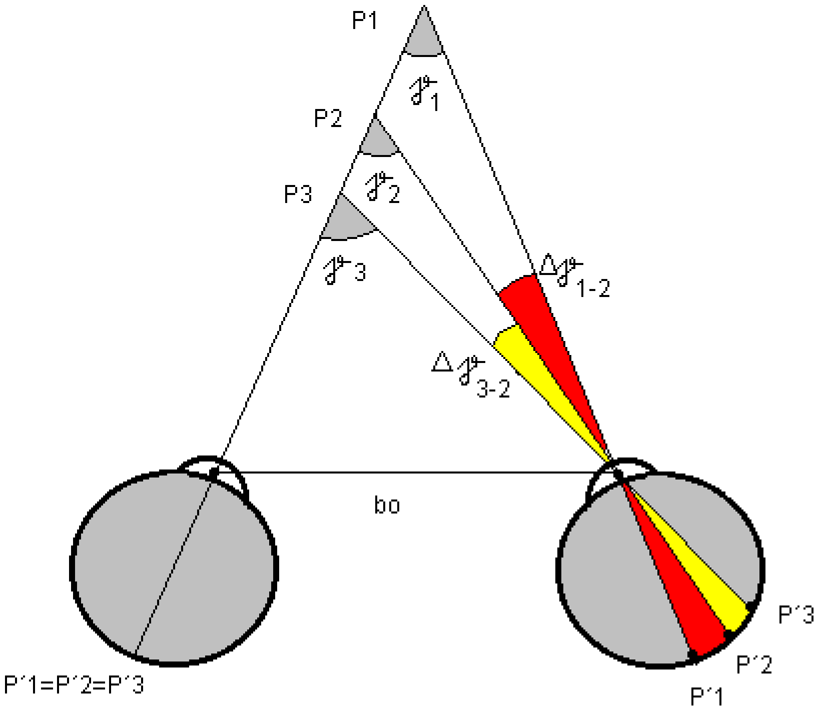
Stereophotogrammetric method introduced at the beginning of the 20th century (Pulfrich)

Evaluation based on stereoscopic perception also for non-signalized points

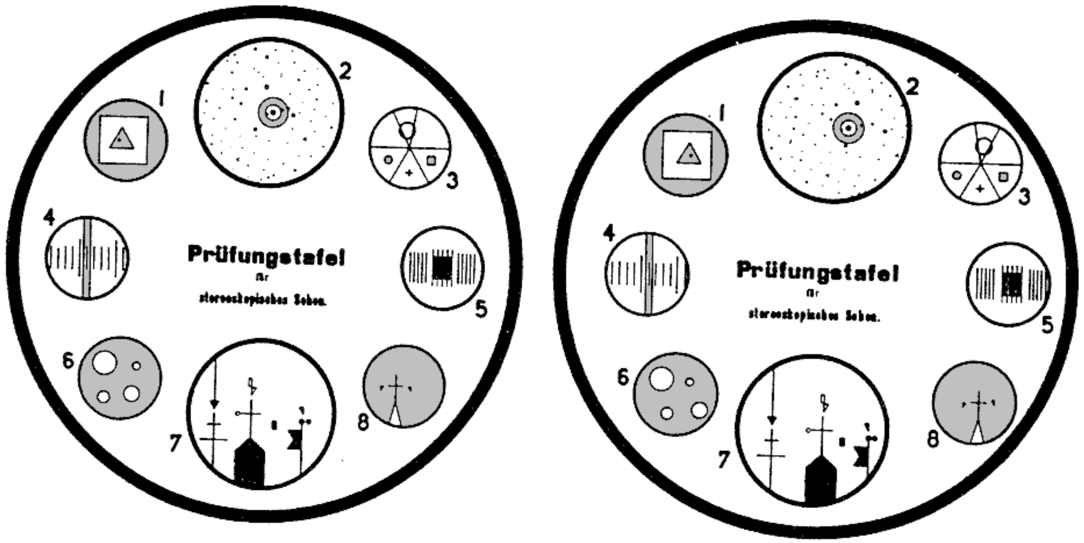
Photogrammetric stereoscopic observation and evaluation

- artificial stereoscopic perception based on the natural perception of healthy eyes

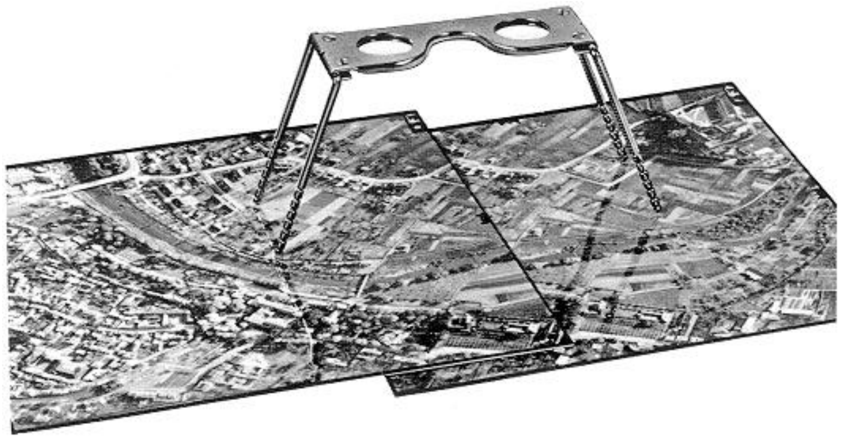


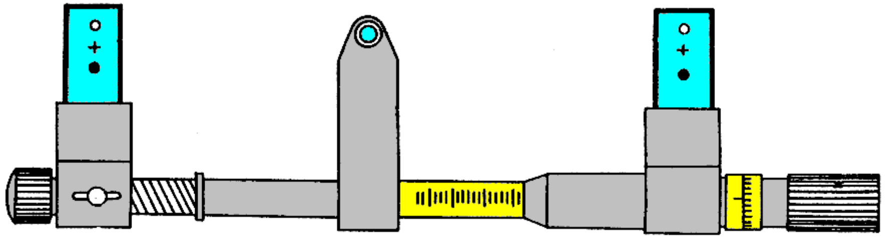


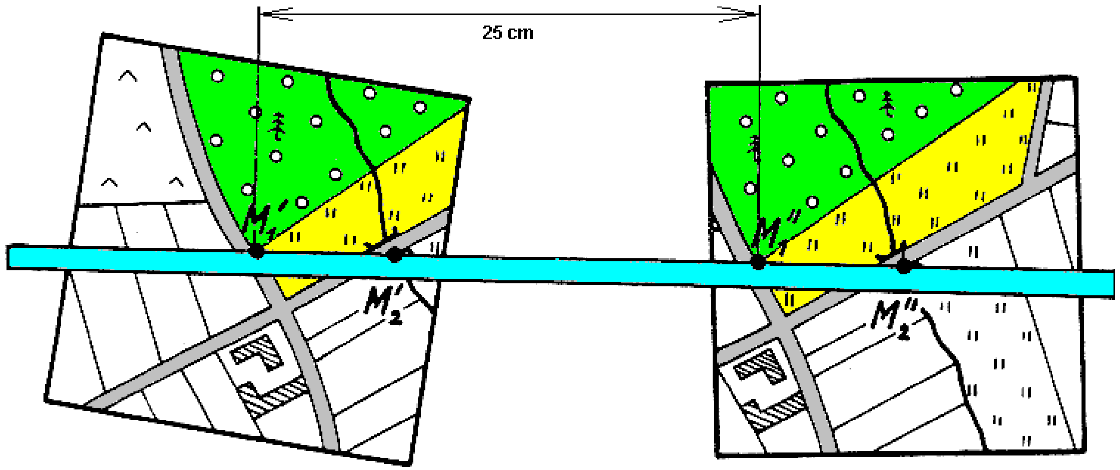
Stereoscopes



Stereoscopes

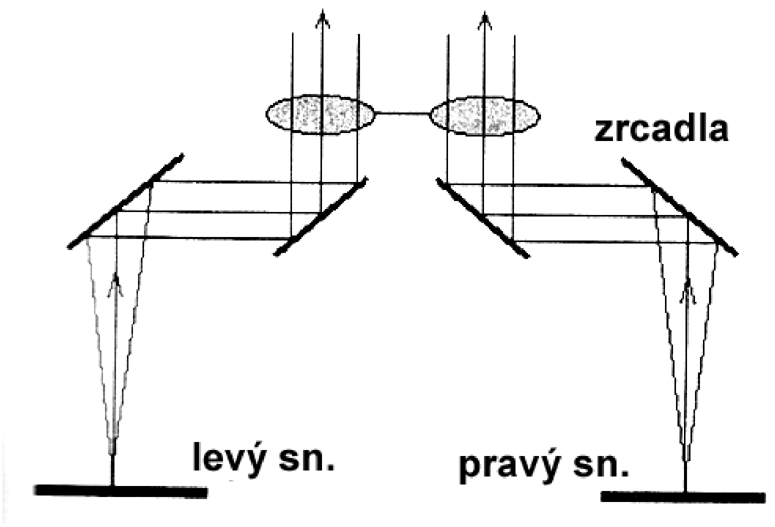


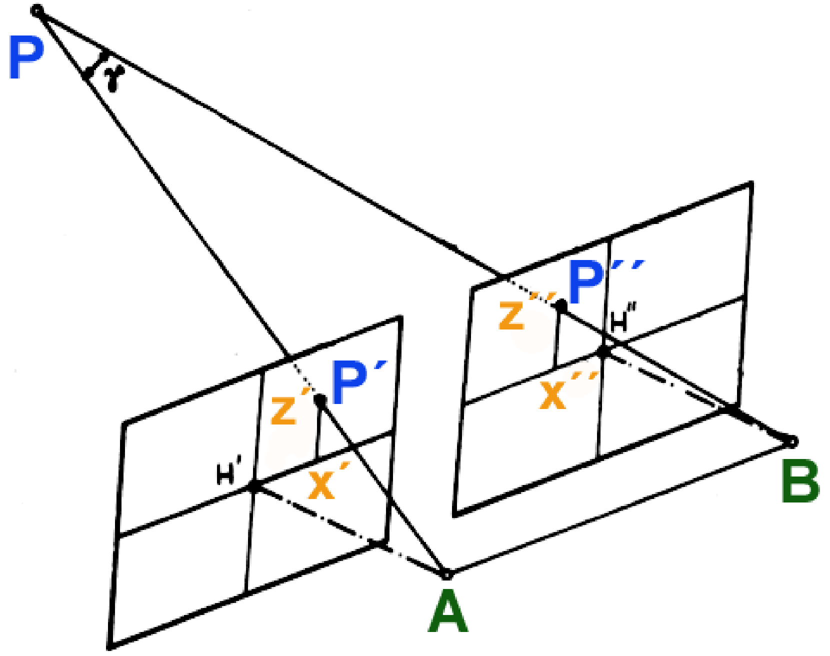




Stereophotogrammetry

Conditions for the formation of artificial stereoscopic perception: I observe both images separately at the same time





1. horizontal parallaxes are non-zero, <u>p = x´ - x´´ = 0</u>

2. vertical parallaxes are zero <u>q = y´ - y´´ (= z´- z´´) = </u>0

Stereophotogrammetry

Evaluation of image content based on stereoscopic perception, conversion of image coordinates to geodetic

older transfer procedure

$x', y', (f)$ x'_F , y'_F , z'_F x, y, z X, Y, Z frame fictitious sn. model geod. system

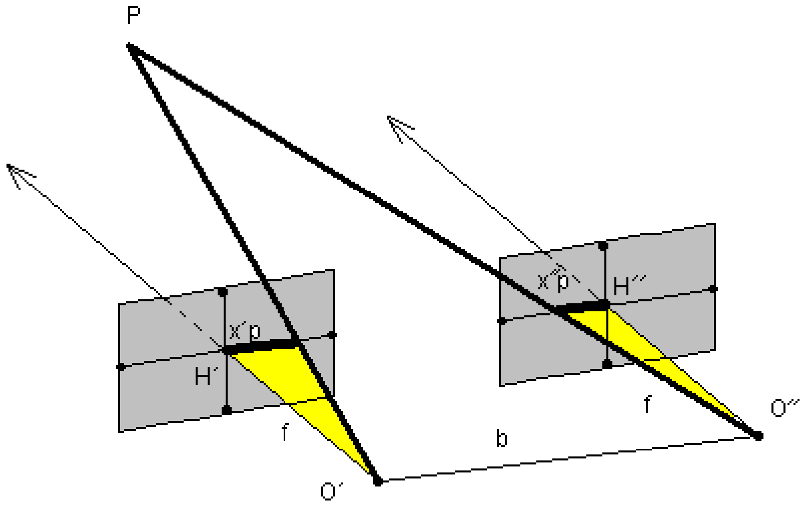
solved by a process of image orientations

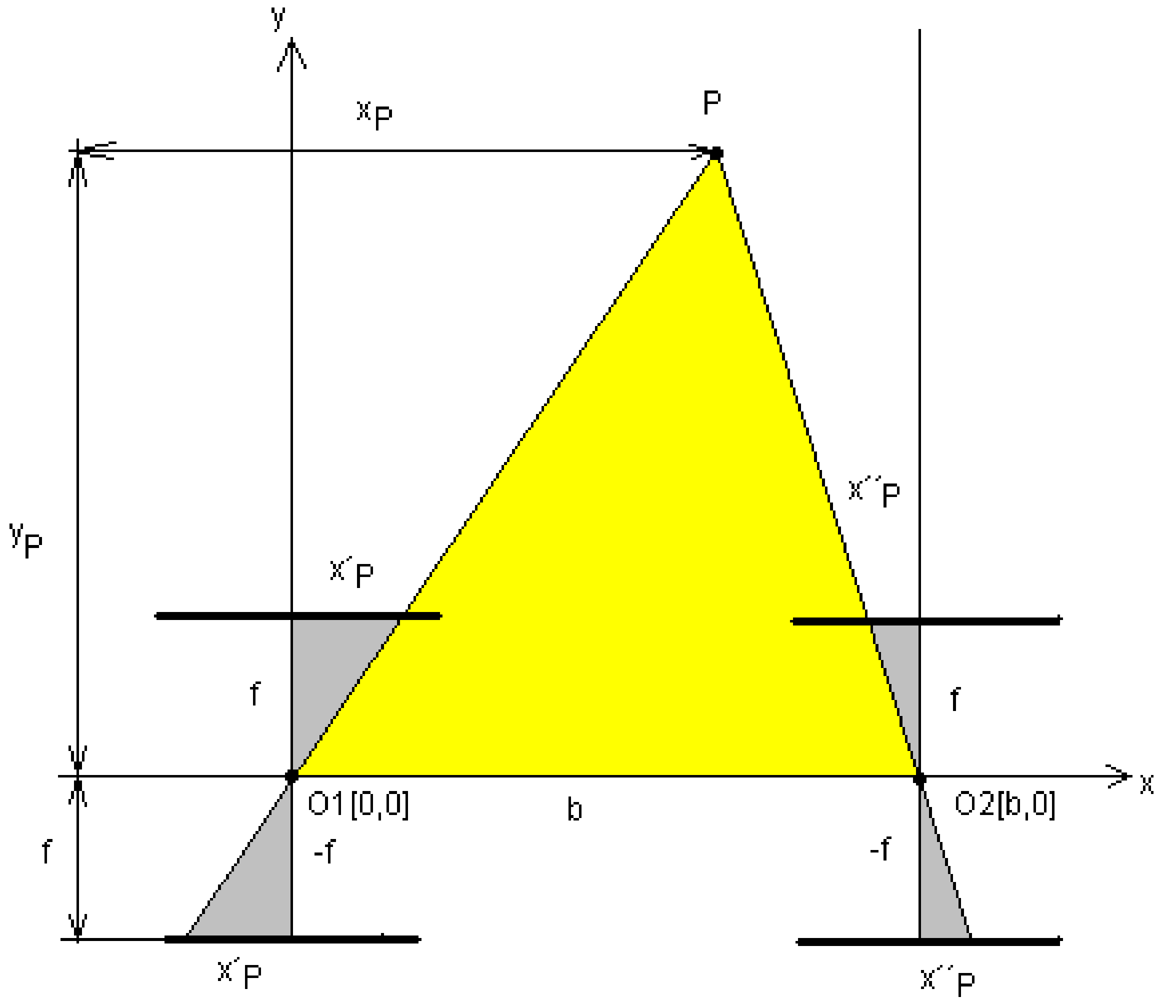
direct relationship

$x', y', (f)$ X, Y, Z frame geod. system

the basis of modern digital photogrammetry

Solution





Terrestrial stereophotogrammetry - normal case

In terrestrial applications, standardization of the external orientation elements can be ensured

R=E

$$x = \frac{b \cdot x'}{p} \quad y = \frac{b \cdot f}{p} \quad z = \frac{b \cdot z'}{p}$$

b (AB) base

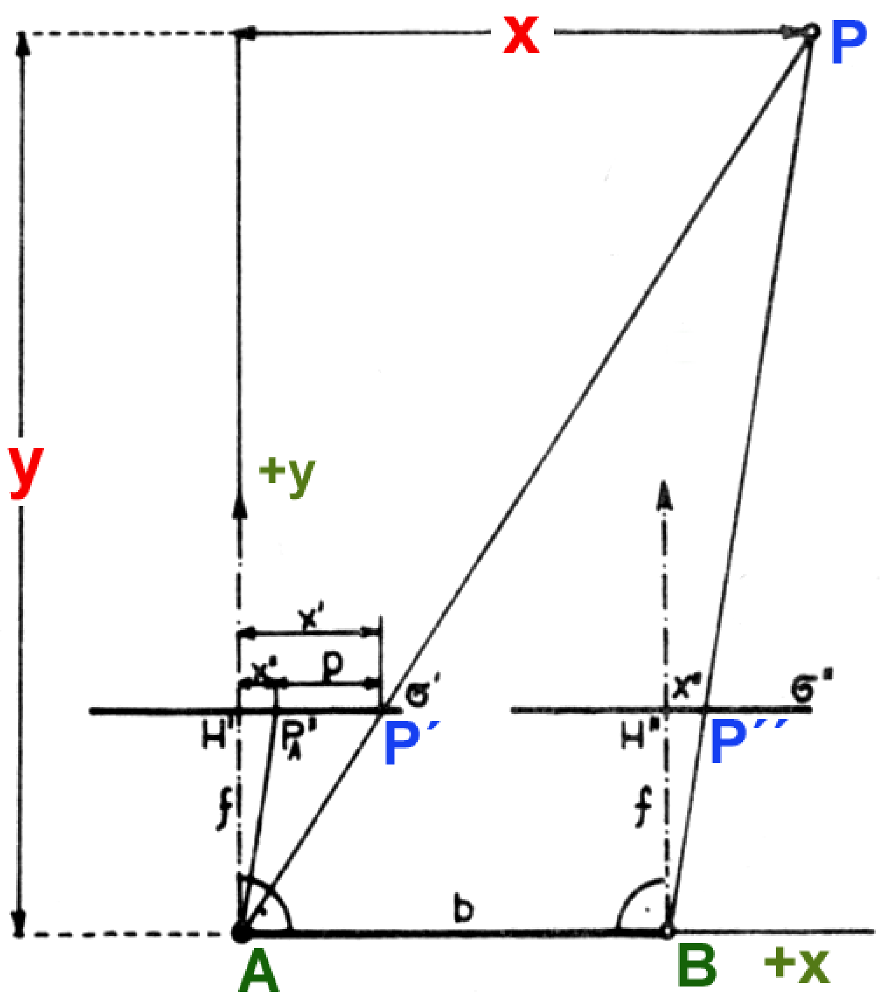
x y (z) model system

´, ´´ image planes

x´, x´´,z´, z´´ image coordinates

p parallax

f chamber constant



Accuracy of photogrammetry

$$\frac{y}{f} = \frac{b}{p}$$

$$x = \frac{b \cdot x'}{p} \qquad y = \frac{b \cdot f}{p} \qquad z = \frac{b \cdot z'}{p}$$

$$x = y\frac{x'}{f} \qquad z = y\frac{z'}{f}$$

$$y = \frac{b \cdot f}{p}, dy = \frac{f}{p}db + \frac{b}{p}df - \frac{bf}{p^2}dp$$

$$dy = -\frac{b \cdot f}{p^2}dp, p^2 = \left(\frac{b \cdot f}{y}\right)^2$$

$$dy = -\frac{y \cdot y}{b \cdot f}dp$$

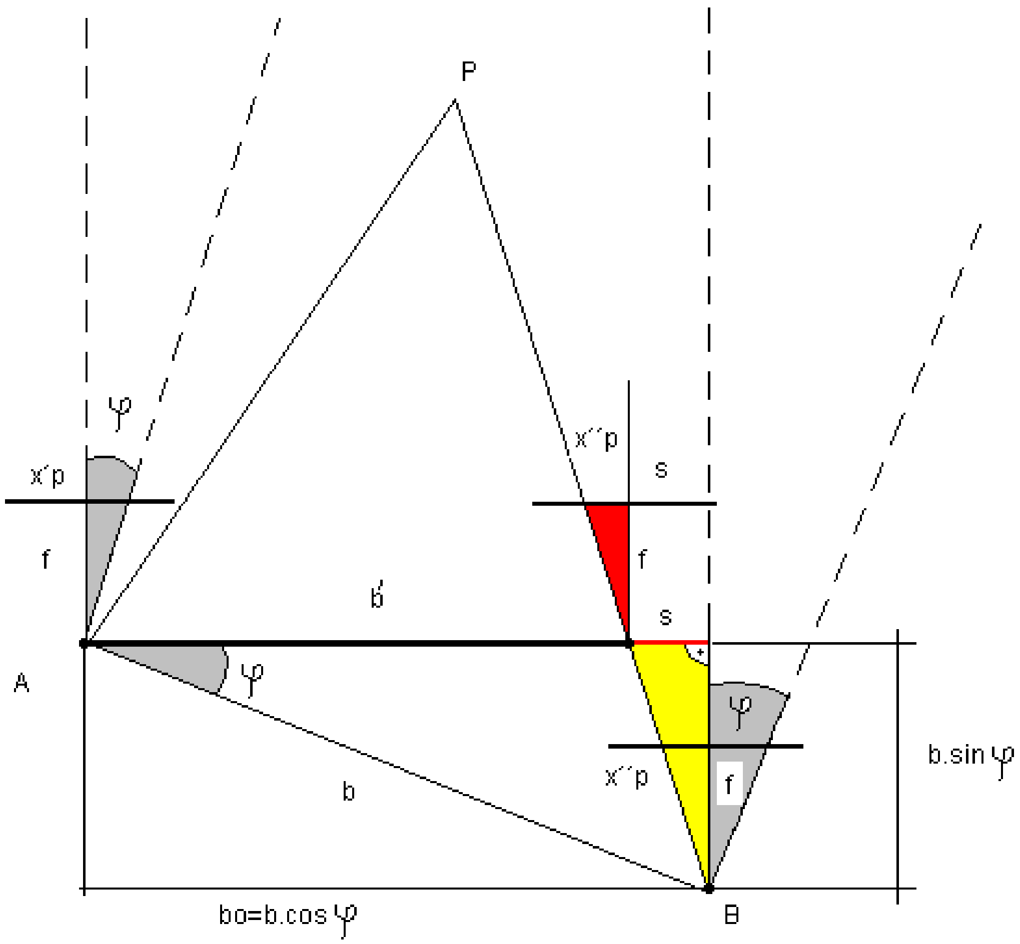
$$m_y = \pm\frac{y \cdot y}{b \cdot f}m_P$$

A twisted case

$$b' = b \cdot \left(\cos\phi \pm \frac{x_p''}{f} \cdot \sin\phi \right) \frac{s}{b \cdot \sin\phi} = \frac{x_p''}{f}$$

$$b' = b_x \pm b_y \cdot \frac{x_p''}{f}$$

$$x = \frac{b' \cdot x'}{p} \qquad y = \frac{b' \cdot f}{p} \qquad z = \frac{b' \cdot z'}{p}$$



a leaning case

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{R}_\omega \cdot \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix}, \qquad \mathbf{R}_\omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{pmatrix}$$

Photogrammetric base

$$y = \frac{b \cdot f}{p} \qquad dy = -\frac{y \cdot y}{b \cdot f} dp$$

$$b = -\frac{y}{dy} \cdot \frac{y}{f} dp$$

where dp = 0.01mm = common mean error of horizontal parallax measurement, dy/y is the required accuracy of evaluation as relative error (e.g. 1/1000), f is the chamber constant;
p_{max} = 40-50mm

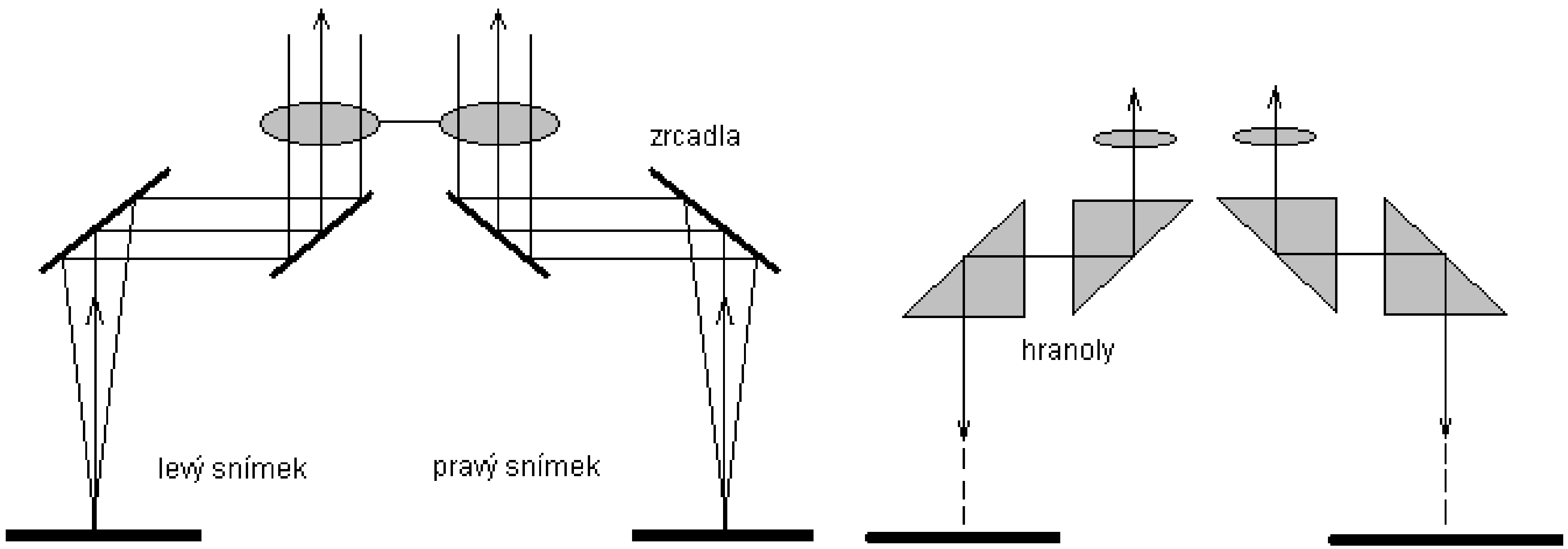
$$b \frac{y}{dy} \frac{dp}{f} \frac{10mm}{f[mm]} \mathrm{max}_{\mathrm{min}} \mathrm{max}_{\mathrm{min}}$$

$$b_{\max} = y_{\min} \cdot \frac{p_{\max}}{f}$$

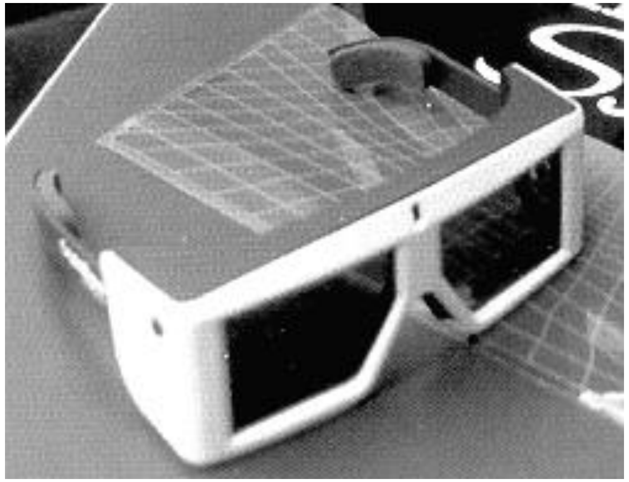
photogrammetric base

Accuracy dy/y = 1/1000	*f [mm]*	*medium b*	*min. b*	*max. b*
PhoTheo 13x18	195	$b_{stř} = 1/10\ y_{stř}$	$b_{min} = 1/20\ y_{max}$	$b_{max} = (1/4 - 1/5)\ y_{min}$
RolleiMetric 6x6	40	1/2	1/4	4/5 - 1/1
RolleiMetric 6x6	80	1/3	1/8	2/3
UMK 10/1318	100	1/5	1/10	1/2
Nikon 100	17	1	1/2	2 - 3
Canon 20D	10	2	1	4 - 5
Canon 20D	22	1	1/2	2 - 3
Nikon 100	35	9/10	1/3	1-3/2
Canon 20D	85	1/3	1/9	1/2

Stereoscopic observation



Device for stereoscopic perception



new solutions

new solutions

Image capture

The origin of the image

Photogrammetry is concerned with extracting measurement information from an image - this is captured by a **detector**

$$M = S^E$$

where E is the number of elements, S is the number of possible states of one element and M is the total number of states (number of combinations). A unit of information is defined as the amount of information needed to write two different states of one element:

$$\log_2 M = E \cdot \log_2 S$$

where $\log_2 M$= amount of information [bit], (1byte=8bits). The basic unit of a digital image is the **pixel** (from the English *picture element*). **Principle : capturing radiation**

$$E = h \cdot f$$

Detectors:
-thermal -photonic
-integral -quantitative

$$Q = \Phi \cdot t \qquad \Phi = \frac{dQ}{dt}$$

where Q is the radiant energy, is the radiant flux and t is the time

The origin of the image

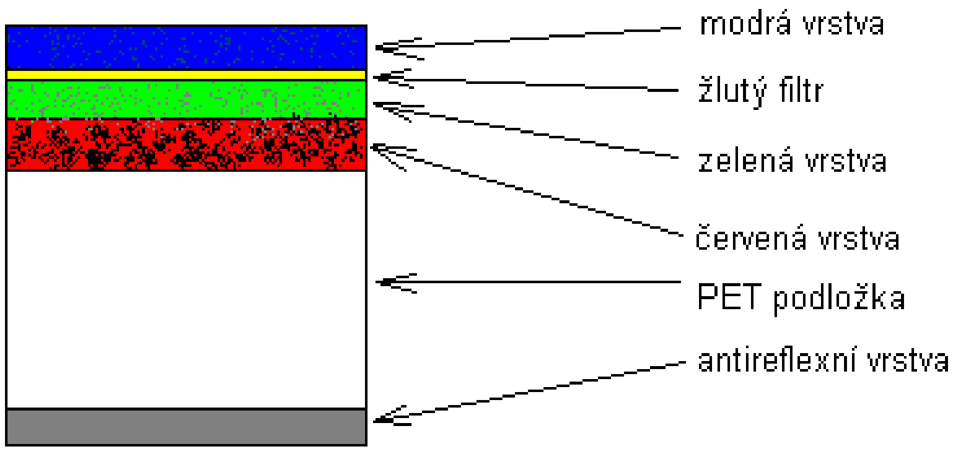
<u>Photographic material</u>

In general, photographic material can be divided into:

positive material, negative material and inverse (slide) material.

Photographic material is further divided into:

- \- coloured
- \- black and white (panchromatic, orthochromatic)
- \- inframaterial
- \- spectrosonal



The origin of the image

- - general sensitivity

$$100\ ASA = 21\ DIN$$
$$200\ ASA = 24\ DIN$$
$$400\ ASA = 27\ DIN$$

- - gradations
- indicates the dependence between the amount of light and the degree of blackening of the sensitive layer, or the blackening rate at constant illumination. The dependence of blackening on exposure is given by *the sensitometric curve*.

- - distinctiveness

$$RS_{max} = \frac{1000 \cdot A}{2.4 \cdot \lambda \cdot f} \text{ /mm}$$

f /A	2.8	8.0	32.0
RS_{max} (line/mm)	298	83	26

sensitometric curve

$$T = \frac{\Phi_{prostup}}{\Phi_{dopad}}$$

where is luminous flux, T is transmittance, 1/T is called transparency

$$D = \log\left(\frac{1}{T}\right) = -\log(T)$$

D is the density (optical density, degree of blackening).

$$E = \frac{\Phi}{S}$$

E is the illuminance [lux], S is the illuminated area.

$$H = E \cdot t$$

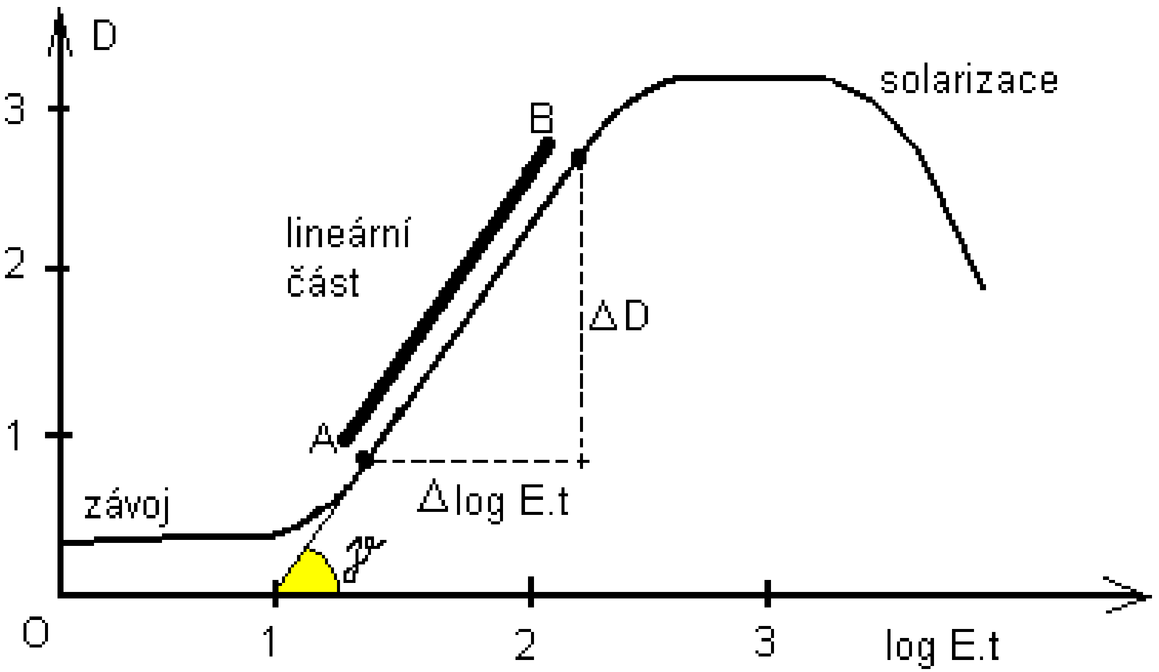
H is the exposure and t is the time (the exposure time).

$$G = \frac{\Delta D}{\Delta \log(H)}$$

The origin of the image

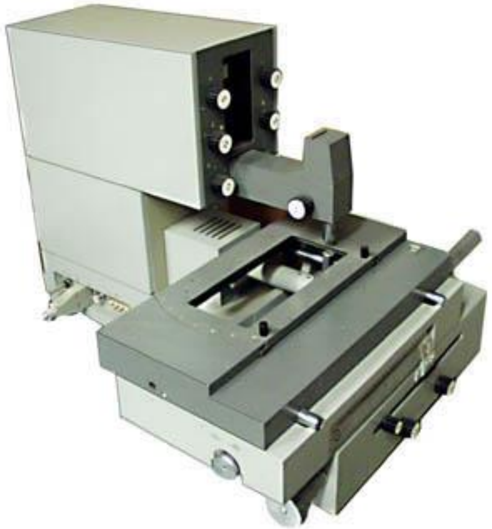
$$G = \mathrm{tg}(\gamma)$$

gradation (steepness) G
Gradation is a linear gradient parts of the sensitometric curve



G<1 ...(<45) ...soft working material
G=1 ...(=45) ...normal working material
G>1 ...(>45) ...steep working material (hard)

The origin of the image



Densitometer Meodenzi TRD01-Meopta (left), Zeiss Jena MD100 (right)

The origin of the image

Processing of photographic materials

Ordinary black and white photographic material is processed in the classical way:

a) material exposure

b) material development ; film strips in tanks, plates based in carriers in tubs; (developer used according to type 5-15 minutes)

c) intermittent bath (normally plain water, rinse)

d) Settling (acid settler, 5-10 minutes)

e) washing (running water, 10-20 minutes)

(f) drying

The emergence of the digital image

A digital image is an image in digital form (expressed in numbers). It is created either primarily by digital capture devices or by scanning analogue images. A digital image consists of individual pixels,

(from English *picture elements*) taking certain values which are not arbitrary (determined by technical possibilities of the computer and coding).

$$P[i,j] = \mathrm{f}(i,j)$$

f(i,j)	f(i,j+1)	f(i, j+2)	f(i, j+3)	f(i, j+4)
f(i+1,j)	f(i+1,j+1)	f(i+1,j+2)	f(i+1,j+3)	f(i+1,j+4)
f(i+2,j)	f(i+2,j+1)	f(i+2,j+2)	f(i+2,j+3)	f(i+2,j+4)
.....				
.....				f(m,n)

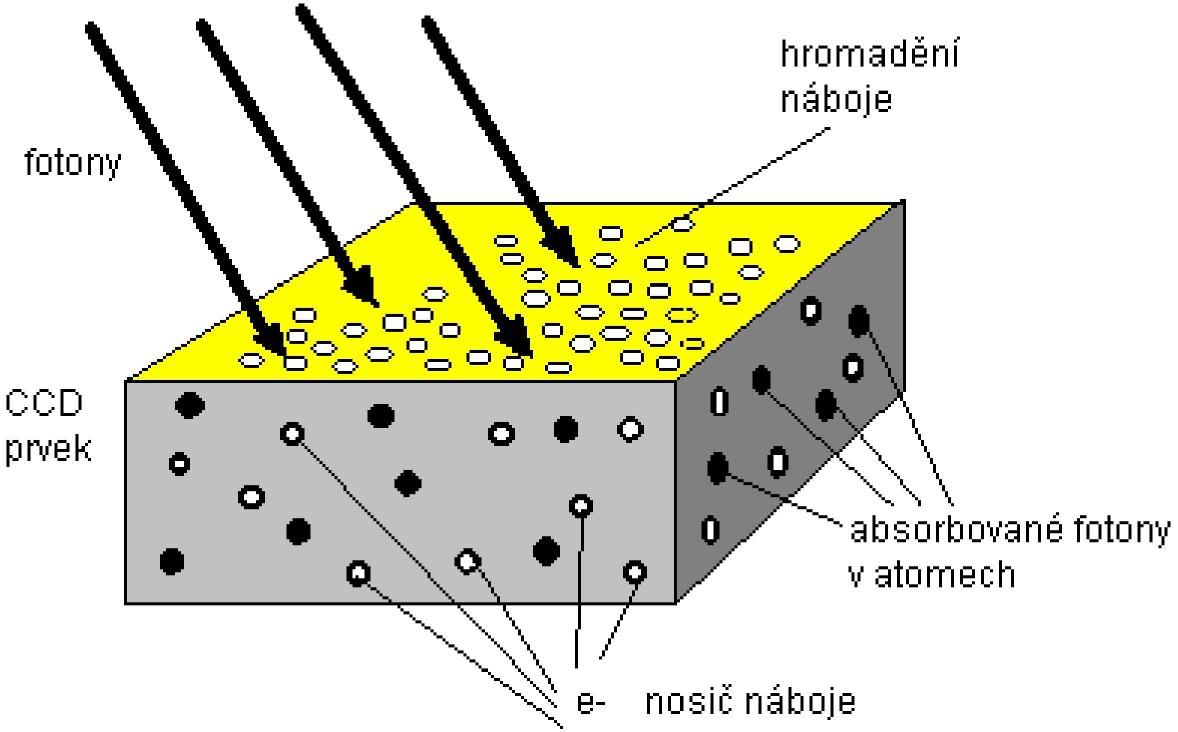
$$M = m \cdot n \cdot e \; [\mathrm{byte}]$$

The origin of the image

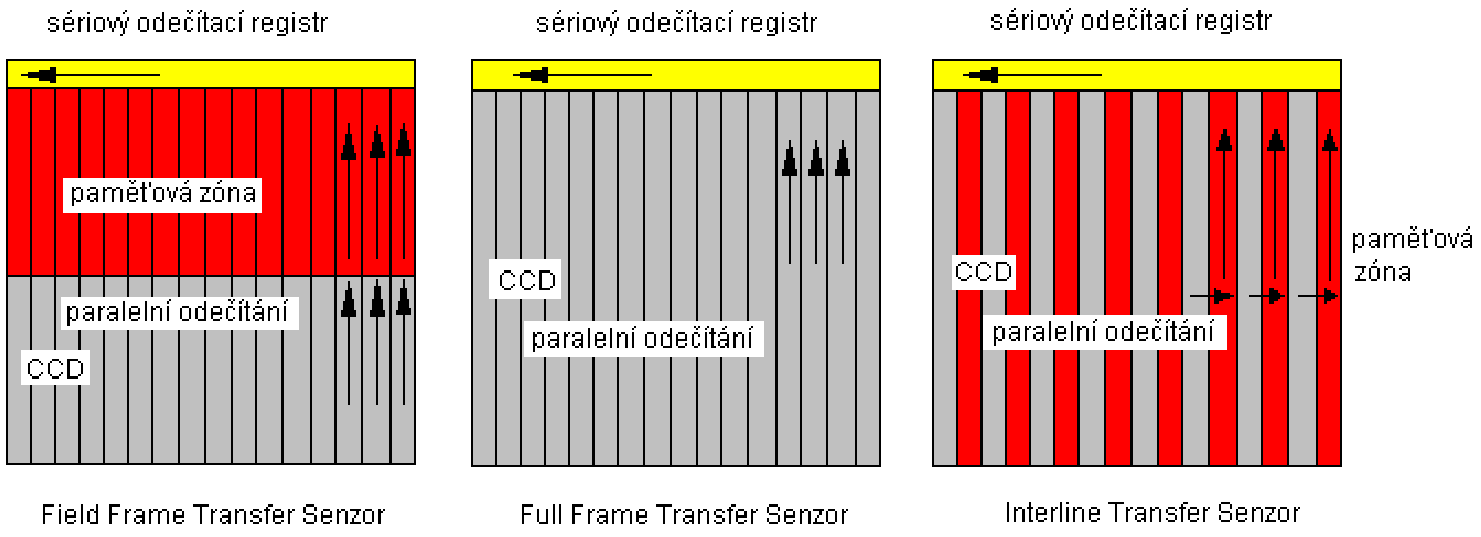
The most common type of detector is the CCD element. The name is derived from the name of the element in English "*Charge Coupled Device*".

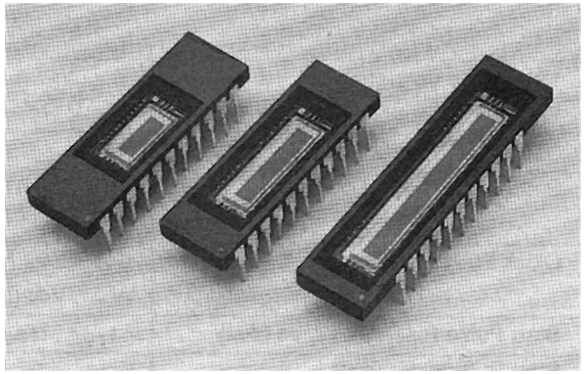
CMOS (*Complementary Metal Oxide Semiconductor*) is a transistor-based electronic component. Compared to a CCD, it is simpler to manufacture, smaller, up to 80% cheaper, and consumes less power than a CCD (only 1%!).

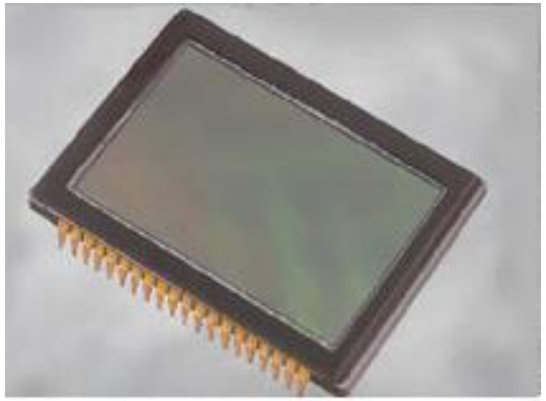
Photocell - the principle of its function is generally the same with CCD detectors, differing mainly in size



The origin of the image



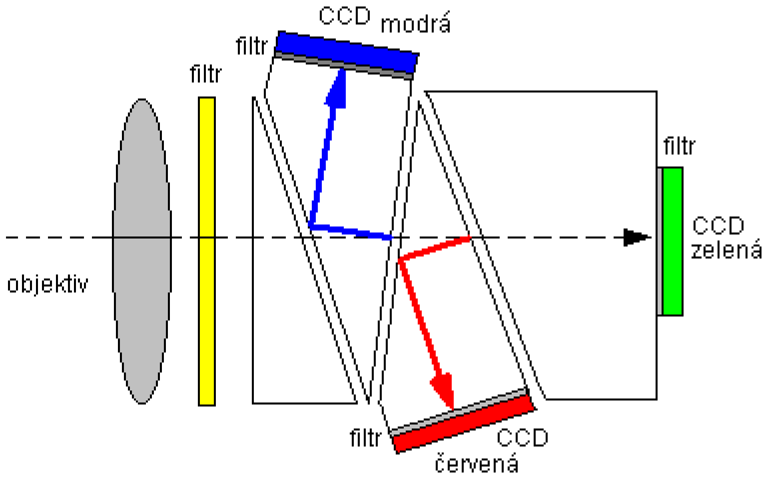




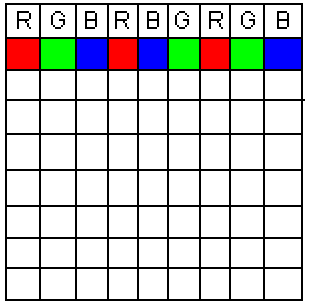
Origin of the colour image

Tree **pass camera**

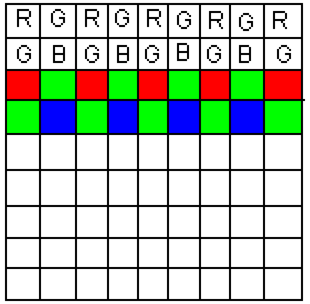
Three-sensor chamber



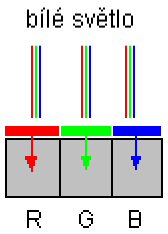
Single sensor (*one shot camera*)

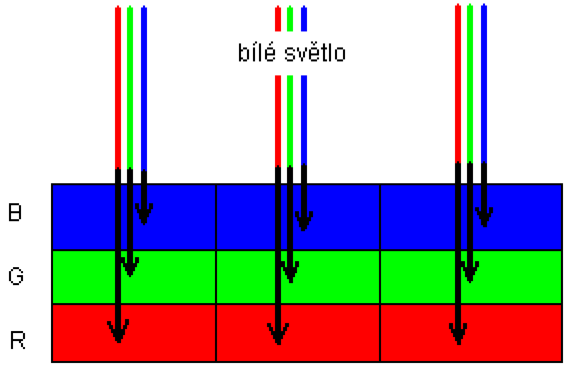


RGB pásová maska



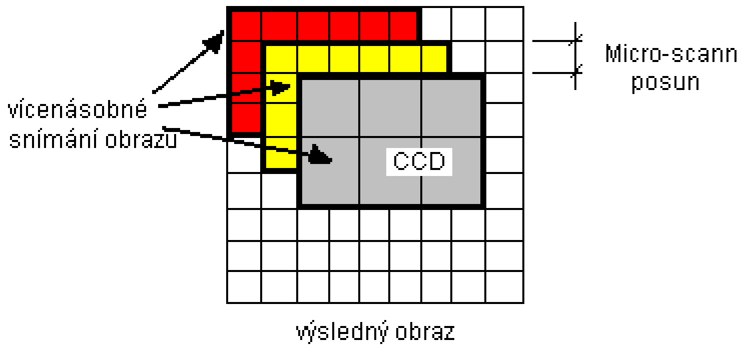
RGBG mozaiková maska



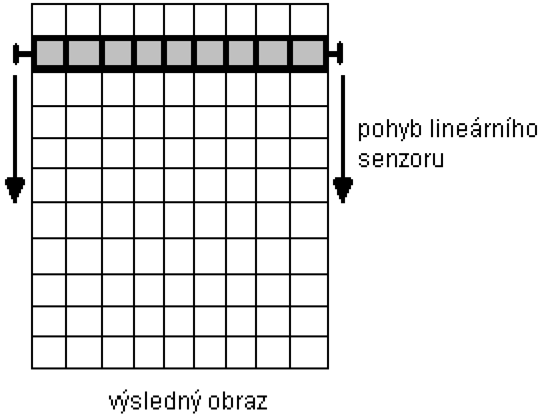


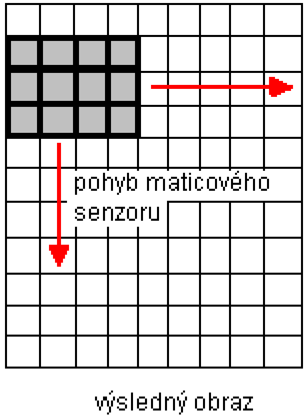
The origin of the image

Micro-scanning

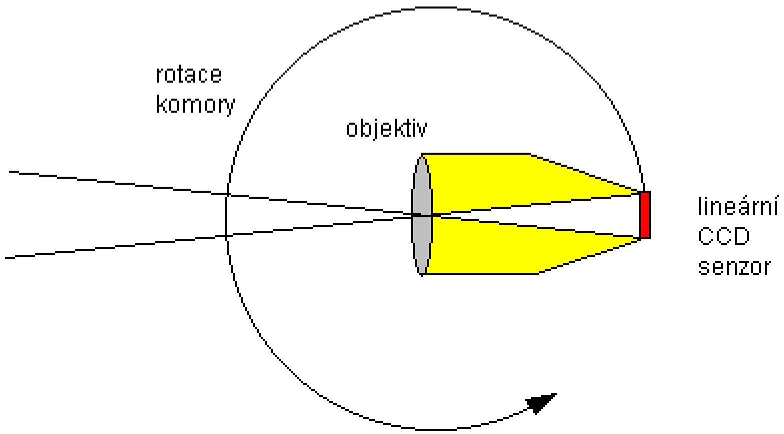


Macro-scanning

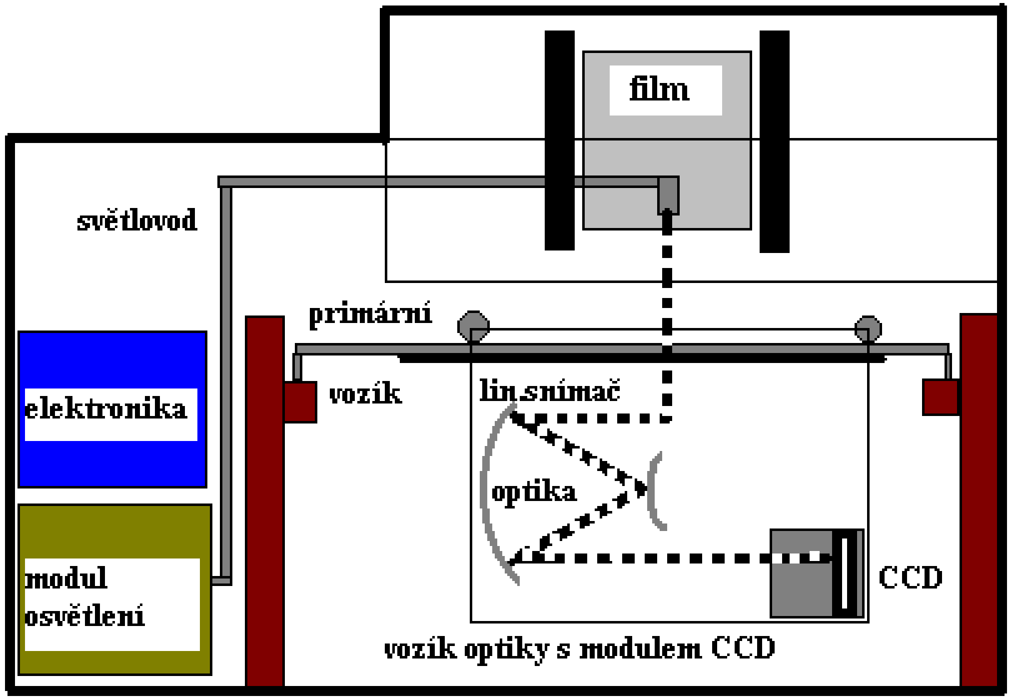




Panoramic chambers



Secondary digital image formation: by scanning

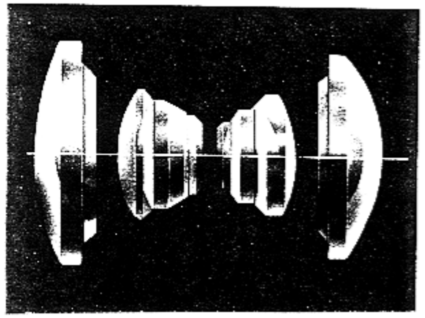


Scanner PhotoScan2001

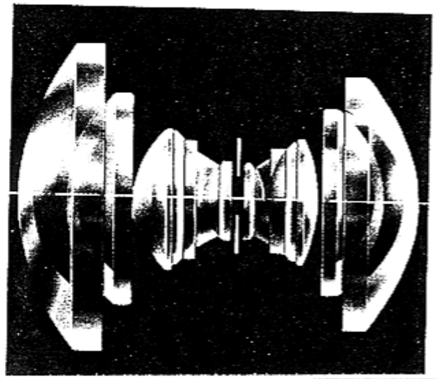
Photogrammetric cameras

Photogrammetric terrestrial cameras

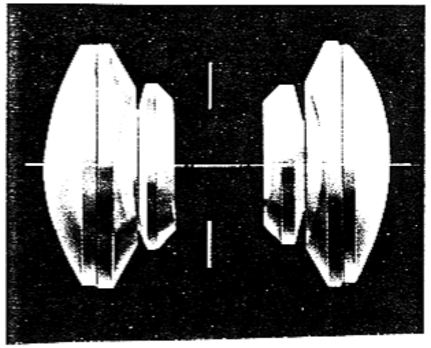
LAMEGON 8/100 B



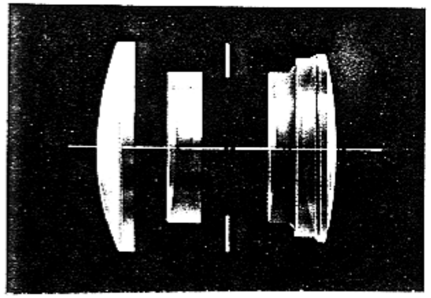
SUPERLAMEGON PI 5,6/64

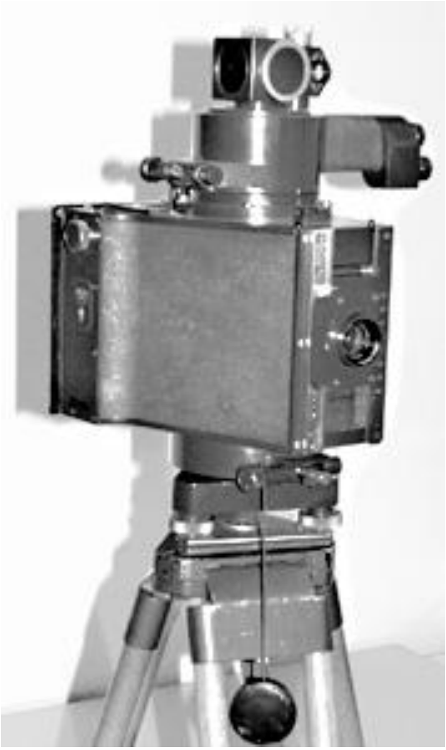


LAMETAR 8/200



LAMETAR 11/300





Photheo 13x18, UMK 201318 and Wild P31 chamber

Terrestrial photogrammetric **cameras**

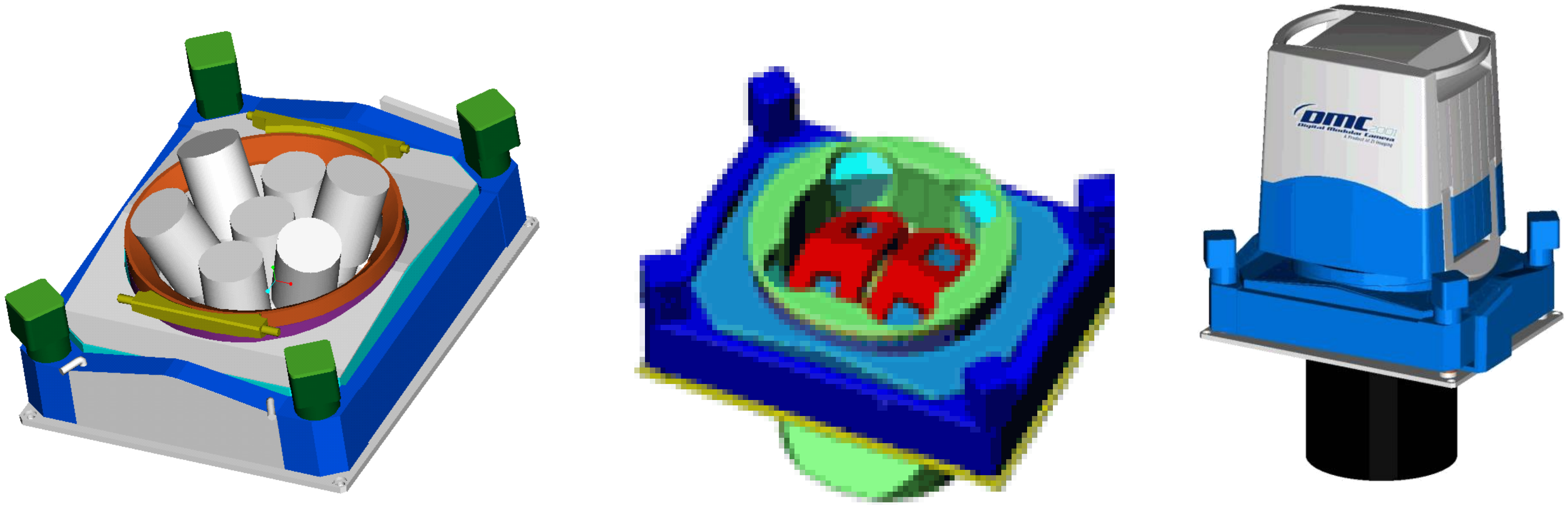
Heavy cameras UMK
(glass plates or planfil 13x18cm

Réseau camera RolleiMetric 6006
(roll film, 6x6cm)

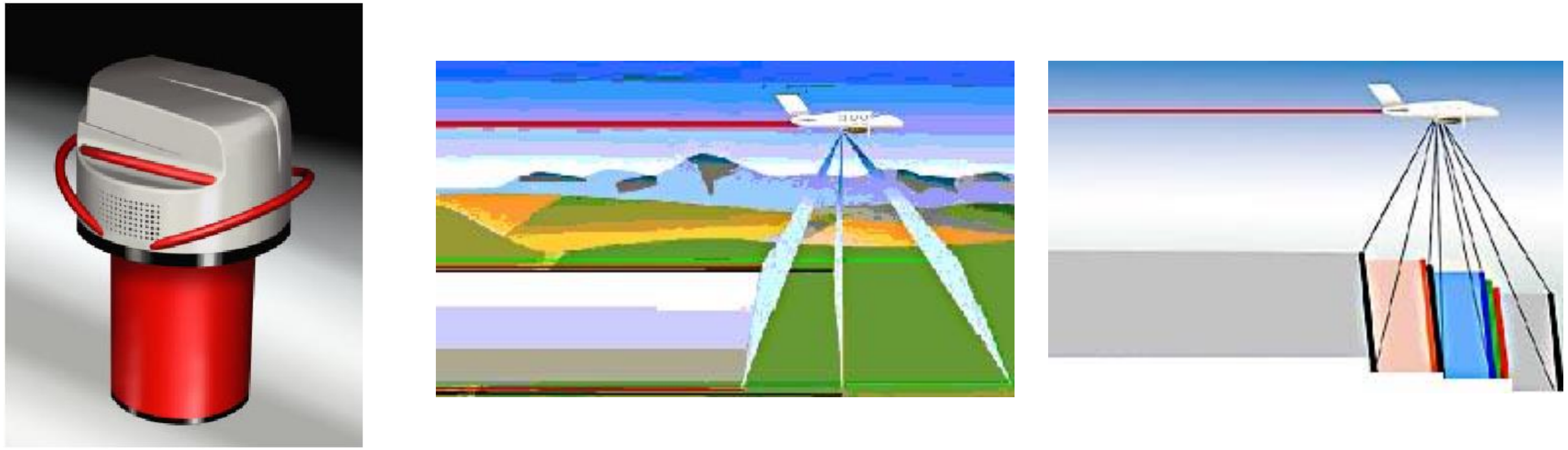
Photogrammetric cameras

Leica RC 30 camera
Zeiss RMK-TOP camera and TAS gyrostabilised platform,
LMK camera (Zeiss Jena)

digital air cameras



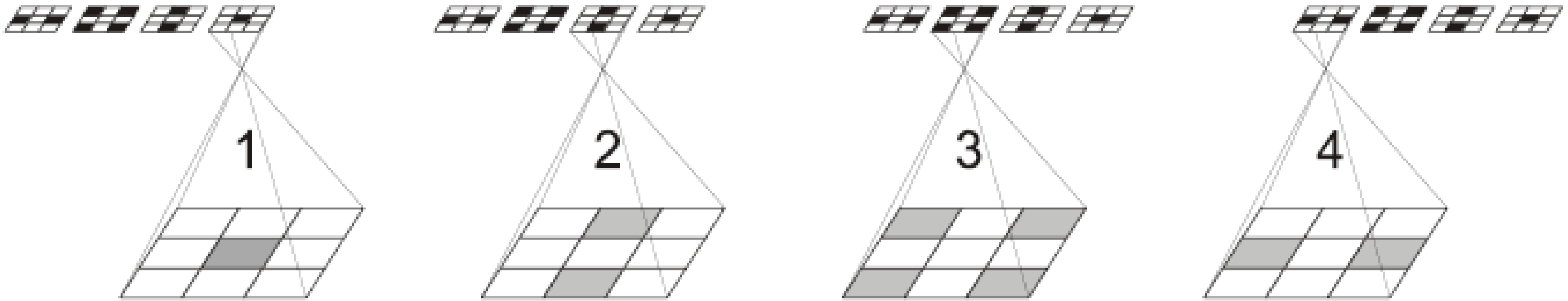
View of the DMC camera (top), original schematic of the gyrostabilized sensor part (left), current schematic (right)



Sensor head ADS40 , ADS40, surface scanning principle

digital air cameras

UltraCam D camera



Professional Digital Cameras

Canon EOS and Sigma cameras, circa 2003, 3.1MPix

- ***Current resolution options 10-30MPix***

Canon EOS 5D Mark IV 2020, 30.4MPix

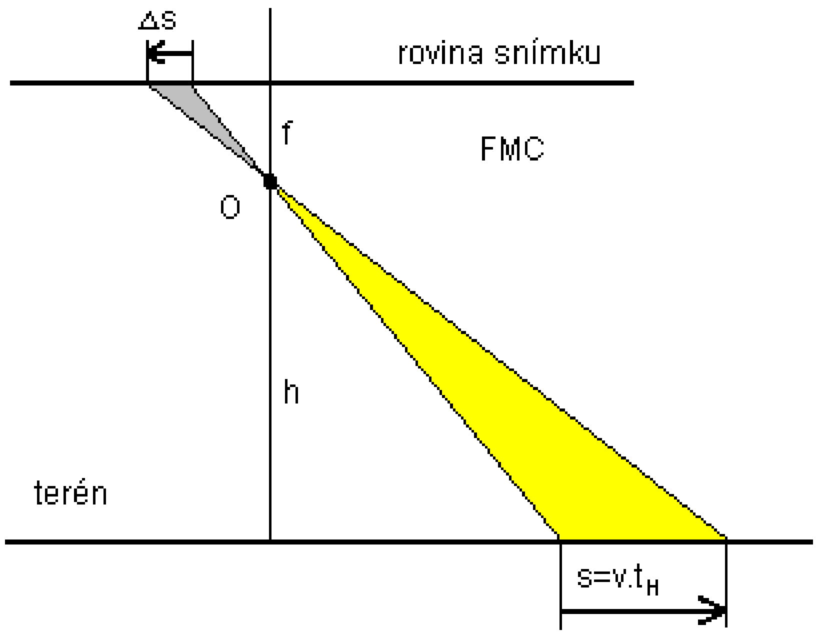
Airlocks - auxiliary equipment

Film *Motion Correction* (**FMC**) **: film movement at the moment of exposure, higher recording sharpness**

INS=IMU/GNSS (Inertial Navigation System, *Inertial Measurement Unit)* **equipment allows to determine the spatial orientation and position of the airlock in time. The values of the exterior orientation and acceleration elements can be determined directly**

Others: altimeter, overlay controller

The origin of the image blurring

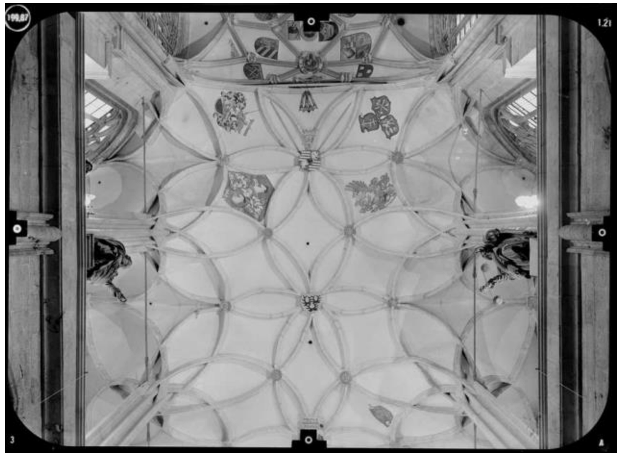


$$m_S = \frac{h}{f}, \Delta s = \frac{t_H \cdot v}{m_S}$$

Ex: m_S =10 000, t_H =0.01s, v=360km.h^{-1} : s=1/10000m=100m, i.e. a compensating shift at 10mm/s is necessary.

Classical photogrammetric records

frame marks



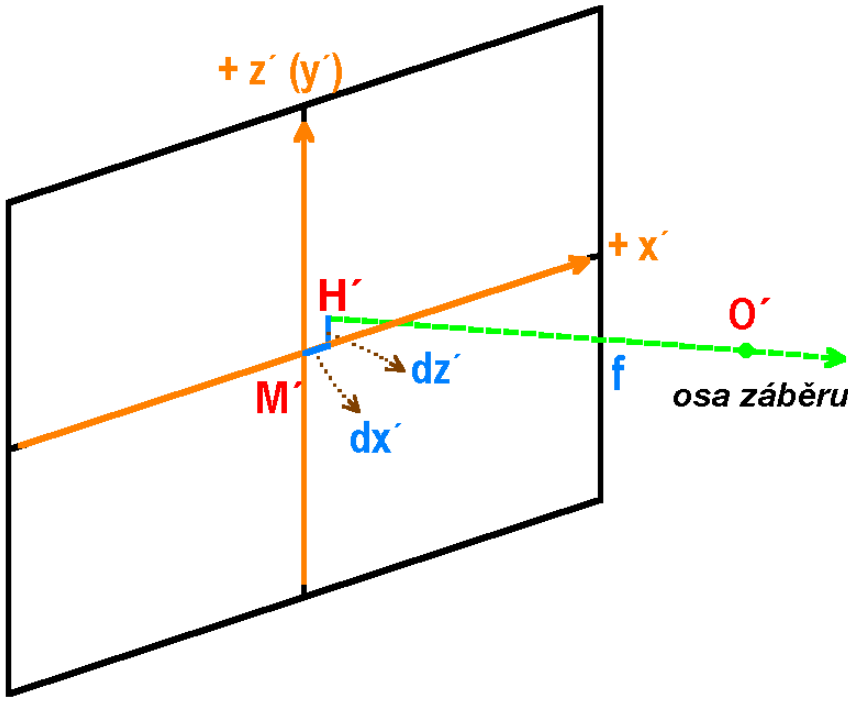
satellite photogrammetry

Ikonos 1, 1999 (1m PAN, 4m MSS);
today's civilian satellites 0.35-0.5m PAN

Calibration and calibration of cameras

Image from metric camera

Known elements of internal orientation, known position of frame markers



Position of the main image point H´

- coordinates dx´, dy´ (dz´)

with respect to M´ (to 0.01 mm)

camera constant f

- (to 0.01mm)

Knowledge of the course of distortion

- (if significant, i.e. greater than 0.01mm)

Calibration of cameras

The main steps of the validation and control for older conventional analogue ground cameras are:

•checking *f*, *dx´*, *dz´* - by calculation or in the optical laboratory

•checking the perpendicularity of the vertical axis to the axis of the libel (level the place where the axis and the centre of the projection point)

•orientation device - the aiming axis of the telescope should be flush with the photocamera's axis of view; the control is performed by aiming at a certain point and photographing it; for measuring its position on the image it should be shown in *H´* (beware of eccentricity!)

•vertical position of the frame (shooting two plumb lines)

•clamping device - checks whether the plate or film is actually pressed accurately against the marking frame; in case of non-adhesion, there are measurable differences in the position of the frame marks (tenths of mm)

•the plane of the image is to be parallel to the plane of the frame ($=0$ should apply), a photograph of two plumb lines is taken and the distances between the hinges at the top and bottom are measured on the images

•rectification of calipers (a common method see geodesy)

•deliberate device - like an ordinary theodolite (see surveying)

•For digital cameras, the whole process is limited to finding the internal orientation elements by calculating them from the calibration field and then usually to professional cleaning of the chamber in the service (cleaning of the optics - lenses are often susceptible to dust particles due to their complexity, cleaning of the sensor from dust)

Methods for determining the elements of internal orientation (pvo)

1. **Laboratory methods**
2. **Measurement and numerical determination of PVO**

a)determination of the elements of internal orientation without alignment

(b) methods for determining the elements of internal orientation with alignment

● Gruber method ● Hugershoff method ● Baeschlin method

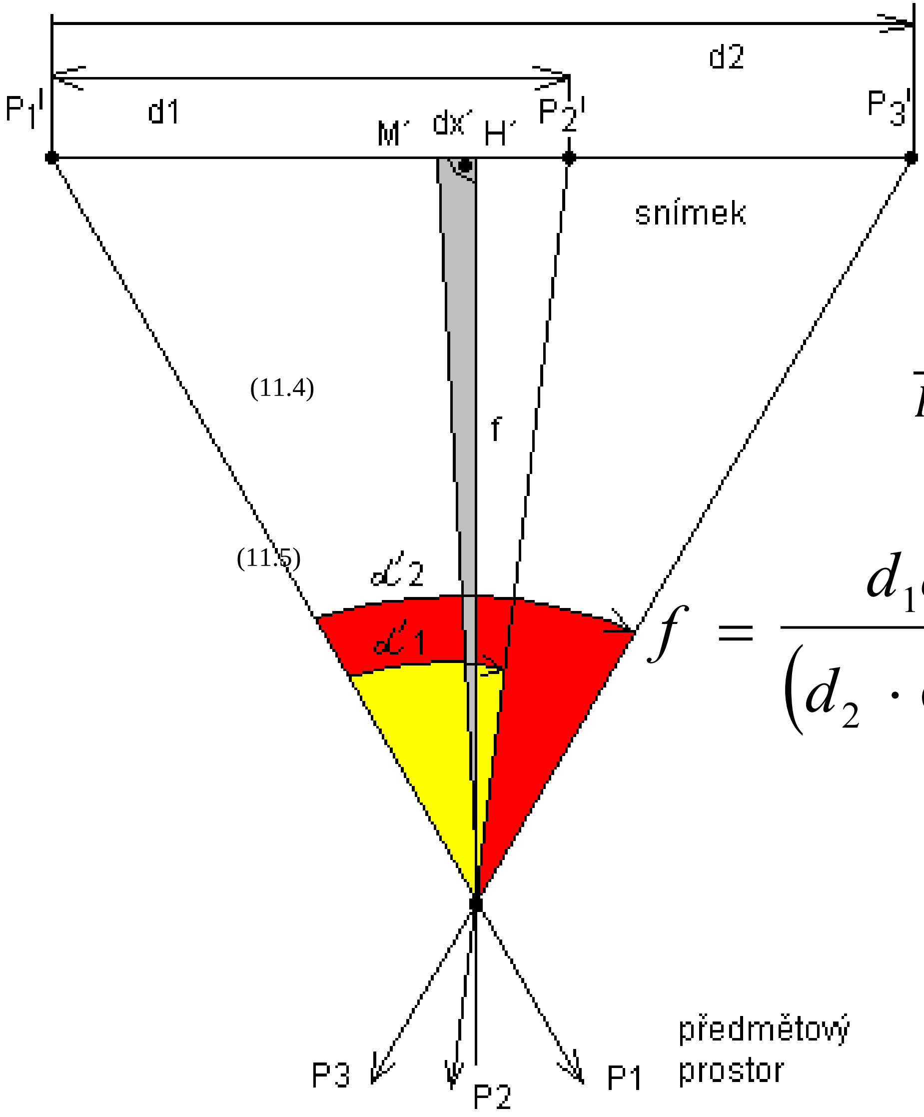
c)simultaneous determination of the elements of internal orientation and external orientation by means of Direct Linear Transformation (*DLT*)

d)simultaneous determination of the internal and external orientation elements in analytical methods (e.g. complex method, see below)

Procedure for determining internal orientation elements for analogue cameras

- Taking a test image
- We carefully horizon the test chamber with the rectified vials and take an image of the space with well identifiable distant points, preferably distributed along the horizon.
- Orientation of the outline of the horizontal directions
- Aim the warp of the horizontal directions at the selected points (with a second theodolite). The theodolite is centered over the center of the entrance pupil of the tested photocamera - but it is not identical to the spin axis of the instrument (i.e., the theodolite cannot simply be replaced by the measuring chamber in the tripod). The tripod with the tripod and the theodolite must be centred over a point displaced in the direction of the axis of view by the value of the eccentricity of the centre of the input pupil from the vertical rotational axis of the photochamber. The eccentricity values are known for commonly used photochambers.
- Measurement of image coordinates
- For the selected points, the image coordinates are measured on the comparator (read to at least 0.01 mm depending on the type of instrument used).
- Calculation
- The actual calculation can be done quickly without alignment, the methods with alignment are done on a personal computer

Determination of elements of internal orientation without alignment



$$d_1 = x_2' - x_1', d_2 = x_3' - x_1'$$

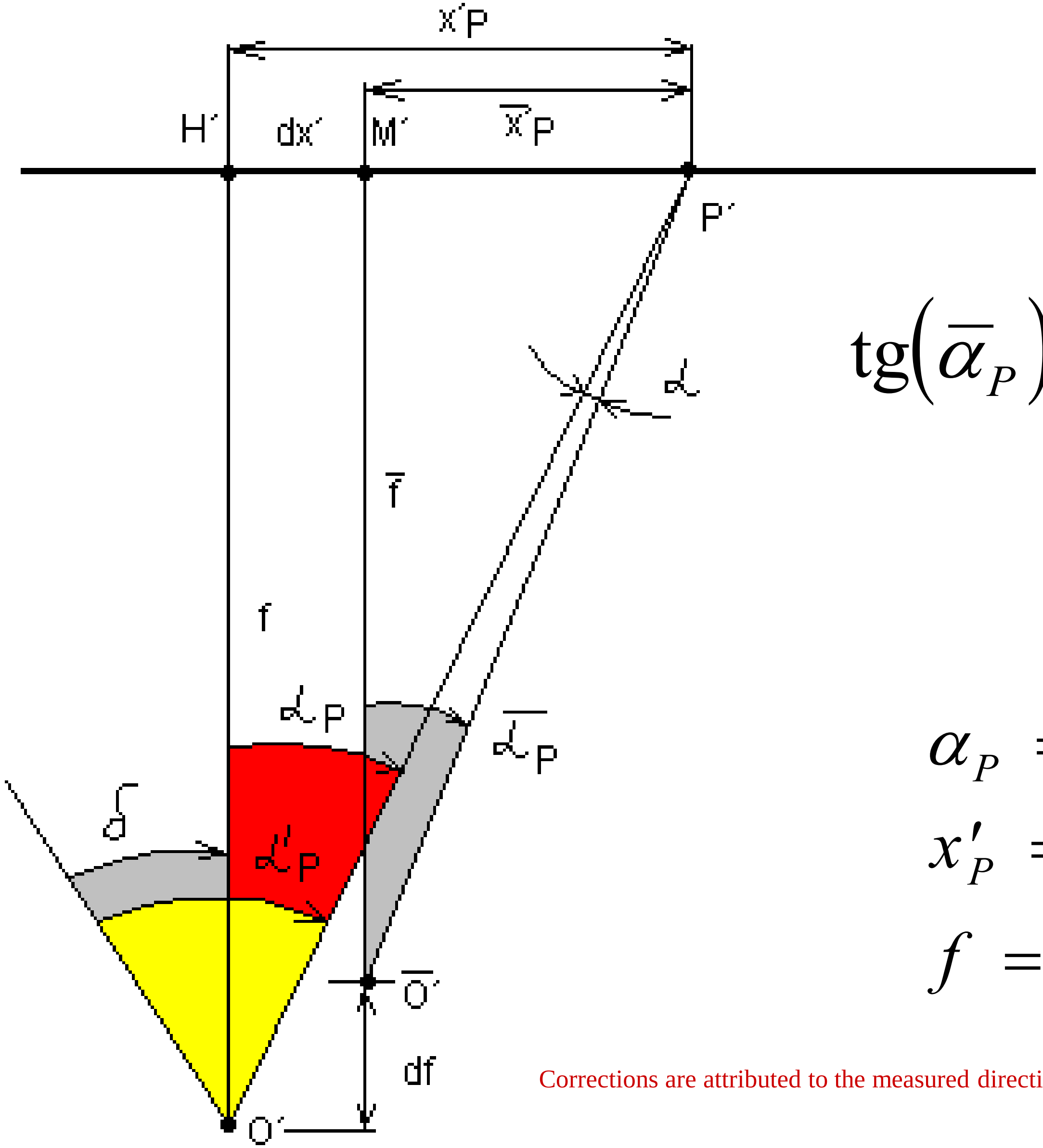
$$\alpha_1 = \psi_2 - \psi_1, \quad \alpha_2 = \psi_3 - \psi_1$$

$$\overline{H'P_1'} = f \cdot \frac{d_2 \cdot \operatorname{cotg}\alpha_2 - d_1 \cdot \operatorname{cotg}\alpha_1}{(d_2 - d_1)}, \overline{M'P_1'} = x_{P_1}'$$

$$f = \frac{d_1 d_2 (d_2 - d_1) \cdot (\operatorname{cotg}\alpha_1 - \operatorname{cotg}\alpha_2)}{(d_2 \cdot \operatorname{cotg}\alpha_2 - d_1 \cdot \operatorname{cotg}\alpha_1)^2 + (d_2 - d_1)}$$

$$d_x' = \overline{H'P_1'} - \overline{M'P_1'}$$

Accurate methods of numerical determination of pvo
Gruber method



$$\mathrm{tg}\left(\bar{\alpha}_P\right) = \frac{\overline{x'_P}}{\bar{f}}, \qquad \mathrm{tg}\left(\alpha_P\right) = \frac{x'_P}{f}$$

$$\alpha_P = \alpha'_P - \delta = \bar{\alpha}_P - d\alpha$$

$$x'_P = \overline{x'_P} + dx'$$

$$f = \bar{f} + df$$

Corrections are attributed to the measured directions

Gruber **method**

$$\operatorname{tg}\left(\alpha_P\right) = \operatorname{tg}\left(\bar{\alpha}_P + d\alpha\right) = \frac{x'_P}{f} = \frac{\overline{x'_P} + dx'}{\bar{f} + df}$$

$$\operatorname{tg}\left(\bar{\alpha}_P\right) + \frac{1}{\cos^2\left(\bar{\alpha}_P\right)}d\alpha = \frac{\overline{x'_P}}{\bar{f}} + \frac{1}{\bar{f}}dx' - \frac{\overline{x'_P}}{\bar{f}^{\,2}}df$$

$$\frac{1}{\cos^2\left(\bar{\alpha}_P\right)} = 1 + \operatorname{tg}^2\left(\bar{\alpha}_P\right) = 1 + \frac{\overline{x'_P}^{\,2}}{\bar{f}^{\,2}} = \frac{\bar{f}^{\,2} + \overline{x'_P}^{\,2}}{\bar{f}^{\,2}}$$

$$\frac{x^{\bar{\prime}}{}_P}{\bar{f}\,\frac{1}{\cos^2(\bar{\alpha}_P)}\,\frac{x^{\bar{\prime}}{}_P}{\bar{f}\,\frac{1'}{\bar{\bar{f}}}\,\frac{x^{\bar{\prime}}{}_P}{\bar{f}^2\,\frac{\bar{f}^2}{\left(x^{\bar{\prime}}{}_P{}^{\bar{2}^2}\,()\right)}}}}$$

$$d\alpha = \frac{\bar{f}}{\left(x^{\bar{\prime}}{}_P{}^{\bar{2}^2}\,()\right)'\,\frac{x^{\bar{\prime}}{}_P}{\left(x^{\bar{\prime}}{}_P{}^{\bar{2}^2}\,()\right)df}}$$

$$\alpha_P = \alpha'_P - \delta = \bar{\alpha}_P - d\alpha$$

$$d\alpha = \alpha'_P - \delta - \bar{\alpha}_P$$

$$v_{\alpha_P} = \delta + \frac{\bar{f}}{\left(x^{\bar{\prime}}{}_P{}^{\bar{2}^2}\,()\right)'\,\frac{x^{\bar{\prime}}{}_P}{\left(x^{\bar{\prime}}{}_P{}^{\bar{2}^2}\,()\right)df + (\bar{\alpha}_P - \alpha'_P)}}$$

Hugershoff method

attributes corrections to the measured image coordinates:

$$\operatorname{tg}\left(\bar{\alpha}_P - \delta\right) = \frac{\overline{x'_P} + v_{x'} + dx'}{\bar{f} + df}$$

$$\operatorname{tg}\left(\alpha'_P\right) - \frac{1}{\cos^2\left(\alpha'_P\right)}\delta = \frac{\overline{x'_P}}{\bar{f}} + \frac{1}{\bar{f}}\left(v_{x'} + dx'\right) - \frac{\overline{x'_P}}{\bar{f}^2}df$$

the following applies:

$$\operatorname{tg}\left(\bar{\alpha}_P\right) = \frac{\overline{x'_P}}{\bar{f}} \qquad \operatorname{tg}\left(\alpha'_P\right) - \operatorname{tg}\left(\bar{\alpha}_P\right) \cong \frac{1}{\cos\left(\bar{\alpha}_P\right)}\left(\alpha'_P - \bar{\alpha}_P\right) \qquad \frac{1}{\cos^2\left(\alpha'_P\right)} \cong \frac{1}{\cos^2\left(\bar{\alpha}_P\right)}$$

$$\left(\alpha'_P - \bar{\alpha}_P - \delta\right)\frac{1}{\cos^2\left(\bar{\alpha}_P\right)} = \frac{1}{\bar{f}}\left(v_{x'} + dx'\right) - \frac{\overline{x'_P}}{\bar{f}^2}df$$

$$\left(\alpha'_P - \bar{\alpha}_P - \delta\right) = \frac{\bar{f}}{\bar{f}^2 + \overline{x'}^2_P}\cdot v_{x'} + \frac{\bar{f}}{\bar{f}^2 + \overline{x'}^2_P}\cdot dx' - \frac{\overline{x'_P}}{\bar{f}^2 + \overline{x'}^2_P}df$$

$$v_{x'} = -dx' + \frac{\overline{x'_P}}{\bar{f}}df - \frac{\bar{f}^2 + \overline{x'}^2_P}{\bar{f}}\cdot\delta - \frac{\bar{f}^2 + \overline{x'}^2_P}{\bar{f}}\left(\alpha'_P - \bar{\alpha}_P\right)$$

Direct Linear Transformation (DLT)

$$x' = x_0' - f\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

$$z' = z_0' - f\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

$$x' = \frac{\hat{a}_1 X + \hat{a}_2 Y + \hat{a}_3 Z + \hat{a}_4}{\hat{c}_1 X + \hat{c}_2 Y + \hat{c}_3 Z + \hat{c}_4}, \hat{a}_1 = x_0' r_{13} - f \cdot r_{11}, \hat{a}_2 = x_0' r_{23} - f \cdot r_{21}, \hat{a}_3 = \ldots$$

$$z' = \frac{\hat{b}_1 X + \hat{b}_2 Y + \hat{b}_3 Z + \hat{b}_4}{\hat{c}_1 X + \hat{c}_2 Y + \hat{c}_3 Z + \hat{c}_4}$$

$$x' = \frac{a_1 X + a_2 Y + a_3 Z + a_4}{c_1 X + c_2 Y + c_3 Z + 1}$$

$$z' = \frac{b_1 X + b_2 Y + b_3 Z + b_4}{c_1 X + c_2 Y + c_3 Z + 1}$$

$$f_x = \sqrt{\left((a_1^2 + a_2^2 + a_3^2)d^2 - {x'}_0^2\right)}$$

$$f_z = \sqrt{\left((b_1^2 + b_2^2 + b_3^2)d^2 - {z'}_0^2\right)}$$

$$d^2 = \frac{1}{c_1^2 + c_2^2 + c_3^2}$$

$$x_0' = dx' = \left(a_1 c_1 + a_2 c_2 + a_3 c_3\right) \cdot d^2$$

$$z_0' = dz' = \left(b_1 c_1 + b_2 c_2 + b_3 c_3\right) \cdot d^2$$

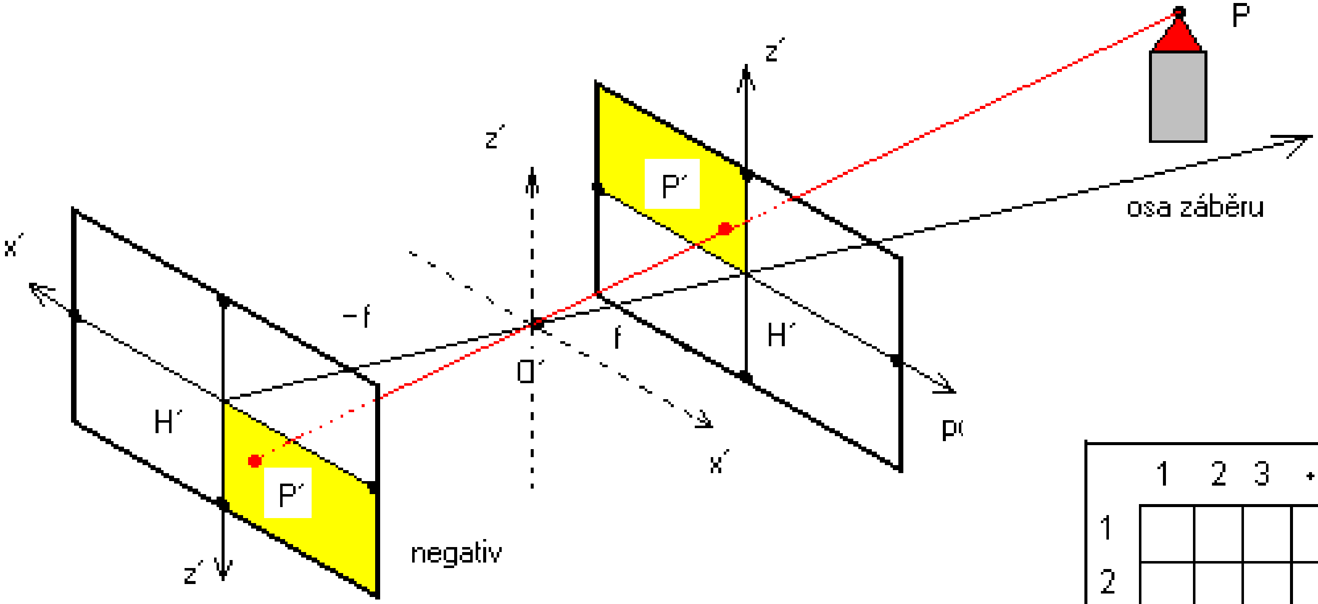
$$f = \frac{f_x + f_z}{2}$$

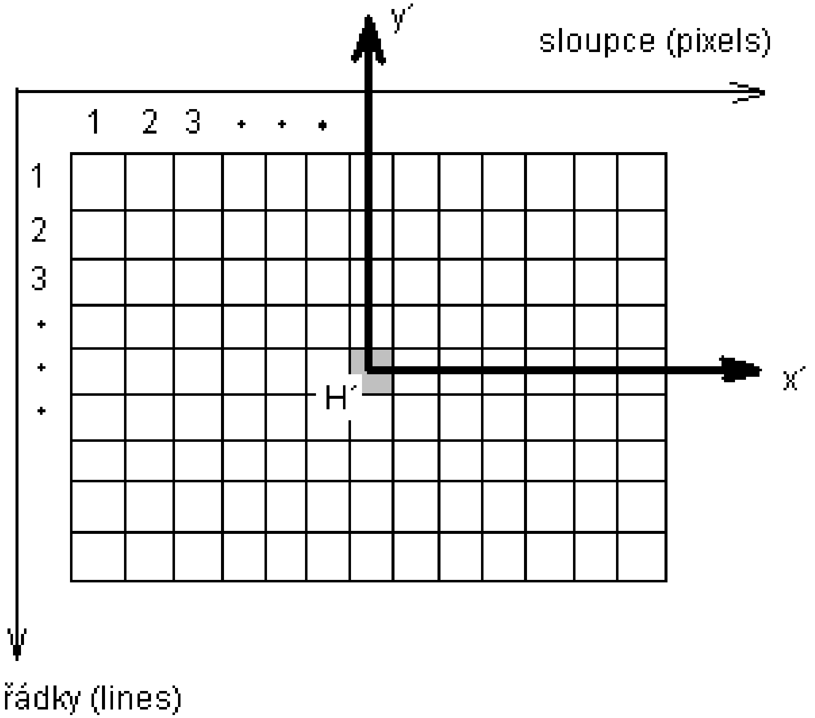
$$v_{x'} = a_1 X + a_2 Y + a_3 Z + a_4 - c_1 X x' - c_2 Y x' - c_3 Z x' - x'$$

$$v_{z'} = b_1 X + b_2 Y + b_3 Z + b_4 - c_1 X z' - c_2 Y z' - c_3 Z z' - z'$$

Digital image

Coordinates are in pixels (columns, rows)





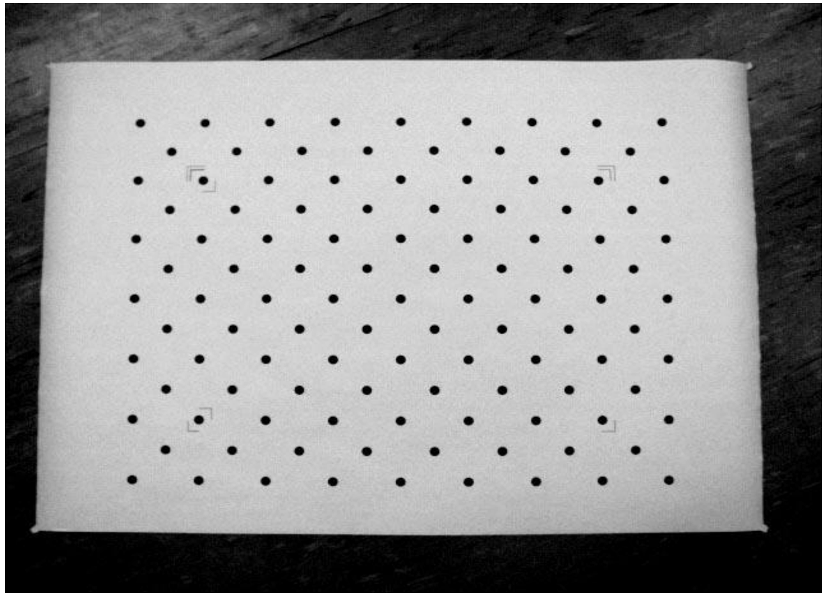
Instead of frame marks: corner pixels

Calibration of the cameras

If we do not know the elements of the internal orientation, we need to calculate them by measuring

Calibration methods

the most common calibration using a calibration field or in a specialized optical laboratory



aerial photogrammetry

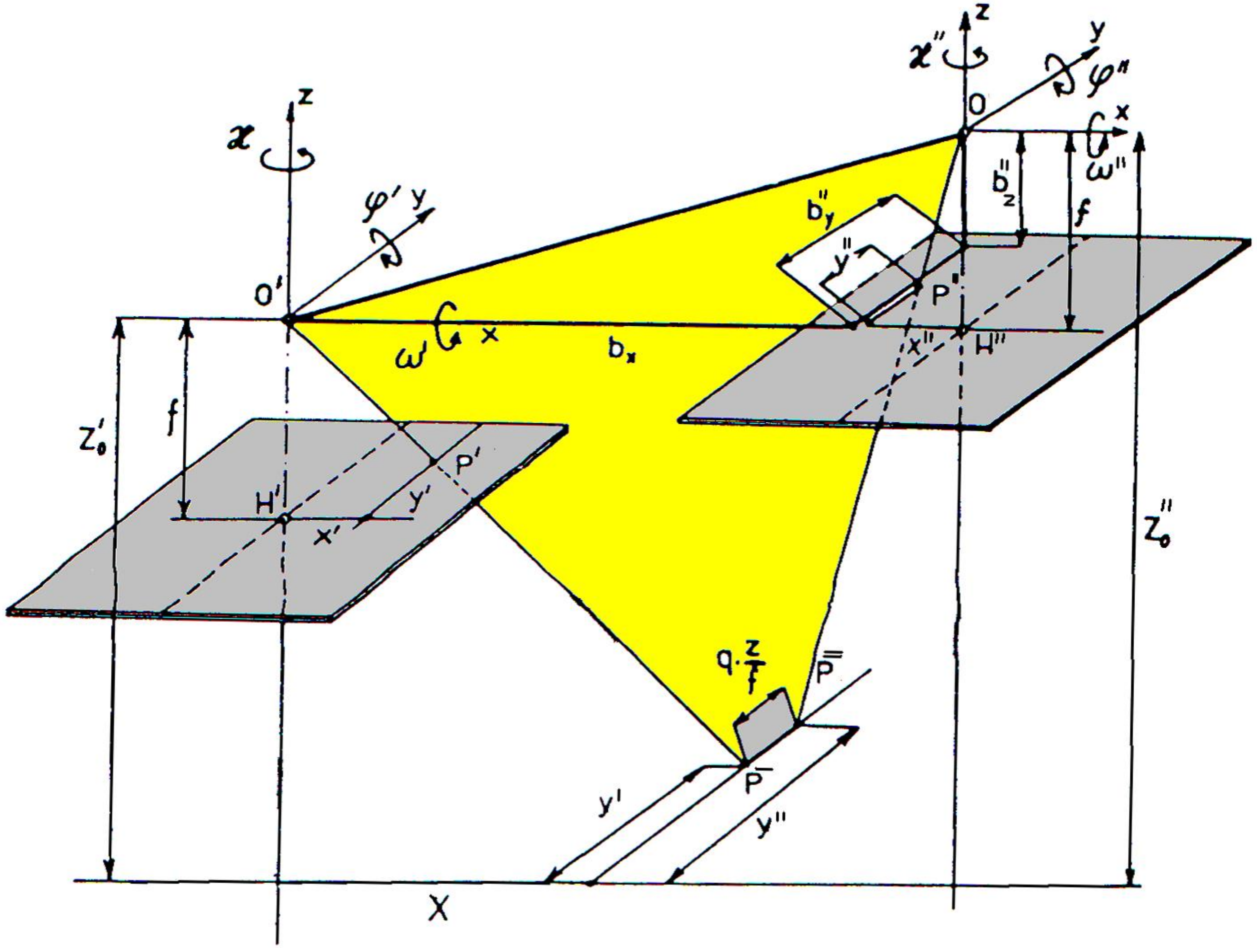
Aerial stereophotogrammetry

In aeronautical applications, standardisation of exterior orientation elements cannot be ensured

$$X = X_o + (Z - Z_o) \cdot \frac{r_{11} \cdot (x' - x'_o) + r_{12} \cdot (y' - y') - r_{13} \cdot f}{r_{31} \cdot (x' - x'_o) + r_{32} \cdot (y' - y') - r_{33} \cdot f}$$

$$Y = Y_o + (Z - Z_o) \cdot \frac{r_{21} \cdot (x' - x'_o) + r_{22} \cdot (y' - y') - r_{23} \cdot f}{r_{31} \cdot (x' - x'_o) + r_{32} \cdot (y' - y') - r_{33} \cdot f}$$

R $\neq$ **E**



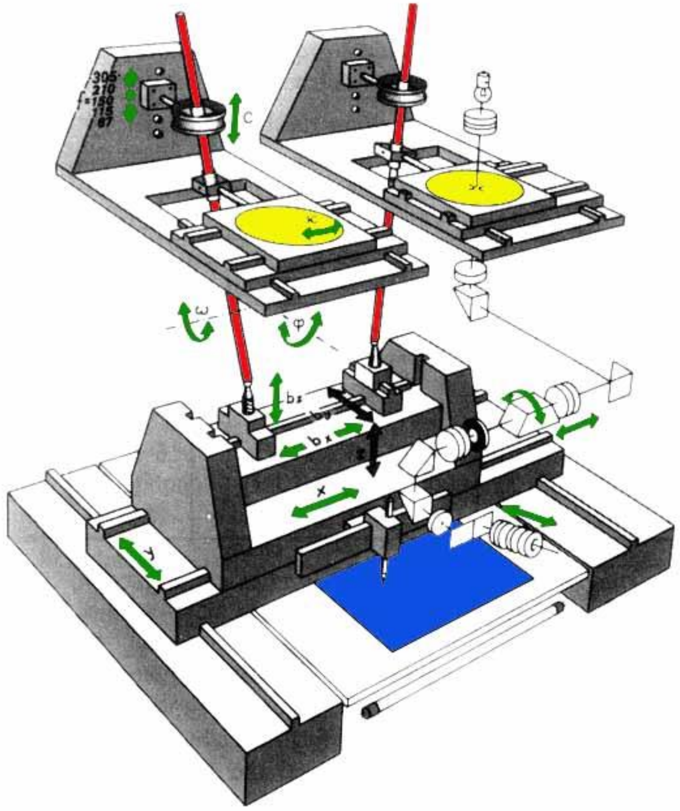
Photogrammetric technologies for image content evaluation

Analog technology
(obsolete, not used)

Numerical (analytical) technology
- using analogue machines and partial counting steps (classical)

Analytical technology
-cannot be solved without a computer
- present

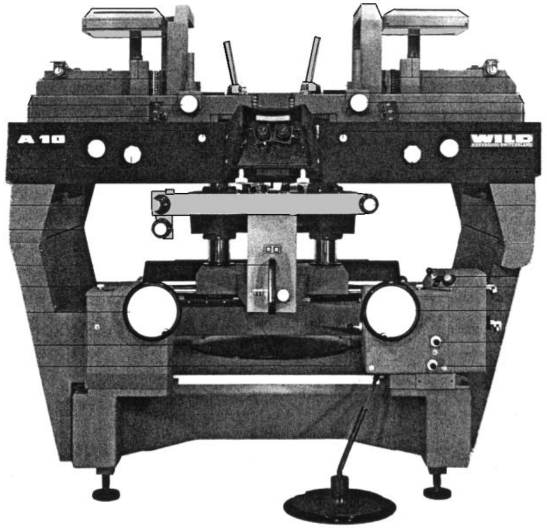
Spatial evaluation using streptophotogrammetric devices

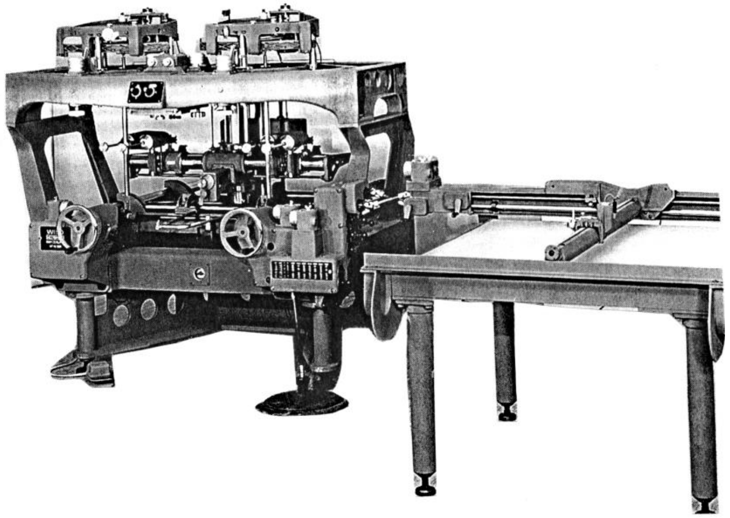


Analog plotters

-produced until 1986 (Wild) and 1990 (Zeis Jena)
- a complex and precise mechanical device, allowing the restoration of the elements of the external orientation
- a realistic stereoscopic model is created by tilting and shifting the images
- latest models with computer support
- model coordinates are controlled
- currently obsolete

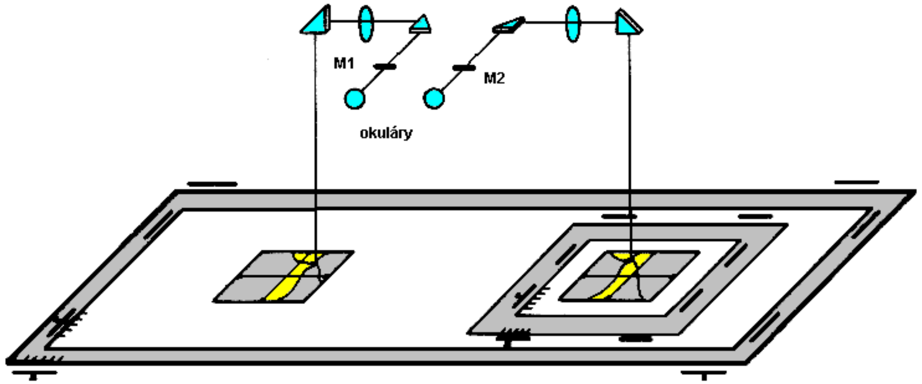
Analog plotters

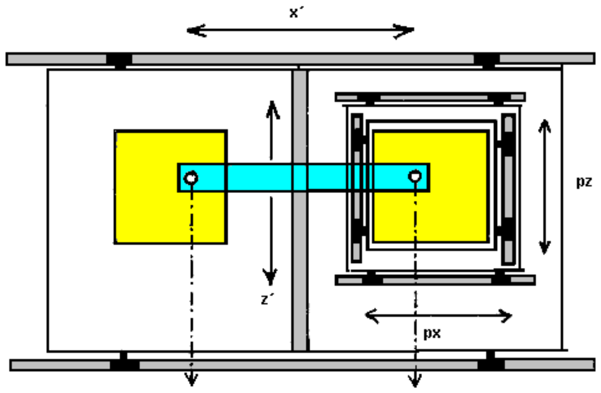


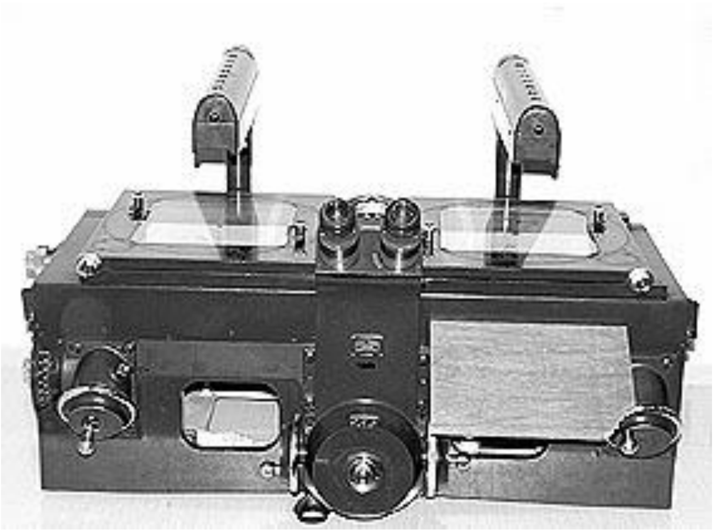


Stereometrograph, Topocart (Zeiss Jena) A-10 and A-7 (Wild)

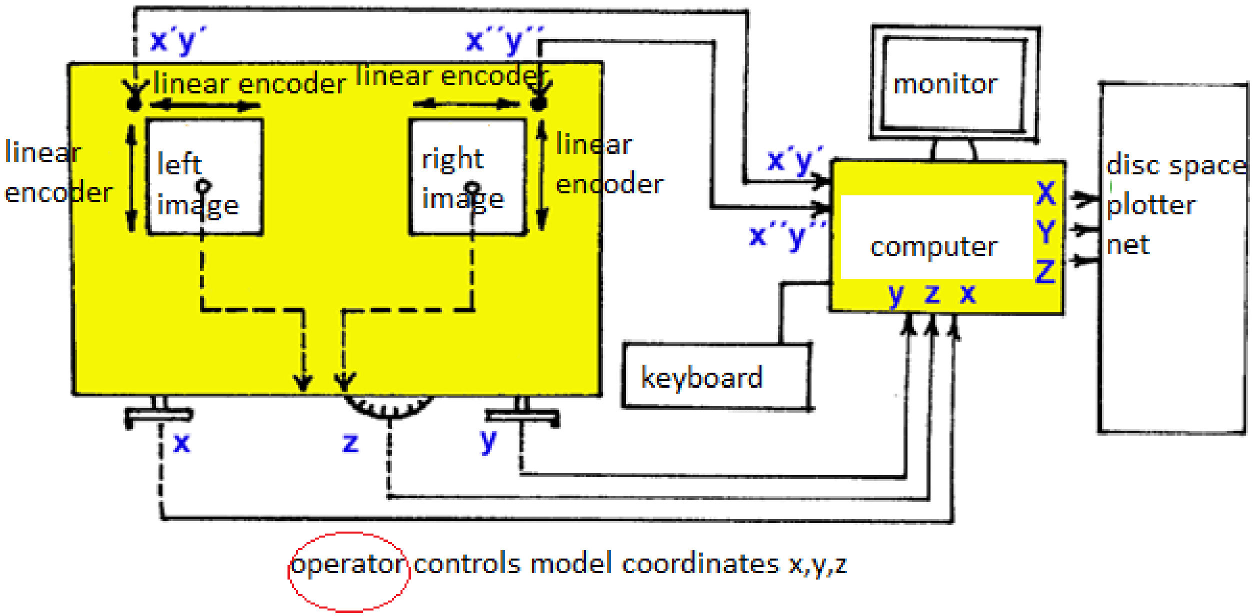
analogue comparators







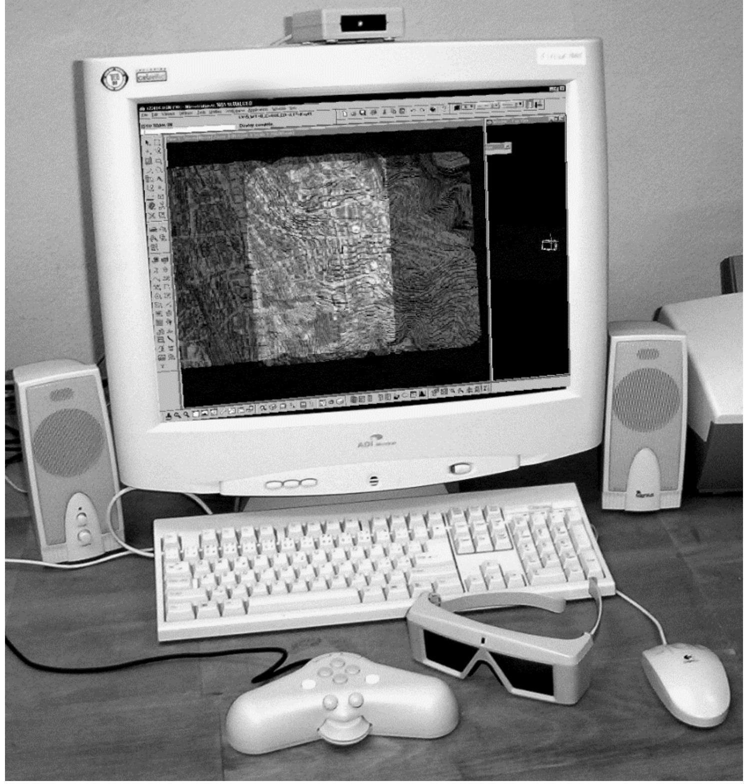
Analytical plotters



- measured on real images, computer required
- no real model is created
- the most accurate fm method
- model coordinates are controlled, converted to frame coordinates on the computer
- mobile image carriers are set to the calculated image coordinates

BC-1 (Wild, 1985) SD 2000 (Leica, 1995)

Digital stations



Imagestation Unix Imagestation SSK
Helava (Leica) VSD

aerial photogrammetry

a)photogrammetric flight project $m_S = \frac{h}{f}$ $m_Z = \frac{h}{b} m_S \cdot m_P,$ $m_{XY} = m_S \cdot m_{x'y'}$

b)Imaging flight design (analogue cameras, digital cameras, add-on devices)

c)Ground works (control points, signalling)

d)Aerotriangulation (the goal of today's aerotriangulation is mainly the creation of new insertion points directly from the images for later detailed evaluation in optimal positions, the alignment of the whole block, which allows the continuity of the evaluation of models in sequence and also the accurate calculation of the elements of the internal orientation of each image).

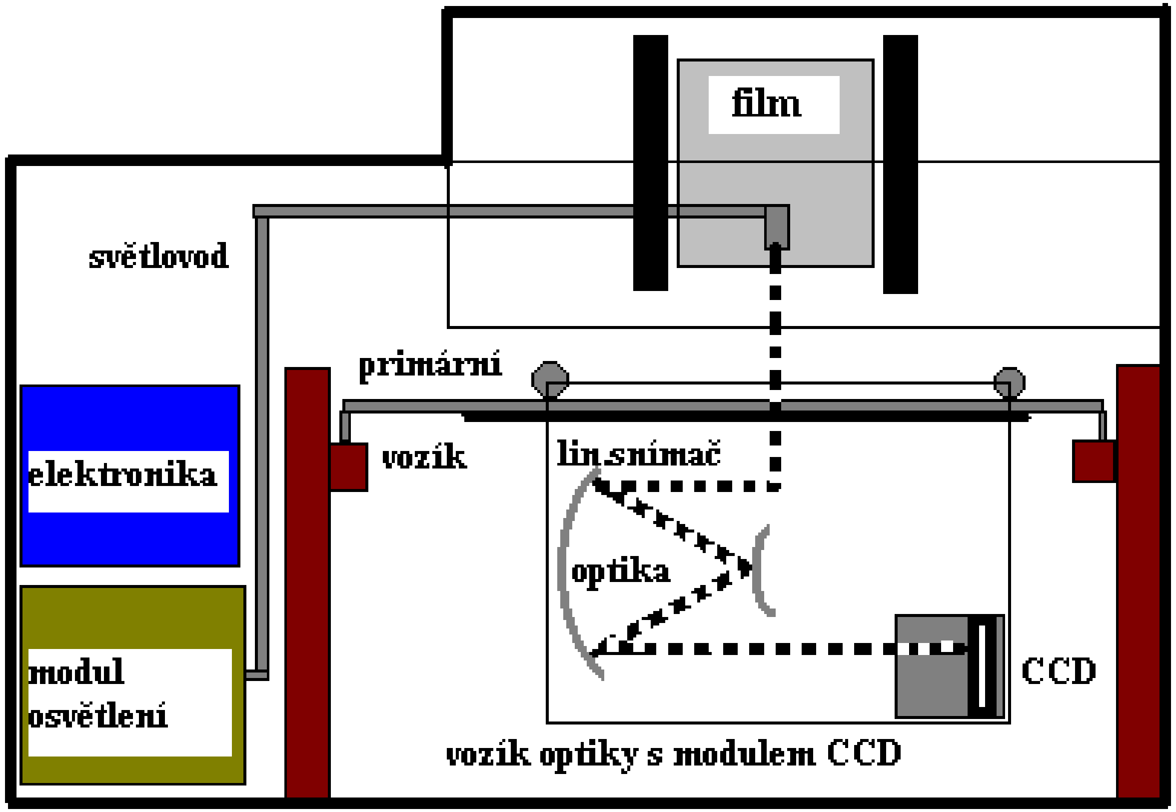
e)Classification and local investigation

f)Evaluation of the image content

- analogue, empirically
- analogically using numerical or semi-analytical methods
- by purely analytical methods
- digitally

aerial photogrammetry

scanning - digitization of images



aerial photogrammetry

Image orientation:

•*interior orientation (elements of internal orientation)*

•*Exterior orientation (coordinates of the centre of the entrance pupil X_0 , Y_0 , Z_0 , then inclinations „)*

For stereo-evaluation of photogrammetric images, the exterior orientation is defined as:

- direct transformation relation based on complex solution - beam alignment
- older step-by-step procedure called:

a) ***relative orientation*** *(relative orientation between the two stereo images, forming an arbitrary spatially oriented virtual stereo model)*
b) ***absolute orientation*** *(scaling, rotation and shifting of the model to the geodetic reference system)*

aerial stereophotogrammetry

Evaluation of stereo images with known exterior orientation parameters

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

$$X = X_0 + (Z - Z_0)\frac{r_{11}(x' - x'_0) + r_{12}(y' - y'_0) - r_{13} f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33} f}$$

$$Y = Y_0 + (Z - Z_0)\frac{r_{21}(x' - x'_0) + r_{22}(y' - y'_0) - r_{23} f}{r_{31}(x' - x'_0) + r_{32}(y' - y'_0) - r_{33} f}$$

$$X = X_{01} + (Z - Z_{01})k_{x1}$$

$$Y = Y_{01} + (Z - Z_{01})k_{y1}$$

$$X = X_{02} + (Z - Z_{02})k_{x2}$$

$$Y = Y_{02} + (Z - Z_{02})k_{y2}$$

$$Z = \frac{X_{02} - Z_{02}k_{x2} + Z_{01}k_{x1} - X_{01}}{k_{x1} - k_{x2}}$$

aerial stereophotogrammetry

Evaluation of stereo images with unknown exterior orientation parameters

In principle, the determination of the external orientation elements can be divided into methods:

<u>Empirical (not used today)</u>

a) relative orientation (the basis is the gradual manual removal of vertical parallaxes on landmarks with direct introduction of empirical corrections)

- relative orientation of the independent pair

- relative orientation when attaching an image

b) absolute orientation (shift, rotate, scale, height adjustment, tilt of the model)

<u>Numerical (classical)</u>

a) Relative orientation (the basis is the measurement of vertical parallaxes at a minimum of five orientation points with a stereocomparator or in an evaluation device, followed by the calculation of unknown elements):

- relative orientation of the independent pair (change of orientation of both images)

- relative orientation when attaching a frame (change the orientation of only one frame)

b) absolute orientation (calculation of elements using linearized relations with subsequent equalization)

<u>Analytical</u>

Use of a direct relationship between the measured image coordinates and the geodetic coordinates of the reference system; the basis is the measurement of image coordinates.

<u>- complex solution </u>(the entire orientation is solved in one step, including calculation of detailed points, bundle alignment)

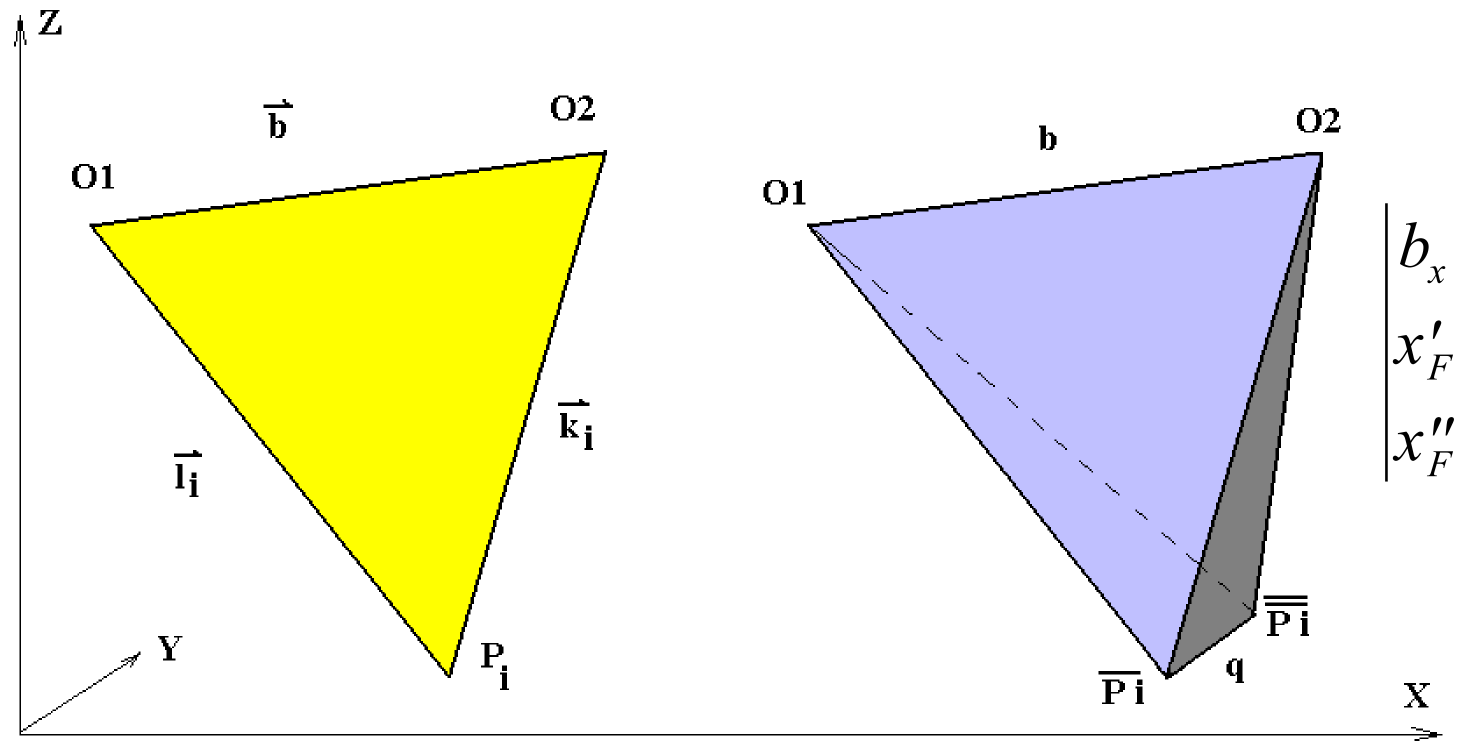
- staged solution (solution is divided into steps, block alignment)

aerial photogrammetry

When determining the elements of the relative orientation numerically, we can rely on various conditions, the satisfaction of which guarantees the solution of the relative orientation. The most well-known methods include the **complanarity condition** and the **zero vertical parallax condition**. In general, in relative orientation we can identify **five** unknown elements for which at least five equations must be constructed.

Condition of complanarity

$$(\mathbf{b}, \mathbf{k_i}, \mathbf{l_i}) = 0, \qquad i = 1, \ldots, 5$$



$$\begin{vmatrix} b_x & b_y & b_z \\ x'_F & y'_F & z'_F \\ x''_F & y''_F & z''_F \end{vmatrix} = 0$$

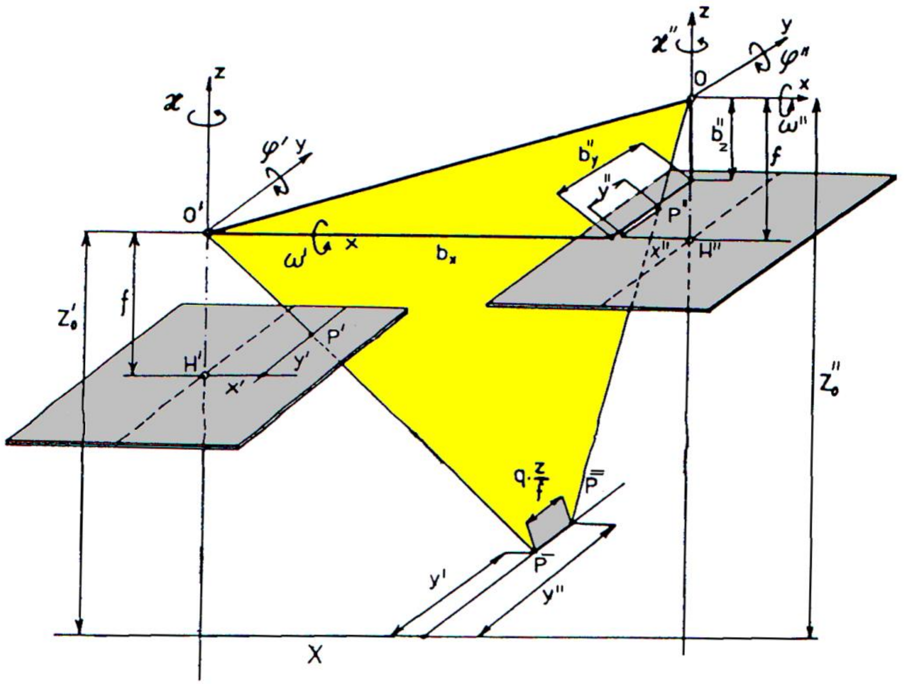
aerial photogrammetry

Zero vertical parallax condition

$$x'_s = -f\,\frac{r_{11}x' + r_{12}y' - r_{13}f}{r_{31}x' + r_{32}y' - r_{33}f}\,, \quad y'_s = -f\,\frac{r_{21}x' + r_{22}y' - r_{23}f}{r_{31}x' + r_{32}y' - r_{33}f}$$

$$\Delta x' = x'_s - x' = -y'd\kappa' - \left(f + \frac{x'^{\,2}}{f}\right)d\varphi' + \frac{x'\,y'}{f}\,d\omega' + db'_x + \frac{x'}{f}\,db'_z$$

$$\Delta y' = y'_s - y' = x'd\kappa' - \frac{x'\,y'}{f}\,d\varphi' + \left(f + \frac{y'^{\,2}}{f}\right)d\omega' + db'_y + \frac{y'}{f}\,db'_z$$



$$\begin{aligned} y'_s - y''_s &= y' + \Delta y' - y'' - \Delta y'' \\ 0 \quad &= q + \Delta y' - \Delta y'' \end{aligned}$$

$$0 = q + x'd\kappa' - \frac{x'y'}{f}\,d\phi' + \left(f + \frac{y'^2}{f}\right)d\omega' + db'_y + \frac{y'}{f}\,db'_z -$$
$$-x''d\kappa'' + \frac{x''y''}{f}\,d\phi'' - \left(f + \frac{y''^2}{f}\right)d\omega'' - db''_y - \frac{y''}{f}\,db''_z$$

In this equation, there are 8 orientation elements in total, but only 5 of them are independent; we can therefore choose 3 elements preferably as zero. Depending on which we choose as independent, we speak of the **relative orientation of the independent pair** or the **relative orientation when the frame is attached**.

aerial photogrammetry

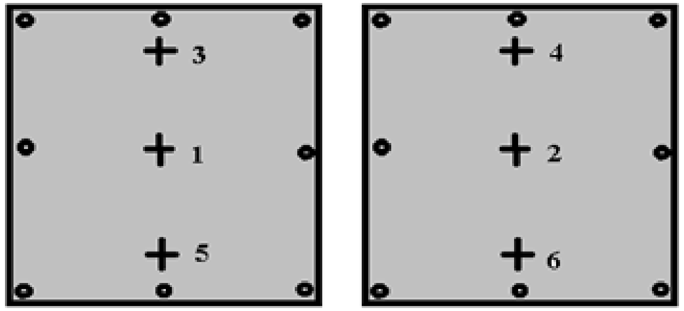
Relative orientation of the independent pair

The rotation *d, d , d, d, =d -d* is chosen as the determinate unknowns. assumption *yy*

$$0 = q + x'd\kappa' - x'd\kappa'' - \frac{x'y'}{f}d\varphi' + \frac{x'y'}{f}d\varphi'' + \left(f + \frac{y'^2}{f}\right)\Delta\omega$$

$$\left(f + \frac{y'^2}{f}\right)d\omega' - \left(f + \frac{y''^2}{f}\right)d\omega'' \cong \left(f + \frac{y'^2}{f}\right)(d\omega' - d\omega'') = \left(f + \frac{y'^2}{f}\right)\Delta\omega$$

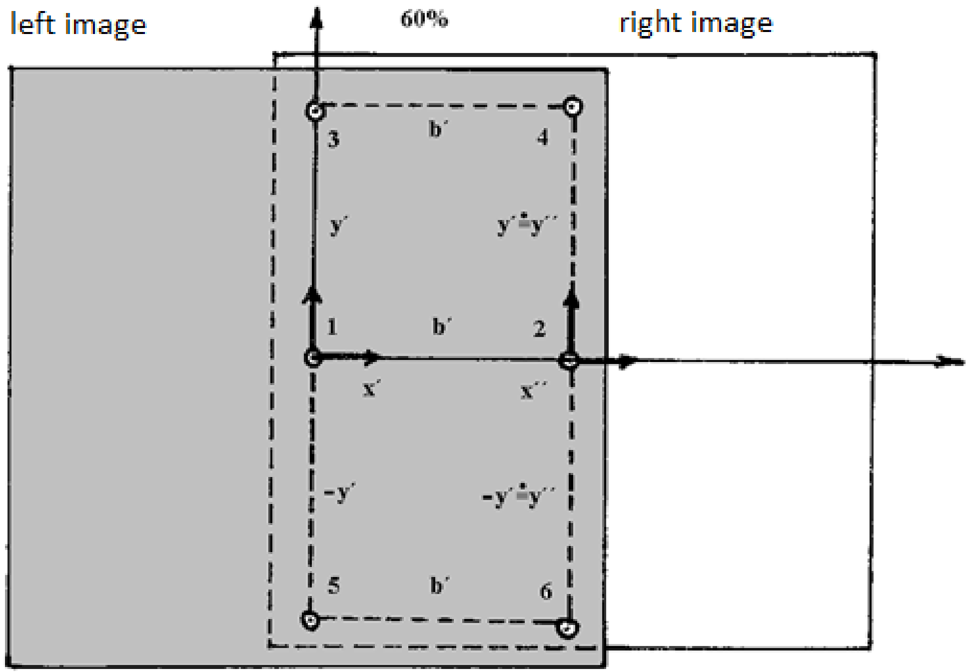
orinthal point	left picture x	left frame y	right picture x	right image y
1	0	0	-b	0
2	+b	0	0	0
3	0	+y	-b	+y
4	+b	+y	0	+y
5	0	-y	-b	-y
6	+b	-y	0	-y



left image

right image

aerial photogrammetry



$$0 = q_1 + b' d\kappa'' + f\Delta\omega$$
$$0 = q_2 + b' d\kappa' + f\Delta\omega$$
$$0 = q_3 + b' d\kappa'' - \frac{b' y'}{f} d\varphi'' + \left(f + \frac{y'^2}{f} \right) \Delta\omega$$
$$0 = q_4 + b' d\kappa' - \frac{b' y'}{f} d\varphi' + \left(f + \frac{y'^2}{f} \right) \Delta\omega$$
$$0 = q_5 + b' d\kappa'' + \frac{b' y'}{f} d\varphi'' + \left(f + \frac{y'^2}{f} \right) \Delta\omega$$
$$0 = q_6 + b' d\kappa' + \frac{b' y'}{f} d\varphi' + \left(f + \frac{y'^2}{f} \right) \Delta\omega$$

aerial photogrammetry

Relative orientation when attaching an image

The rotations d, d, db_y , db_z are chosen as the determinate unknowns.

$$0 = q - x''d\kappa'' + \frac{x''y''}{f}d\varphi'' - \left(f + \frac{y''^{\,2}}{f}\right)d\omega'' - db_y'' - \frac{y''}{f}db_z''$$

$$0 = q_1 + b'd\kappa'' - fd\omega'' - db_y''$$

$$0 = q_2 - fd\omega'' - db_y''$$

$$0 = q_3 + b'd\kappa'' - \frac{b'\,y''}{f}d\varphi'' - fd\omega'' - \frac{y''^{\,2}}{f}d\omega'' - db_y'' - \frac{y''}{f}db_z''$$

$$0 = q_4 - fd\omega'' - \frac{y''^2}{f}d\omega'' - db_y'' - \frac{y''}{f}db_z''$$

0

$$= q_5 + b'd\kappa'' + \frac{b'y''}{f}d\phi'' - fd\omega'' - \frac{y''^2}{f}d\omega'' - db_y'' + \frac{y''}{f}db_z''$$

$$0 = q_6 - fd\omega'' - \frac{y''^2}{f}d\omega'' - db_y'' + \frac{y''}{f}db_z''$$

aerial photogrammetry

Absolute orientation

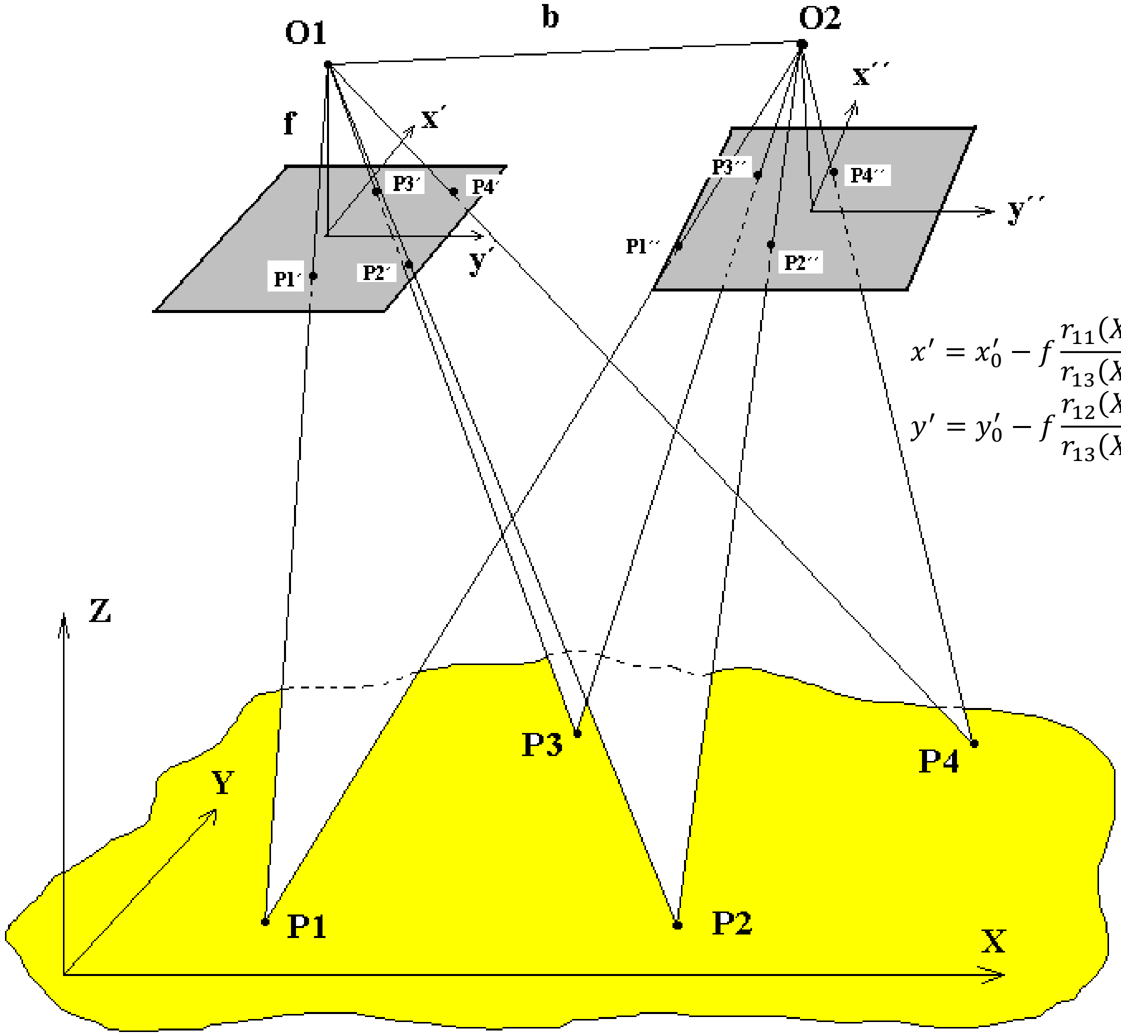
$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} + m.\mathbf{R}.\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

where X,Y,Z are geodetic coordinates, X_0 , Y_0 , Z_0 are the geodetic coordinates of the origin of the model coordinate system x,y,z , m is the scale of the model and $\boldsymbol{R}$ is the spatial rotation matrix of the model coordinate system in the geodetic coordinate system, containing the three angles ,,. The relation (14.12) is a spatial similarity transformation with unknowns X_0 , Y_0 , $Z_{0,}$ m, , .

$$M = \begin{pmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & m_z \end{pmatrix}$$

Analytical methods

***Comprehensive** solution*



$$x' = x_0' - f\frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} = x_0' - f\frac{\mathbf{R_1} \cdot \mathbf{X}}{\mathbf{R_3} \cdot \mathbf{X}}$$

$$y' = y_0' - f\frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} = y_0' - f\frac{\mathbf{R_2} \cdot \mathbf{X}}{\mathbf{R_3} \cdot \mathbf{X}}$$

Analytical methods

Comprehensive solution

$$
\begin{aligned}
x'_{i1} &= F_x(f, x'_0 = d'x, X_{01}, Y_{01}, Z_{01}, \omega_1, \phi_1, \kappa_1, X_i, Y_i, Z_i)\\
y'_{i1} &= F_y(f, y'_0 = d'y, X_{01}, Y_{01}, Z_{01}, \omega_1, \phi_1, \kappa_1, X_i, Y_i, Z_i)\\
x''_{i2} &= F_x(f, x''_0 = d'x, X_{02}, Y_{02}, Z_{02}, \omega_2, \phi_2, \kappa_2, X_i, Y_i, Z_i)\\
y''_{i2} &= F_y(f, y''_0 = d'y, X_{02}, Y_{02}, Z_{02}, \omega_2, \phi_2, \kappa_2, X_i, Y_i, Z_i)
\end{aligned}
$$

$$
f(x_1, \ldots, x_n) = f(x_1^0, \ldots, x_n^0) + \left(\frac{\partial f}{\partial x_1}\right)^0 dx_1 + \ldots + \left(\frac{\partial f}{\partial x_n}\right)^0 dx_n
$$

$$
\begin{aligned}
v_{xij} = &\left(\frac{\partial x'}{\partial X_{0j}}\right)^0 dX_{0j} + \left(\frac{\partial x'}{\partial Y_{0j}}\right)^0 dY_{0j} + \left(\frac{\partial x'}{\partial Z_{0j}}\right)^0 dZ_{0j} +\\
&+ \left(\frac{\partial x'}{\partial \omega_j}\right)^0 d\omega_j + \left(\frac{\partial x'}{\partial \varphi_j}\right)^0 d\varphi_j + \left(\frac{\partial x'}{\partial \kappa_j}\right)^0 d\kappa_j +\\
&+ \left(\frac{\partial x'}{\partial X_i}\right)^0 dX_i + \left(\frac{\partial x'}{\partial Y_i}\right)^0 dY_i + \left(\frac{\partial x'}{\partial Z_i}\right)^0 dZ_i - \left(x'_{ij} - x'^{0}_{ij}\right)
\end{aligned}
$$

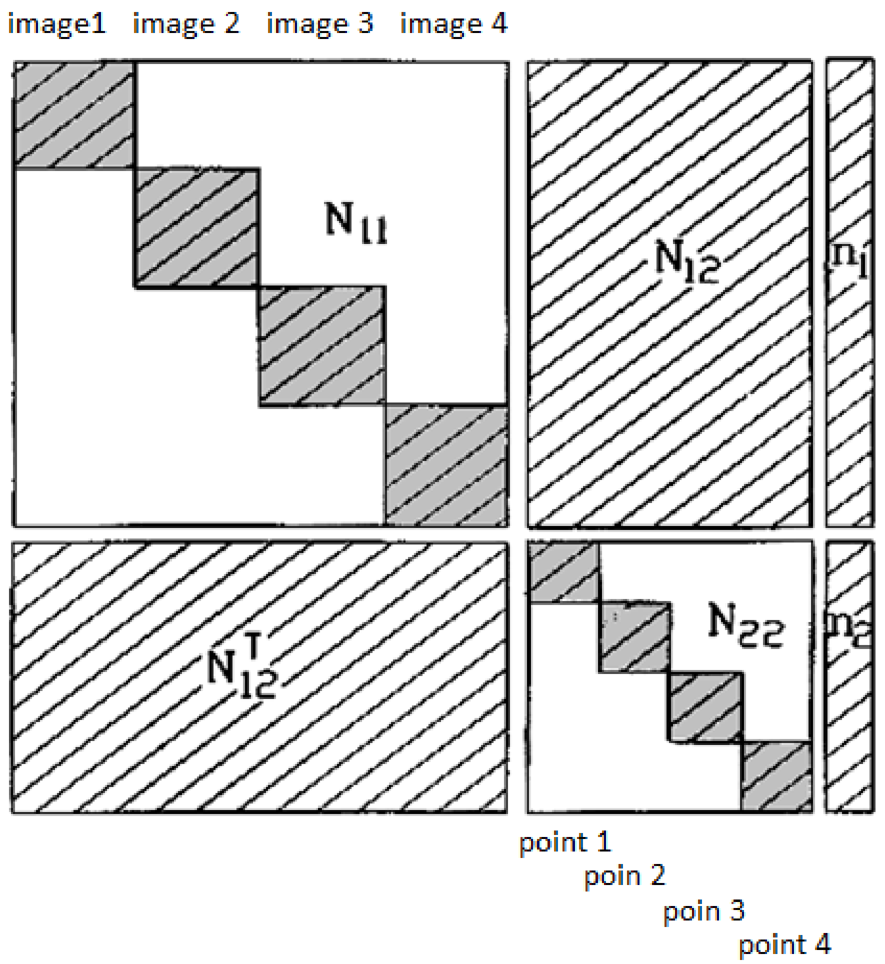
Analytical methods

Comprehensive solution

$$\mathbf{v} = \mathbf{A}_1 \cdot \mathbf{x}_1 + \mathbf{A}_2 \cdot \mathbf{x}_2 - \mathbf{l} \quad \leftrightarrow \quad (\mathbf{v} = \mathbf{A} \cdot \mathbf{x} - \mathbf{l})$$

$$\mathbf{x} = \left(\mathbf{A}^T \mathbf{A}\right)^{-1} \cdot \mathbf{A}^T \mathbf{l}$$

$$\mathbf{A}^T \mathbf{A} = \mathbf{N} \qquad \mathbf{A}^T \cdot \mathbf{l} = \mathbf{n}$$



$$\mathbf{v} = \mathbf{A} \cdot \mathbf{x} - \mathbf{l}$$

$$\Downarrow$$

$$\mathbf{N} \cdot \mathbf{x} = \mathbf{n}$$

$$\begin{pmatrix} \mathbf{N}_{11} & \mathbf{N}_{12} \\ \mathbf{N}^T{}_{12} & \mathbf{N}_{22} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{pmatrix}$$

$$\left(\mathbf{N}_{11} - \mathbf{N}_{12} \cdot \mathbf{N}_{22}{}^{-1} \cdot \mathbf{N}^T{}_{12}\right)\mathbf{x}_1 = \mathbf{n}_1 - \mathbf{N}_{12} \cdot \mathbf{N}_{22}{}^{-1} \cdot \mathbf{n}_2$$

Analytical methods

Image triangulation

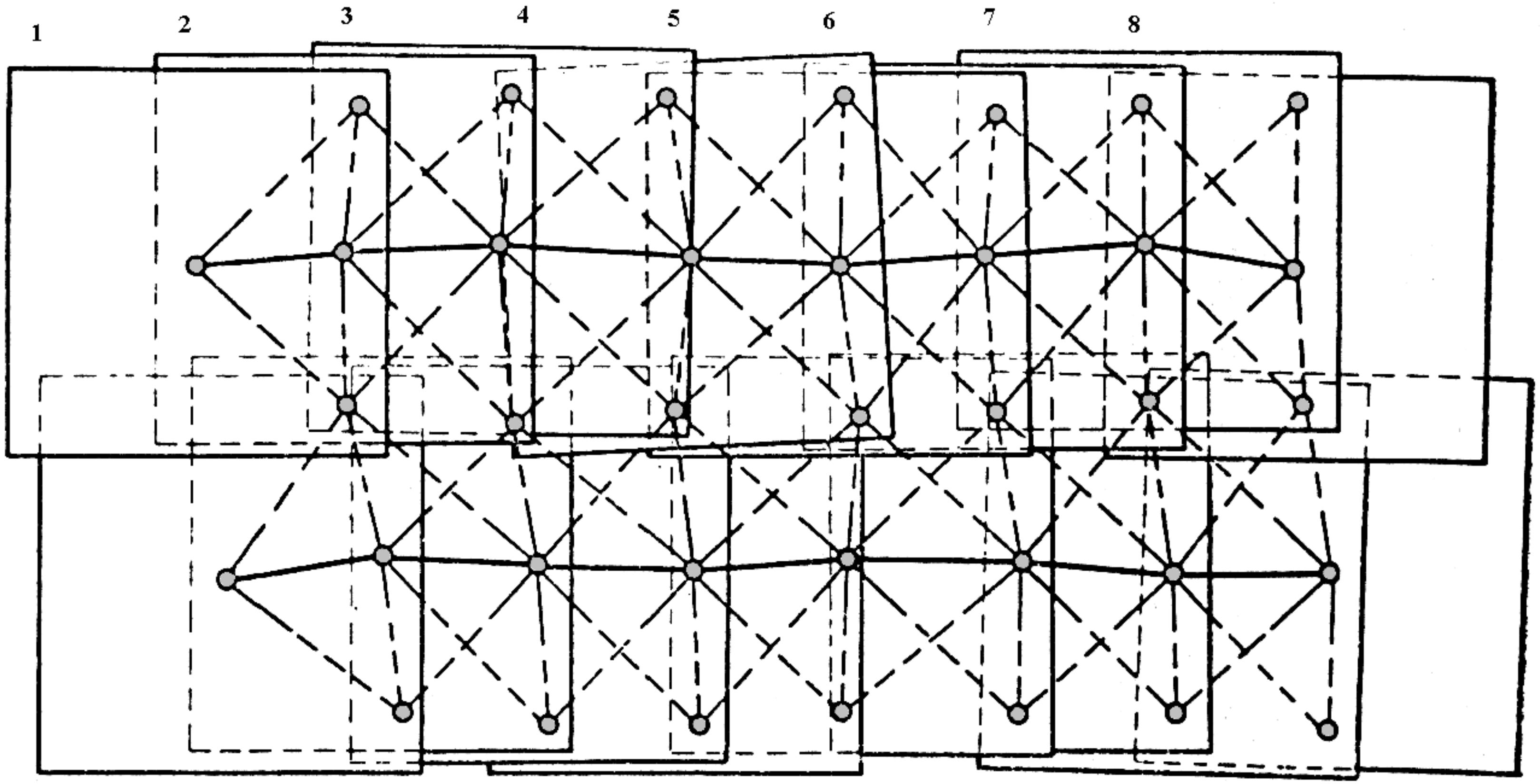
The goal of today's analytical aerotriangulation is:

- ***obtaining germination points for detailed evaluation***
- ***Alignment of a set of images or models to ensure continuous evaluation***
- ***accurate calculation of exterior orientation elements for all images***

Historically, there are radial, analoque,semi-analyticaland analytical technology of aerotriangulation

*•Today only automated (digital) aerotriangulation is used(**AAT**)*

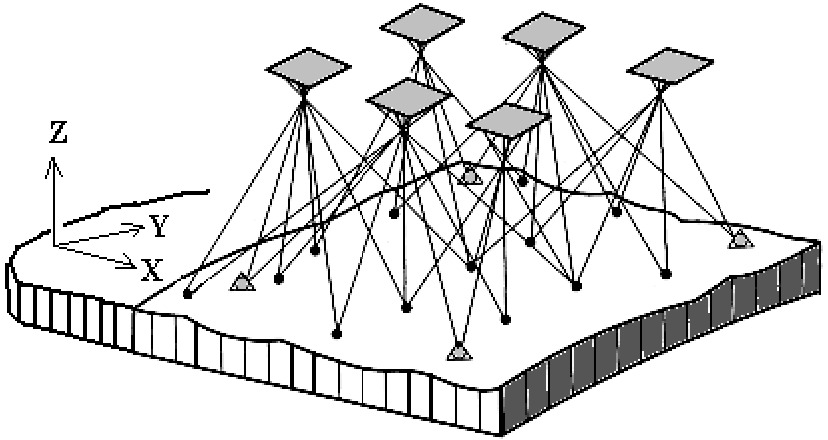
Image triangulation



Full triangular radial mesh and normal image block in radial triangulation

Image triangulation

Bundle block adjustment (complex solution)



integrated INS

INS (*Inertial Navigation System*) or GNSS/IMU (*Inertial Measurement Unit*).

digital photogrammetry

Theory of image correlation

$$\rho\,(A,B)=\frac{\operatorname{cov}(A,B)}{\rho(A)\cdot\rho(B)}$$

If we want to calculate the correlation coefficient for two equally sized digital images (or their cuts),
we will use to calculate the pixel value $p(A)_{i,j}$ for image **A** and $p(B)_{i,j}$ for image **B**. We obtain the expression:

$$r(A,B)=\frac{C(A,B)}{\sqrt{C(A)\cdot C(B)}}$$

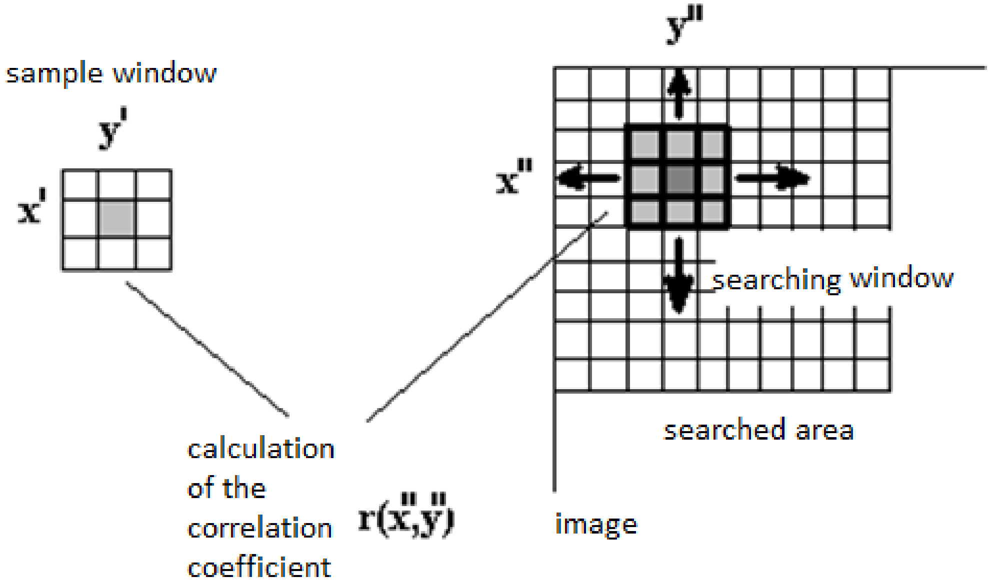
where the individual expressions are (*n* is the number of pixels in the side of the square window)

$$C(A,B)=\frac{1}{n^2-1}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p(A)_{i,j}-\overline{p}(A)\right)\cdot\left(p(B)_{i,j}-\overline{p}(B)\right)$$

$$C(A)=\frac{1}{n^2-1}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p(A)_{i,j}-\overline{p}(A)\right)^2,\quad \overline{p}(A)=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p(A)_{i,j}\right)$$

$$C(B)=\frac{1}{n^2-1}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p(B)_{i,j}-\overline{p}(B)\right)^2,\quad \overline{p}(B)=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p(B)_{i,j}\right)$$

digital photogrammetry



We are looking for its maximum with subpixel precision. Since it reaches its maximum only in a limited region, we can replace the discrete correlation function by a continuous function and describe it by, for example, a second degree polynomial:

$$r = \bar{r} + v = a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2$$

where x_i , y_i is the position of the search window for the calculated value of the correlation coefficient $r_{i\,;}$ if we have a matrix of 3x3 correlation coefficients, we get 9 values. We need a total of 6 to calculate the coefficients and$_i$ and

digital photogrammetry

Subpixel transformation

$$p_B(x) = p_A(x + a_1)$$

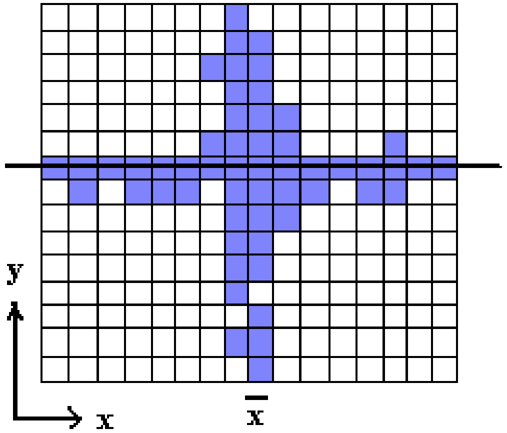
$$p_B(y) = p_A(y + a_2)$$

$$v_x + p_B(x) = p_A(x + a_1)a_3 + a_5$$

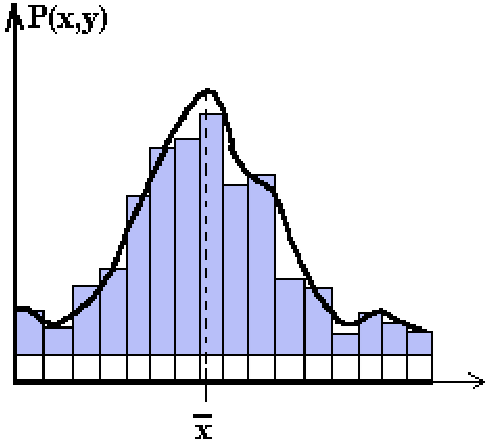
$$v_y + p_B(y) = p_A(y + a_2)a_4 + a_6$$

$$v_x + p_B(x) = p_A \cdot a_3(x) + p'_A \cdot a_3 a_1(x) + a_5$$

$$v_y + p_B(y) = p_A \cdot a_4(y) + p'_A \cdot a_4 a_2(y) + a_6$$



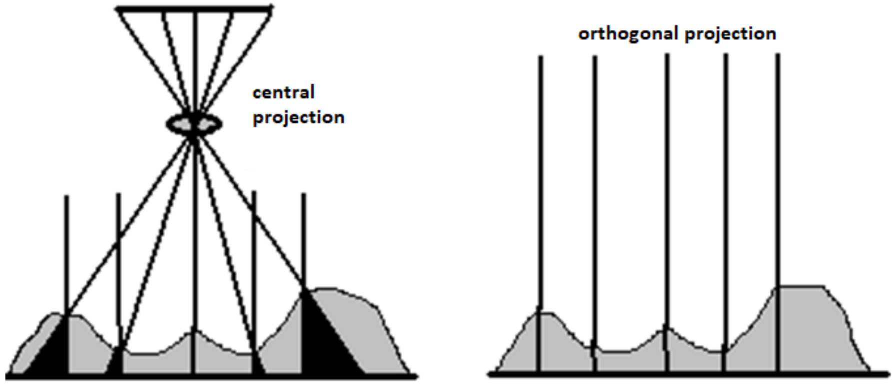
a detail of displayed cross
(for example fiducial mark)



its probable centre

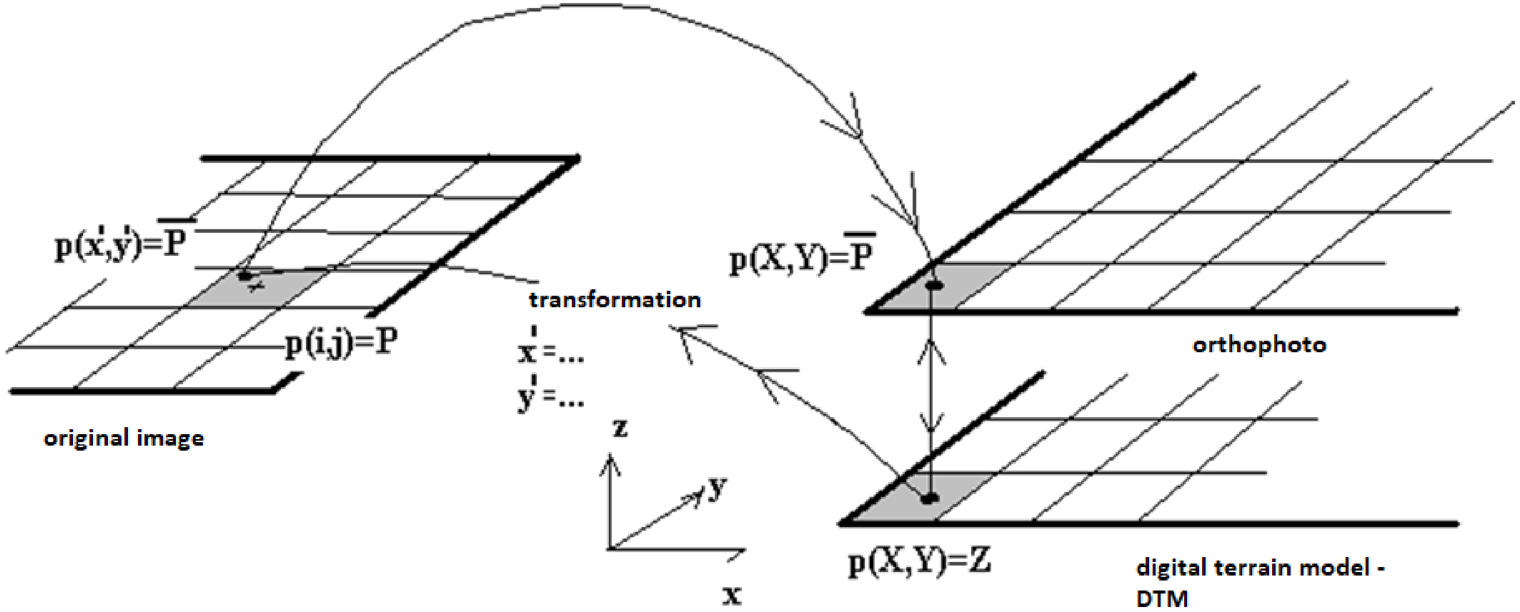
digital photogrammetry

Digital orthophoto



$$x' = x'_0 - f\frac{r_{11}(X-X_0)+r_{21}(Y-Y_0)+r_{31}(Z-Z_0)}{r_{13}(X-X_0)+r_{23}(Y-Y_0)+r_{33}(Z-Z_0)}$$

$$y' = y'_0 - f\frac{r_{12}(X-X_0)+r_{22}(Y-Y_0)+r_{32}(Z-Z_0)}{r_{13}(X-X_0)+r_{23}(Y-Y_0)+r_{33}(Z-Z_0)}$$

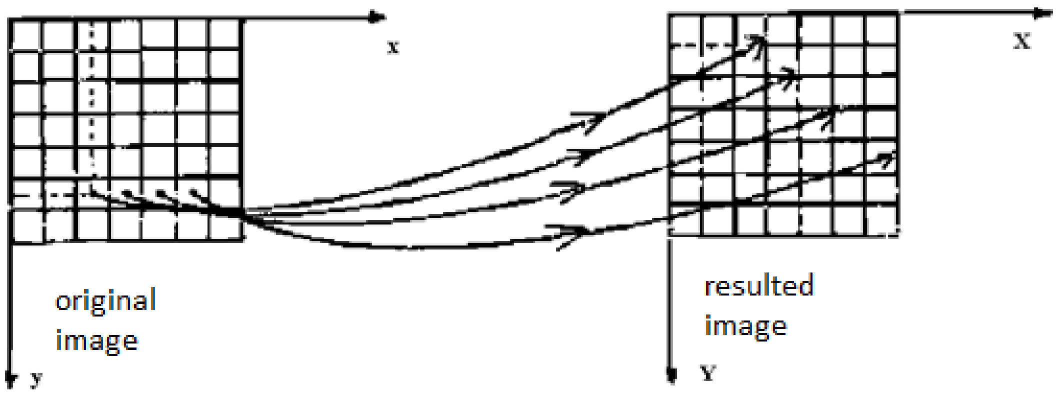


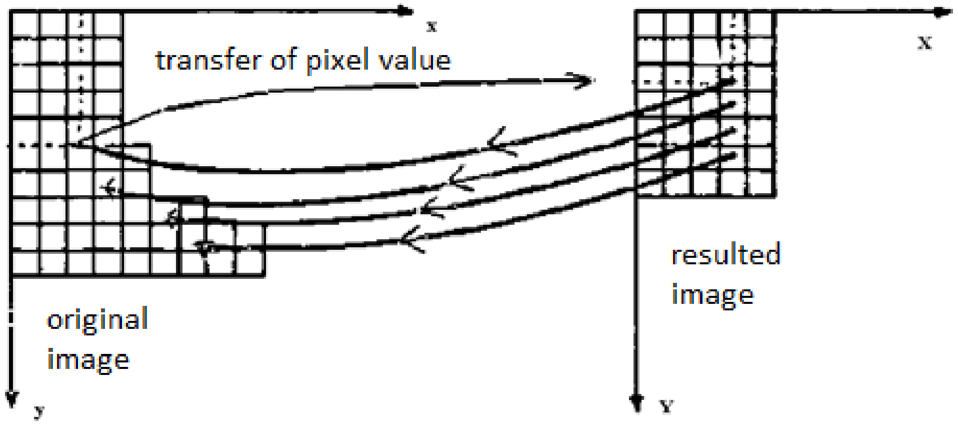
Geometric transformation

The aim of geometric transformation is either to remove image distortion caused by the instability of geometric conditions during measurement and to convert the data into a suitable projection (especially in DPZ) or to create a new image (e.g. orthophoto) based on transformation relations. Geometric correction can be performed in three different ways:

- ***data transformation based on precisely known carrier trajectory parameters***
- ***direct geometric transformation based on embedding points or vectors***

- direct geometric transformation based on embedding points or vectors





- data transfer from the original image matrix to the corrected matrix

Nearest Neighbour Method

- ***Bilinear transformation***
- ***Bicubic Convolution***

End